

PCPs and Instance Compression from a Cryptographic Lens

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Abstract

Modern cryptography fundamentally relies on the assumption that the adversary trying to break the scheme is computationally bounded. This assumption lets us construct cryptographic protocols and primitives that are known to be impossible otherwise. In this work we explore the effect of bounding the adversary’s power in other information theoretic proof-systems and show how to use this assumption to bypass impossibility results.

We first consider the question of constructing *succinct* PCPs. These are PCPs whose length is polynomial only in the length of the original NP witness (in contrast to standard PCPs whose length is proportional to the non-deterministic verification time). Unfortunately, succinct PCPs are known to be impossible to construct under standard complexity assumptions. Assuming the sub-exponential hardness of the learning with errors (LWE) problem, we construct *succinct probabilistically checkable arguments* or PCAs (Kalai and Raz 2009), which are PCPs in which soundness is guaranteed against efficiently generated false proofs. Our PCA construction is for every NP relation that can be verified by a small-depth circuit (e.g., SAT, clique, TSP, etc.) and in contrast to prior work is *publicly verifiable* and has *constant* query complexity. Curiously, we also show, as a proof-of-concept, that such publicly-verifiable PCAs can be used to derive *hardness of approximation* results.

Second, we consider the notion of *Instance Compression* (Harnik and Naor, 2006). An instance compression scheme lets one compress, for example, a CNF formula φ on m variables and $n \gg m$ clauses to a new formula φ' with only $\text{poly}(m)$ clauses, so that φ is satisfiable if and only if φ' is satisfiable. Instance compression has been shown to be closely related to succinct PCPs and is similarly highly unlikely to exist. We introduce a *computational analog* of instance compression in which we require that if φ is unsatisfiable then φ' is *effectively* unsatisfiable, in the sense that it is computationally infeasible to find a satisfying assignment for φ' (although such an assignment may exist). Assuming the same sub-exponential LWE assumption, we construct such computational instance compression schemes for every bounded-depth NP relation. As an application, this lets one compress k formulas ϕ_1, \dots, ϕ_k into a single short formula ϕ that is *effectively* satisfiable if and only if at least one of the original formulas was satisfiable.

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1 Introduction

The question of *instance compression*, introduced by Harnik and Naor [27], asks whether it is possible to compress long instances of an NP relation down to their witness length, which can be significantly shorter, while preserving the “correctness” of the statement. In more detail, an instance compression scheme for an NP relation \mathcal{R} is a polytime computable function f , such that for every instance x , of length n , with corresponding witness length $m \ll n$, it holds that (1) the compressed instance $f(x)$ has length $|f(x)| = \text{poly}(m)$, and (2) x is a true statement (i.e., exists w such that $(x, w) \in \mathcal{R}$) if and only if $f(x)$ is a true statement. The definition can be extended to allow for f to map instances of \mathcal{R} to a different NP relation \mathcal{R}' and to allow for a poly-logarithmic dependence on the instance length n .

For example, consider the relation SAT of all satisfiable Boolean formulas (in conjunctive normal form). An instance compression scheme for SAT can compress a long formula φ over m variables with $n \gg m$ clauses to a new formula φ' with only $\text{poly}(m)$ clauses, such that φ is satisfiable if and only if φ' is satisfiable.¹

Harnik and Naor as well as followup works [22, 18, 9, 5], have shown that instance compression scheme can be quite useful. Some of their diverse applications include natural ones such as efficiently storing instances in order to solve them later (see [27]), or as a preprocessing step for solving algorithmic problems (see [9]), as well as more surprising applications such as establishing the existence of a variety of fundamental cryptographic primitives (see [27, 5] for details).

Unfortunately, Fortnow and Santhanam [22] showed that, under standard complexity assumptions (namely, that the polynomial hierarchy does not collapse), instance compression cannot exist for some NP languages. More specifically, they showed that the existence of instance compression for SAT implies that $\text{NP} \subseteq \text{coNP}/\text{poly}$ (and therefore the polynomial hierarchy collapses).

To establish their result, Fortnow and Santhanam showed that instance compression is closely related (and, in a sense, *equivalent*) to the notion of *succinct probabilistically checkable proofs* (succinct PCPs) [34, 22]. Recall that a PCP is a special format for writing proofs that can be verified by only reading a few of the bits from the proof. The PCP theorem [4] shows that every NP language has a constant-query PCP whose length is polynomial in the non-deterministic verification time of the language.² In contrast, a PCP is *succinct* if its length is polynomial only in the *witness length*. For example, the PCP theorem guarantees that SAT on m variables and n clauses has a PCP of length $\text{poly}(n, m)$, whereas a *succinct* PCP would have length roughly $\text{poly}(m)$.

Fortnow and Santhanam showed that the existence of a succinct PCP for SAT implies that SAT also has an instance compression scheme. In particular, this implies that SAT cannot have a succinct PCP, unless the polynomial hierarchy collapses.

This leaves us at the unfortunate state of affairs that (under a widely believed assumption) these two natural objects (i.e., instance compression schemes and succinct PCPs) do not exist. In this work we show how to bypass these negative results by taking a cryptographic perspective and considering *computational analogs* of PCPs and instance compression. We further show that some of the applications of instance compression and succinct PCPs remain valid also for these relaxed notions.

¹ We denote the number of variables by m and the number of clauses by n (rather than vice-versa, as typically done) to better align with the notation for general NP relations, in which the instance length is denoted by n and the witness length by m .

² The dependence of the length of the PCP on the non-deterministic verification time has since been improved to *quasi-linear* [8, 17]

1.1 Our Results

To get around the barriers posed by [22], we consider *probabilistically checkable arguments* (PCAs), which are similar to PCPs except that soundness is only guaranteed against proof-strings that are *efficiently generated*. The notion of PCAs was first considered by Zimand [49] in the context of constructing a lightweight version of the PCP theorem and by Kalai and Raz [35] with the exact same motivation as in our work: constructing PCPs whose length is proportional to the witness size.

We also introduce *computational instance compression* (CIC), a relaxation of instance compression in which for false statements x it may be the case that the compressed statement $f(x)$ is true, but it should be *computationally infeasible* to find a witness for $f(x)$.

We first discuss our results for PCAs (in Section 1.1.1) and then the results for computational instance compression (in Section 1.1.2).

1.1.1 Probabilistically Checkable Arguments

As noted above, PCAs are a natural computational analog of PCPs. Similarly to a PCP verifier, a PCA verifier is a probabilistic machine that can only read a few bits from the proof, and is required to accept correct proofs (with high probability). However, while a PCP verifier is guaranteed to reject *any* proof for a false statement (with high probability), a PCA verifier is only required to reject false proofs generated by *computationally bounded* malicious provers. In other words, there can exist accepting proofs for false statements, but it should be *intractable* to find them. Since an accepting proof can be non-uniformly hardwired into the adversary, PCAs are defined in the *common reference string* model. In this model, a common reference string (CRS) is sampled before the beginning of the protocol, and is available both to the PCA prover and the verifier. In analogy to PCPs, we say that a PCA is succinct if the proof length is proportional to the witness length.³

Kalai and Raz [35] constructed succinct PCAs for a large class of NP relations. However, their construction suffers from two significant caveats. First, their PCA is only *privately-verifiable*. That is, the PCA verifier is given a secret trapdoor to the CRS which can be used for verification. We emphasize that for soundness to hold in this model it is imperative that the prover does not know the trapdoor. The fact that the PCA is only privately-verifiable limits its applicability. In particular, only the *designated verifier* that has the trapdoor can verify the proof. In contrast, we consider *publicly-verifiable* PCAs. In other words, the verifier does *not* get a trapdoor to the CRS, and *anyone* can verify the PCA proof, given the corresponding CRS and by making a few queries (see Definition 6 for the formal definition). As shall be discussed further below, public-verifiability is crucial for some of the applications of PCAs.

The second major caveat of the [35] construction is that the query complexity, while small, is super-constant. More specifically, while typical PCP verifiers only need to make a constant number of queries in order to obtain constant soundness error, the PCA verifier of [35] needs to make $\text{poly}(\log n, \lambda)$ queries, where n is the instance length and λ is the computational security parameter.

³ It is well known that PCPs, or any other interactive proof for that matter, cannot be *shorter* than the witness length [24, 25]. In contrast, under extremely strong cryptographic assumptions we *can* construct PCAs (with non-trivial query complexity) that are *shorter* than the witness length. We refer to such PCAs (i.e., whose proof length is shorter than the witness) as *super-succinct*. Since our focus is on standard cryptographic assumptions, we leave the study of super-succinct PCAs for future work. See further discussion at the end of the current subsection.

In this work we resolve both of these caveats. Assuming the sub-exponential hardness of the learning with errors problem (LWE), we construct a *publicly-verifiable* PCA for a rich subclass of NP relations, with *constant* query complexity.

► **Theorem 1** (Informally stated, see Theorem 8). *Assume the sub-exponential hardness of LWE. Then, for every NP relation \mathcal{R} , for which membership can be decided in logspace uniform NC, there exists a succinct constant-query publicly-verifiable PCA with constant soundness error.*

We remark that the subclass of NP relations for which we construct PCAs is identical to the subclass considered by Kalai and Raz [35]. It should be noted that this class seems quite rich and in particular contains many well-known NP relations, such as SAT, k-Clique, 3Col, etc.⁴

Our proof of Theorem 1 follows the approach of [35] while replacing a privately-verifiable non-interactive argument that they use (based on [34, 26]) with a new *publicly-verifiable* non-interactive argument of Jawale et al. [30]. Actually making this approach work, while obtaining constant query complexity, turns out to be non-trivial. See Section 1.2 for details.

PCAs and Hardness of Approximation. As an additional contribution, we also show that publicly-verifiable PCAs can be used to obtain *hardness of approximation results*. We find this surprising since, typically, the NP-hardness of approximation problems is *equivalent* to the existence of PCPs.⁵

While we only give a proof-of-concept construction, we believe that this aspect of PCAs could have important implications for hardness of approximation. We note that results establishing hardness of approximation based on cryptography (or more generally, average-case hardness assumptions) have been demonstrated in the past. For example, lattice based cryptography inherently relies on hardness of approximation (of geometric properties of integer lattices), whereas “local cryptography” is tightly related to approximating CSPs (see in particular the recent work of Applebaum [2] and references therein). Nevertheless, we find the approach for using PCAs for hardness of approximation more generic and hope that it may allow to bypass known limitations of PCPs. See Section 1.3 for details.

Beyond the Witness Length. It is natural to ask if the witness length is indeed a barrier for PCAs. In particular, a so-called “SNARG for NP” (see Section 1.4) achieves length that is *shorter* than the witness, albeit with relatively poor query complexity, as the SNARG verifier has to read its entire proof. We believe that our techniques can possibly also be useful in the context of converting SNARGs to be highly-efficient PCAs but leave the study of such “super-succinct” PCAs to future work.

1.1.2 Computational Instance Compression

In order to bypass the impossibility result of Fortnow and Santhanam for instance compression, we introduce a new computational variant, which we call *computational instance compression* (CIC). In a nutshell, a CIC scheme also compresses the instance down to the witness length, but we require that if the instance x is false, then it should be *computationally infeasible*

⁴ The Cook-Levin theorem implies that every NP language has an NP relation that can be verified by a bounded depth circuit (in fact, a CNF). However, this transformation blows up the witness size. Here we refer to the *natural* NP relations for these well-known problems.

⁵ For example, the NP completeness of, e.g., GapSAT, is equivalent to the PCP theorem.

(but not necessarily impossible) to find a witness for the compressed instance $f(x)$. It may be useful to contrast CICs with cryptographic hashing. Such hash functions can similarly be used to compress an instance x but the result is a “cryptographic string” that mainly serves as a commitment that can later be opened by someone who knows x . In contrast the result of compression by a CIC is a new instance x' which can be used independently of the original input x , while preserving its *effective correctness*.

In more detail, let \mathcal{R} and \mathcal{R}' be NP-relations, where the instance length of \mathcal{R} is denoted by n and the witness length by m . A *computational instance compression* (CIC) scheme from \mathcal{R} to \mathcal{R}' is an efficiently computable function f such that $|f(x)| = \text{poly}(m, \log n)$ with the following requirements:

1. If x is true then $x' = f(x)$ is true.
2. If x is false, then it is infeasible for a polynomial-size adversary, given $x' = f(x)$, to find a witness w' such that $(x', w') \in \mathcal{R}'$.
3. In contrast, there is a polynomial-time algorithm that given a true statement x , and a corresponding witness w (i.e., $(x, w) \in \mathcal{R}$), finds a witness w' for $x' = f(x)$.

(Our actual definition of CICs is in the common reference string model - that is, all algorithms are further given a common reference string. For simplicity we ignore this aspect here.)

The notion of CIC is indeed suitable for the main application of instance compression: efficient storage of long instances: The compressed output x' is a short instance of \mathcal{R}' which preserves the *effective correctness* of the original input x .

At first glance, it may not be entirely clear why we insist that it is possible to efficiently find a witness for the compressed instance given a witness for the original witness (i.e., requirement 3 above). However, this requirement is indeed crucial for many applications. For example, for efficient storage, if one has a witness for the original instance then the third requirement lets one preserve this witness for the compressed instance. Moreover, if we omit this requirement then there is a trivial construction that satisfies the first two requirements (in a non-interesting way): to compress an instance x , simply ignore x , and output some x' which is a hard instance of \mathcal{L}' (i.e., an instance which is computationally indistinguishable from $x'' \notin \mathcal{L}'$). Such an instance x' is true but a satisfying witness cannot be efficiently found.

In this paper we construct CICs for the same subclass of NP relations as in Theorem 1 and under the same assumption - the sub-exponential hardness of LWE.

► **Theorem 2** (Informally stated, see Theorem 10). *Assume the sub-exponential hardness of LWE. Then, for every NP relation \mathcal{R} , for which membership can be decided in logspace uniform NC, there exists a CIC scheme from \mathcal{R} to SAT.*

As explained above, the class of source NP-relations, from which we can compress, includes many natural relations. As for the target relation, the choice of SAT is somewhat arbitrary (but convenient). In particular, we can replace SAT with any NP relation \mathcal{R}' for which (x, w) , an instance-witness pair for SAT can be efficiently translated to an instance-witness pair (x', w') for \mathcal{R}' , and vice-versa. This includes most natural NP complete problems that we are aware of.

An Application: OR-SAT. Consider the following problem: we are given as input k different formulas $\varphi_1, \dots, \varphi_k$ and want to generate a new formula φ' such that φ' is satisfiable if and only if at least one of the φ_i 's is satisfiable.

This problem, referred to by Harnik and Naor [27] as OR-compression, can be viewed as a special case of instance compression where the source relation $\text{OR-SAT} = \{((\varphi_1, \dots, \varphi_k), w) : \exists i \in [k], (\varphi_i, w) \in \text{SAT}\}$ consists of k formulas out of which at least one is satisfiable, and the

target relation is simply SAT. In their aforementioned work, Fortnow and Santhanam [22] also rule out the existence of OR-instance compression for SAT, assuming the polynomial hierarchy does not collapse.

We bypass this impossibility result and obtain a *computational* OR instance compression scheme. In such a scheme the compressed formula is *effectively* sound if and only if at least one of the original formulas was sound. Indeed, leveraging the fact that OR-SAT has witnesses of length that is independent of k (namely a witness that satisfies at least one of the k formulas), the following corollary is immediate from Theorem 2.

► **Corollary 3.** *Assume the sub-exponential hardness of LWE. Then, there exists a CIC scheme from OR-SAT to SAT.*

We find Corollary 3 particularly appealing. It gives a method to reduce k formulas to a single formula while preserving the *effective* satisfiability.

On AND-compression. It is natural to ask whether a similar statement to Corollary 3 holds when replacing OR with AND - namely, the resulting formula should be satisfiable if and only if *all* of the source formulas are satisfiable. This is called AND-instance compression, and has been shown not to exist in the information theoretic setting, assuming the polynomial hierarchy does not collapse [18, 16].

Unfortunately, since the witnesses in the case of AND do grow linearly with k , Theorem 1 does not seem to imply anything meaningful in this case. We leave the study of AND-computational instance compression for future work (this question seems closely related to that of *non-interactive batch verification*, see Section 1.4).

1.2 Technical Overview

The main technical contribution of this work is a construction of a publicly-verifiable constant-query succinct PCA. We give an overview of this construction in Section 1.2.1. Then, in Section 1.2.2, building on techniques from [22], we show how to use our PCA construction to build CICs.

1.2.1 Publicly-Verifiable PCAs from Publicly-Verifiable PSNARGs

We construct our publicly-verifiable PCAs by using a recent publicly-verifiable Succinct Non-interactive ARGument (SNARG) constructed by Jawale et al. [30]. A SNARG, as its name suggests, is a non-interactive argument-system (in the common reference string model) in which the verifier runs in time that is sublinear in the computation.⁶

Usually, SNARGs are considered in the context of NP languages, and the succinctness requirement implies, in particular, that the proof string be shorter than the length of the NP witness. In contrast, we will focus on non-interactive arguments for languages in \mathcal{P} . To avoid confusion we refer to such arguments as PSNARGs. Note that since we insist on sublinear verification, it is (highly) non-trivial to construct PSNARGs (even for problems in \mathcal{P}).

Moreover, in the following we will sometimes refer to PSNARGs for an NP relation \mathcal{R} . What we mean by this is that we view the membership problem in \mathcal{R} (i.e., given an instance *and* a witness - check if they satisfy the relation) as a problem in \mathcal{P} . Therefore, the key

⁶ Actually somewhat different definitions of the notion exist in the literature. Some works require only short communication but allow a long verification time, and other works insist on strictly poly-logarithmic verification time.

difference between a SNARG or a PSNARG for an NP relation \mathcal{R} is that in the latter the verifier gets access to the instance *and* the witness, whereas in the former it gets only the instance.

The starting point for our construction is a recent publicly-verifiable PSNARG for logspace-uniform NC due to Jawale et al. [30], based on a secure application of Fiat-Shamir transform [20] to the [26] doubly-efficient interactive proof-system. This result relies on the sub-exponential hardness of LWE (via [12, 42]).

With this result in hand, consider an NP relation \mathcal{R} for which membership can be decided in logspace-uniform NC. Given an instance x of \mathcal{R} , consider a proof string that consists of the witness w for x appended with a PSNARG proof π showing that $(x, w) \in \mathcal{R}$.

The proof string (w, π) is very short but it is still not locally checkable. Thus, a natural idea is for the prover to further append a standard PCP proof π' attesting that (w, π) are such that π is an accepting PSNARG proof for $(x, w) \in \mathcal{R}$. Intuitively we seem to have made progress since the PCP proof length corresponds to the verification time, which is short due to the efficient PSNARG verification.

Before proceeding, a small correction is in order. Since a PCP verifier needs to read its entire main input (which in our case is all of w and π) what we will actually use is a PCP of *proximity* (PCPP). In the current context it suffices to think of a PCP of proximity as a PCP in which the verifier only needs to read a few bits from the input (as well as the proof), as long as the input is encoded under an error-correcting code.

Thus, as our first attempt, consider a PCA proof-string that consists of $(E(w, \pi), \pi')$, where E is a suitable error-correcting code (i.e., with polynomial or even linear block length). The PCA verifier emulates the PCPP verifier while using $(E(x), E(w, \pi))$ as the input oracle and π' as the proof oracle (note that the verifier can generate $E(x)$ by itself).

While natural, this construction runs into several problems that we have to resolve and are discussed next.

Adaptivity. The first problem we encounter with the construction is that of adaptivity. Namely, a malicious PCA prover can choose a witness w^* , which serves as part of the input for the PSNARG, *after* seeing the sampled CRS. In particular, the PSNARG of [30], is non-adaptive: soundness only holds if the input is fixed *before* the CRS is given to the prover.

Thankfully, there is a relatively standard and simple solution to this problem via complexity leveraging. Namely, we use complexity leveraging to transform sub-exponentially secure PSNARGs to ones which are adaptively sound in the choice of the witness. In a nutshell this is done by increasing the security parameter as to ensure that the soundness error of the PSNARG is so small that we can afford to take a union bound over all possible witnesses. Crucially, the overhead incurred by this transformation is a fixed polynomial only in the witness length m (rather than the instance size n).

Having resolved the adaptivity issue, we turn to a more pressing problem that arises when examining the PCA proof length more closely.

The PCA Proof Length. The proof-string consists of (an encoding of) w which has length m , and π which has length $\text{poly}(\log(n), \lambda)$, where λ is the security parameter. In addition, the PCA contains π' , a PCPP proof that the PSNARG verifier would accept. While the PSNARG verifier runs in time that is sublinear in the verification time of \mathcal{R} , at very least the verifier needs to read its input (x, w) which has length $n + m$. This means that the length of the PCPP proof string π' is at least $\text{poly}(n)$, whereas we were aiming for a polynomial dependence only on m .

Holographic Proofs to the Rescue? As a first idea one might try to get around this by leveraging the fact that the [30] verifier is *holographic*. A PSNARG is said to be *holographic* if the verifier can run in time that it is *sublinear* in the input, as long as it is given oracle access to an *encoding* of the input. The verifier in the protocol [30] is indeed holographic, with respect to the low degree extension code.⁷ Indeed, given oracle access to the low degree extension of the input, the PSNARG verifier of [30] runs in time $\text{polylog}(n)$.

One would hope that this observation suffices to reduce the PCPP length to $\text{poly}(m, \log(n))$. Unfortunately, this is not the case. The difficulty that we encounter now is that PCPPs are not designed for computations involving an oracle. In particular, it is known that the PCP theorem does not relativize [3, 21] and it is unclear how to construct a PCPP in which the input is given as an oracle (without materializing the input).

Sublinear Verification via Crypto. To cope with this difficulty we take, yet again, a cryptographic approach. In particular, we propose a simple PSNARG construction in which the PSNARG verifier truly runs in sublinear time, after a suitable pre-processing step. Our construction is inspired by, and closely related to, ideas arising in the context of *memory delegation* [15, 31].

The sublinear PSNARG is obtained by having the verifier first, as a pre-processing step, generate the low degree extension of its input and hash the low degree extension using a Merkle tree. At this point all that the verifier needs to remember is the root of the hash tree. In the online phase, the prover provides a standard PSNARG proof, but in addition also provides authentication paths for all of the verifier’s queries to the input. This results in a pre-processing sublinear PSNARG construction.

We note that the fact that this solution works relies on two critical facts. First, the fact that the PSNARG prover knows, already when sending the proof, which locations of encoding of the input the holographic verifier is going to query. Indeed, this follows from the fact that the PSNARG is publicly-verifiable (and we could not apply similar reasoning, e.g., to the [35] construction). Second, we need that it be possible for the verifier to fully materialize the low degree extension of the input in $\text{poly}(n)$ time. This is actually non-trivial since the low degree extension has length that is polynomial in the input only for *some* parameter settings. In particular, using the parameter setting of [30] results in *sub-exponential* length (due to their use of extremely large finite fields). To solve this we rely on the followup work of Holmgren et al. [29] who extend the [30] protocol to work also for small fields and in particular, when the block length of the low degree extension is polynomial.

Using this observation, we can reduce the PSNARG verification time, and therefore the PCPP length to be sublinear. Having done so, the entire PCA proof length becomes polynomial only in the witness length, as desired.

Relation to Fiat-Shamir of Kilian’s Protocol. Given that the [30, 29] protocols rely on a secure application of the Fiat-Shamir transform, and we are applying this transform based on a Merkle hash of the instance, our construction bears some resemblance to a secure application of Fiat-Shamir to Kilian’s [38] celebrated protocol. Since the question of securely applying Fiat-Shamir to Kilian’s protocol (as envisioned by Micali [40]) in the standard-model

⁷ Recall that [30] is constructed by applying the Fiat-Shamir transform to the [26] protocol, which is known to be holographic. The holographic property is preserved by the Fiat-Shamir transform because, since [30] are only aiming for non-adaptive soundness, they do not include the instance as an input to the Fiat-Shamir hash function.

is wide open (with mainly negative evidence thusfar [6]) this may seem surprising. In a nutshell the key difference that we leverage is the fact that the verifier has a *trusted* Merkle hash of the instance. We refer the reader to Appendix 1 of the full version [11] for a more detailed discussion.

1.2.2 CICs from Succinct Publicly-Verifiable PCAs

Fortnow and Santhanam [22] show how to convert a succinct PCP into an instance compression scheme for SAT. We follow their approach in order to convert our succinct PCA construction into a CIC for SAT.

Let \mathcal{V} be a PCA verifier for SAT, and φ be an instance (i.e., formula) of SAT. Fixing φ , the verifier \mathcal{V} tosses random coins, reads $q = O(1)$ bits from its proof π and runs some predicate over the answers that it read in order to decide whether to accept or reject.

The compressed formula φ' is generated in the following manner: The variables for φ' correspond to the bits of the PCA string π . To construct the clauses of φ' , we enumerate over all possible random strings of \mathcal{V} , and for each such random string ρ , we consider the residual decision predicate $\mathcal{V}_{\varphi,\rho}$ of \mathcal{V} when both the random string and input formula φ are fixed. Assuming that the verifier makes non-adaptive queries (this is indeed the case for our PCA construction and is also true without loss of generality whenever the query complexity is constant), each random string is associated with a sequence of q indices from the proof and the predicate $\mathcal{V}_{\varphi,\rho} : \{0, 1\}^q \rightarrow \{0, 1\}$.

Note that each $\mathcal{V}_{\varphi,\rho}$ can be represented as a CNF on 2^q clauses. We let our compressed formula $\varphi' = \bigwedge_{\rho} \mathcal{V}_{\varphi,\rho}$ simply be the conjunction of all of these clauses. The length of φ' is exponential in the randomness complexity of \mathcal{V} . Since our \mathcal{V} has $O(\log m + \log \log n)$ randomness complexity, the output formula has length $|\varphi'| = \text{poly}(m, \log n)$ as desired.⁸

For $\varphi \in \text{SAT}$, by the completeness of the PCA, there exists a proof π satisfying all predicates, meaning $(\varphi', \pi) \in \mathcal{R}_{\text{SAT}}$. Thus, φ' is satisfiable (as per the first CIC requirement), and using the witness w for φ , it is possible to efficiently find $w' = \pi$ a witness for φ' (as per the third CIC requirement). For $\varphi \notin \text{SAT}$, by the soundness of the PCA, it is computationally infeasible to find a proof string π^* satisfying over half of the predicates $\{\mathcal{V}_{\varphi,\rho}\}$. This means that it is intractable to find a satisfying assignment for the compressed formula φ' , as per the second CIC requirement.

1.3 Applications to Hardness of Approximation and Future Directions

PCPs have had an incredible impact on the field of hardness of approximation. The seminal work of Feige et al. [19] showed how to use PCPs to derive hardness of approximation results, and this result has blossomed into a major technique in the field, leading to optimal inapproximability results for many important problems (see, e.g., [28]). This is achieved by constructing suitable PCPs which enable us to show the NP-completeness of gap versions of natural optimization problems.

We show that the weaker notion of a PCA can, in principle, also yield hardness of approximation results. Moving forward, we believe that this observation has significant potential for bypassing limitations of PCPs in the context of hardness of approximation.

⁸ We note that even if our verifier had larger randomness complexity, it would have still been possible to build a CIC from a succinct-PCA, similarly to [22].

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The main difference between using PCPs and PCAs to show hardness of approximation is that we will only be able to derive hardness of *search* versions of the gap problems, and the hardness will be based on a Cook reduction rather than a Karp reduction. Nevertheless, these results suffice to demonstrate that the underlying optimization problem is hard to approximate.

As a proof-of-concept we next sketch how to use PCAs to rule out a polynomial-time approximation algorithm for MaxSAT. Namely, an algorithm that, given a formula φ finds an assignment that satisfies *approximately* the maximal number of satisfiable clauses in φ . We emphasize that the conclusion is not surprising since it follows from the PCP theorem that GapSAT (deciding whether a formula is satisfiable or if any assignment satisfies at most a constant fraction of the clauses) is NP-complete. What we find surprising is that the notion of a PCA suffices to establish this result. We note that for this result we do not need *succinct* PCAs, but believe that the fact that such PCAs exist may be useful for establishing new hardness of approximation results, e.g., in the context of fine-grained complexity (see more below). We also emphasize that this result crucially relies on the PCA being *publicly verifiable* - we do not know how to establish any hardness of approximation results based on privately verifiable PCAs (such as those of [35]) - see further discussion below.

Given a CNF formula $\varphi : \{0, 1\}^m \rightarrow \{0, 1\}$ over n clauses, and an assignment $z \in \{0, 1\}^m$, let $|\varphi(z)|$ denote the number of clauses satisfied by z in φ . Recall that in the MaxSAT problem, the goal is to find an assignment z that maximizes $|\varphi(z)|$. Consider the following approximation problem:

► **Definition 4** (Approximate MaxSAT $_\epsilon$). *In the Approximate MaxSAT $_\epsilon$ problem, the input is a CNF formula φ . Denote $\alpha = \max_{z \in \{0, 1\}^m} \frac{|\varphi(z)|}{n}$ the maximally satisfiable fraction of clauses of φ . The goal is to output an assignment z , such that $\frac{|\varphi(z)|}{n} \geq (1 - \epsilon) \cdot \alpha$.*

► **Theorem 5.** *Assume $P \neq NP$. If there exists a publicly-verifiable constant-query PCA for SAT, then there exists $\epsilon > 0$ for which there does not exist a polynomial-time algorithm solving Approximate MaxSAT $_\epsilon$.*

Since Theorem 5 is only a proof-of-concept, we do not make any attempt to optimize the constant ϵ .

Proof Sketch of Theorem 5. Assume for contradiction that there exists \mathcal{A} , a polynomial time algorithm solving Approximate MaxSAT $_\epsilon$, for a constant $\epsilon > 0$ specified below. We show a polynomial-time decision procedure for SAT.

Let \mathcal{V} be a constant-query PCA verifier for SAT, and φ an instance of SAT over m variables and n clauses. With φ fixed, and a fixed CRS sampled from the generator, the verifier \mathcal{V} samples a random string ρ and runs a predicate $\mathcal{V}_{\varphi, \rho} : \{0, 1\}^q \rightarrow \{0, 1\}$, over the $q = O(1)$ values it reads from its proof π . The predicate $\mathcal{V}_{\varphi, \rho}$ can be represented as CNF formula with 2^q clauses. We denote by $\varphi' = \bigwedge_{\rho} \mathcal{V}_{\varphi, \rho}$ the conjunction of all these formulas.

Denote by n' and m' the number of clauses and variables in φ' , respectively.

If $\varphi \in \text{SAT}$, then by the completeness of the PCA, there exists a proof satisfying every predicate $\{\mathcal{V}_{\varphi, \rho}\}_\rho$. This means that φ' is satisfiable, i.e., has $\alpha = \max_{z \in \{0, 1\}^{m'}} \frac{|\varphi'(z)|}{n'} = 1$. Therefore, by the definition of \mathcal{A} , for $z = \mathcal{A}(\varphi')$ it holds that $\frac{|\varphi'(z)|}{n'} \geq (1 - \epsilon) \cdot \alpha = 1 - \epsilon$.

If $\varphi \notin \text{SAT}$, then by the computational soundness of the PCA, no polynomial-time algorithm can find a proof satisfying half of the verifier's predicates $\{\mathcal{V}_{\varphi, \rho}\}_\rho$. For each such rejecting predicate, at least one of its 2^q clauses has to evaluate to False. Therefore, for $\epsilon = \frac{1}{2^{q+1}}$, it holds that no polynomial-time algorithm can find an assignment satisfying a

$(1 - \epsilon)$ -fraction of the clauses in φ' . Consequently, if for some $\varphi \notin \text{SAT}$ the algorithm \mathcal{A} outputs $z = \mathcal{A}(\varphi')$ for which $\frac{|\varphi'(z)|}{n'} \geq 1 - \epsilon$, then \mathcal{A} breaks the computational soundness of the PCA for instance φ , which is a contradiction.

Therefore, for every $\varphi \notin \text{SAT}$ and $z = \mathcal{A}(\varphi')$ it holds that $\frac{|\varphi'(z)|}{n'} < 1 - \epsilon$. In this case, it is possible to decide SAT in polynomial time. This is done in the following manner: Given a CNF formula φ , compute φ' as above, run the algorithm $z = \mathcal{A}(\varphi')$, and consider the value $\beta = \frac{|\varphi'(z)|}{n'}$.

- If $\beta \geq 1 - \epsilon$, output $\varphi \in \text{SAT}$. As discussed above, if $\varphi \in \text{SAT}$ then $\varphi' \in \text{SAT}$, and $\beta \geq 1 - \epsilon$. Thus, $\varphi \in \text{SAT}$ is accepted.
- If $\beta < 1 - \epsilon$, output $\varphi \notin \text{SAT}$. If $\varphi \notin \text{SAT}$ and \mathcal{A} doesn't break the PCA, then $\beta < 1 - \epsilon$. Thus, $\varphi \notin \text{SAT}$ is rejected.

Therefore, it is possible to decide SAT in polynomial time, contradicting the assumption that $\text{P} \neq \text{NP}$. ◀

We discuss two important points regarding the reduction above: The first, is that unlike the hardness of approximation results using PCPs, the reduction using PCAs is a Cook reduction rather than a Karp reduction. The second point is about the importance of the public-verifiability of the PCA (mentioned briefly above). The reduction above does not work with privately-verifiable PCAs, in which the verification relies on a secret trapdoor τ for the CRS. Specifically, generating the formula φ' is problematic: Hard-wiring τ to φ' prevents it from being a “hard” instance, as breaking the privately-verifiable PCA is trivial given the trapdoor. On the other hand, it is not clear how to compute β when omitting τ from φ' , as the PCA proof is only verifiable given its secret key. Thus, public-verifiability seems essential to the reduction above.

Hardness of approximation in Fine-Grained Complexity: A potential use-case. The field of fine-grained complexity aims to understand the precise complexity of fundamental problems that are already known to be solvable in polynomial time: for example, whether one can compute the edit distance of two strings in sub-quadratic time. Negative results here are usually shown by assuming a precise hardness for a core problem, and using so-called “fine-grained reductions” to prove lower bounds for the problem at hand [48].

A recent line of work in this field [1, 13] uses proof systems, such as PCPs and interactive proofs, to show the fine-grained *inapproximability* of problems in P . We believe succinct PCAs may have applications in this field as well. To demonstrate a possible use case, consider the 3SUM problem, in which we are given a list of integers a_1, \dots, a_n , each over $\text{polylog}(n)$ bits, and need to decide whether there exist $i, j, k \in [n]$ such that $a_i + a_j + a_k = 0$. This problem can be solved in time $\tilde{O}(n^2)$, but it is conjectured that there are no substantially faster algorithms.

Observe that while it seems that 3SUM may be hard to solve in time $O(n)$, it is easy to *verify* given an NP witness - namely the indices i, j and k . Applying, e.g., the [8, 17] short PCP yields a constant-query PCP for 3SUM of length $\tilde{O}(n)$. In contrast, since the NP witness has *logarithmic* length, our succinct PCA for 3SUM (which follows from Theorem 1) has *poly-logarithmic* length and constant-query complexity. We believe that this use of PCAs has potential for deriving new inapproximability results in the context of fine-grained complexity.

1.4 Additional Related Works

Succinct Non-interactive Arguments. SNARGs for NP have drawn considerable attention recently (see, e.g., [47] for a recent survey). We note that a SNARG for NP, with say poly-logarithmic proof length can be viewed as an (extremely short) PCA with poly-logarithmic query complexity (i.e., the verifier reads the entire proof).

Unfortunately, all known SNARG constructions rely on assumptions that are not-falsifiable [41] (e.g., knowledge-of-exponent or heuristic instantiations of Fiat-Shamir) and this is known to be inherent for blackbox constructions (achieving adaptive soundness) [23].

Nevertheless, there has recently been significant progress on constructing SNARGs for languages in P. For example, the PSNARG of [30] that we use in our construction. We also note that a publicly-verifiable PSNARGs for *all* of P was constructed by Kalai, Paneth and Yang [32] based on a new, but falsifiable, assumption on groups with bilinear maps. However, since the [33] construction does not appear to be *holographic*, it does not seem to be directly usable in our construction.

In a very recent and exciting independent work, Choudhuri et al. [14] constructed a PSNARG for all of P, based on the LWE assumption. If their construction can be shown (or modified) to be holographic (using sufficiently small fields) then it could potentially be used to extend our PCA and CIC constructions to all of NP and to relax the assumption to plain LWE rather than sub-exponential LWE. A challenge that remains to be overcome in order to do so is the fact that Choudhuri et al. apply the Fiat-Shamir hash function also on the main input x , which makes their construction non-holographic.

Batch Verification. Given an NP language \mathcal{L} it is interesting to ask what is the complexity of verifying that k different instances x_1, \dots, x_k all belong to \mathcal{L} . This question seems closely related to the question of AND-compression in which one wants to compress x_1, \dots, x_k into a short instance x' that is true if and only if $x_1, \dots, x_k \in \mathcal{L}$. In the statistical setting general purpose batch verification is known for any language in UP (i.e., NP relations in which the YES instances have a unique witness) [43, 44, 46] and for *statistical zero-knowledge proofs* [36, 37]. In the cryptographic setting, *non-interactive* batch verification is known for language in P [10], albeit in the designated-verifier model. The aforementioned work of [14] constructs *publicly-verifiable* batch verification for NP. Lastly, we note the related notion of *folding schemes* which allows one to perform a type of AND-compression, by interacting with the prover in order to perform the compression itself [39].

Interactive PCPs and IOPs. Kalai and Raz [34] introduced the model of *interactive PCPs* as a different way to circumvent the impossibility result for succinct PCPs. In an interactive PCP, the prover first sends a PCP proof string but afterwards the prover and verifier engage in a standard (public-coin) interactive proof in order to verify the PCP. This model can also be seen as a special case of the more recent notion of *interactive oracle proof* (IOP) [7, 43]. Kalai and Raz constructed interactive PCPs whose length is polynomial in the witness length. More recently, Ron-Zewi and Rothblum [45] constructed interactive PCPs whose length is only *additively* longer than the witness length.

2 Our Results

In this section we state our main results. First, in Section 2.1 we define the notion of *publicly-verifiable probabilistically checkable arguments* (PCAs). We then state a theorem showing the existence of *succinct* publicly-verifiable PCAs for a large subclass of NP, assuming the sub-exponential hardness of LWE. Then, in Section 2.2 we introduce the notion of *computational instance compression* (CICs). Assuming the sub-exponential hardness of LWE, we show the existence of CIC for a large subclass of NP.

2.1 Probabilistically Checkable Arguments

Much like a PCP, a *probabilistically checkable argument* (PCA) is a special format for writing a proof in which the verifier only has to read a few bits from the proof. Unlike PCPs, in which *every* alleged proof for a false statement is rejected with high probability, for PCAs, accepting proofs may exist, but we require that it is *intractable* to find them. This relaxation enables us to build PCAs of length polynomial in the length of the *witness*, rather than the non-deterministic verification time, a result which is unlikely to be possible for PCPs [22]. In contrast to PCPs, PCAs are constructed with respect to a *common reference string* (CRS), which can be accessed both by the prover and the verifier.

PCAs were originally introduced by Kalai and Raz [35]. We consider a natural restriction of their definition that mandates that the PCA is *publicly-verifiable* - that is, given the PCA proof-string π (and the CRS) *anyone* can verify the proof by making only a few queries to π .

► **Definition 6** (Publicly verifiable PCA). *A publicly-verifiable probabilistically checkable argument (PCA) for an NP relation \mathcal{R} , with adversary size $T : \mathbb{N} \rightarrow \mathbb{N}$ and soundness error $s : \mathbb{N} \rightarrow [0, 1]$, is a triplet of $\text{poly}(n, m, \lambda)$ -time algorithms $(\mathcal{G}, \mathcal{P}, \mathcal{V})$, with deterministic \mathcal{P} and probabilistic \mathcal{G} and \mathcal{V} , such that for every instance length n and witness length m the following holds:*

- **Completeness:** *For every $x \in \{0, 1\}^n$ and $w \in \{0, 1\}^m$, such that $(x, w) \in \mathcal{R}$, every $\lambda \in \mathbb{N}$, and every $\text{CRS} \leftarrow \mathcal{G}(1^n, 1^m, 1^\lambda)$ it holds that*

$$\Pr \left[\mathcal{V}^\pi(x, \text{CRS}) = 1 \right] = 1, \quad (1)$$

where $\pi = \mathcal{P}(x, w, \text{CRS})$, and the probability in Equation (1) is only on \mathcal{V} 's coin tosses.

- **Computational soundness:** *For every $x \in \{0, 1\}^n$ s.t. $\mathcal{R}(x) = \emptyset$, every $\lambda \in \mathbb{N}$, and every \mathcal{P}^* of size $\leq T(\lambda)$, with all but $s(\lambda)$ probability over the choice of $\text{CRS} \leftarrow \mathcal{G}(1^n, 1^m, 1^\lambda)$ it holds that*

$$\Pr \left[\mathcal{V}^{\pi^*}(x, \text{CRS}) = 1 \right] \leq \frac{1}{2}, \quad (2)$$

where $\pi^* = \mathcal{P}^*(x, \text{CRS})$, and again the probability in Equation (2) is only on \mathcal{V} 's coin tosses.

The length of π , as a function of n, m , and λ is called the *proof length*. In order to verify its oracle, the verifier \mathcal{V} tosses $r = r(n, m, \lambda)$ random coins, and makes $q = q(n, m, \lambda)$ queries to π . The functions r and q are called the *randomness complexity* and *query complexity*, respectively.

We reiterate that in contrast to the definition of [35], our PCA verifier is *not* given a trapdoor to the CRS. In this work we only discuss publicly-verifiable PCAs, and in what follows whenever we say PCA we refer to the publicly-verifiable variant. Also, by default we assume that PCAs are secure against malicious $T = \text{poly}(\lambda)$ size provers, with soundness error $s = \text{negl}(\lambda)$.

Note that in Definition 6 we distinguish between the randomness of \mathcal{G} (used to generate the CRS) and that of \mathcal{V} (used to check the proof). This separation is done since we would like the CRS to be “good” with overwhelming probability, but since the verifier only makes a small number of queries, we can only guarantee that its checks will fail (in case $x \notin \mathcal{L}$) with constant probability. In particular, this distinction allows us to easily reduce the soundness error of \mathcal{V} arbitrarily, and without increasing the proof length. We do so in the natural way - by simply having the verifier generate more query sets.

► **Fact 7.** Let $(\mathcal{G}, \mathcal{P}, \mathcal{V})$ be a PCA for relation \mathcal{R} , with adversary size T and soundness error s . Denote by \mathcal{V}_k the verifier which runs the verifier k times and accepts if and only if every run accepts. Then, for every $n, m, \lambda \in \mathbb{N}$, every $x \in \{0, 1\}^n$ such that $\mathcal{R}(x) = \emptyset$, and \mathcal{P}^* of size $\leq T(\lambda)$, with all but $s(\lambda)$ probability over the choice of $\text{CRS} \leftarrow \mathcal{G}(1^n, 1^m, 1^\lambda)$ it holds that

$$\Pr \left[\mathcal{V}_k^{\pi^*}(x, \text{CRS}) = 1 \right] \leq \frac{1}{2^k},$$

where $\pi^* = \mathcal{P}^*(x, \text{CRS})$.

Succinct PCAs. A PCA for an NP relation \mathcal{R} , which is decidable in some time $t = t(n, m) \geq n$, is said to be *succinct* if the PCA proof is of length $\text{poly}(m, \lambda, \log(t))$, where poly refers to a fixed *universal* polynomial (that does not depend on the relation \mathcal{R}).

Our first main result shows the existence of succinct publicly-verifiable PCAs for a large subclass of NP.

► **Theorem 8.** Assume the sub-exponential hardness of LWE . Let \mathcal{R} be an NP relation where membership can be decided in logspace-uniform NC, given the instance of length n and witness of length m . Then, there exists a succinct constant-query publicly-verifiable PCA for \mathcal{R} . Furthermore, the verifier runs in time $\text{poly}(n, \log(m), \lambda)$ and has randomness complexity $O(\log(m) + \log \log(n) + \log(\lambda))$.

(Note that the furthermore clause is non-trivial as the requirement for the PCA verifier runtime is $\text{poly}(n, m, \lambda)$, where as we achieve $\text{poly}(n, \log(m), \lambda)$.)

The proof of Theorem 8 is given in Section 3 of the full version [11].

2.2 Computational Instance Compression

Instance compression [27] is a mechanism for compressing a long instance with a short witness, down to (roughly) the witness length. Formally, an *instance compression from NP-relation \mathcal{R} to NP-relation \mathcal{R}'* is a poly-time computable function f , such that for every x of length n with corresponding witness length m , the compressed value $x' = f(x)$ is an instance of \mathcal{R}' with length and witness length $\text{poly}(m, \log(n))$, and $\mathcal{R}(x) = \emptyset \iff \mathcal{R}'(x') = \emptyset$.

Unfortunately, instance compression for SAT and many other NP-complete problems does not exist, unless $\text{NP} \subseteq \text{coNP}/\text{poly}$, in which case the polynomial hierarchy collapses [22]. In this work we consider a relaxation of instance compression which we call *computational instance compression (CIC)*. A CIC consists of three polynomial-time algorithms: a *generator \mathcal{G}* , an *instance compressor \mathcal{IC}* , and a *witness transformer \mathcal{WT}* . Loosely speaking, we require that:

- The *generator \mathcal{G}* which, given the security parameter and instance and witness lengths, generates a CRS.
- If $\mathcal{R}(x) \neq \emptyset$, then the (deterministic) *instance compressor \mathcal{IC}* outputs $x' = \mathcal{IC}(x, \text{CRS})$ such that $\mathcal{R}'(x') \neq \emptyset$. Moreover, given a witness w for x , the (deterministic) *witness transformer \mathcal{WT}* produces a corresponding witness w' for x' .
- If $\mathcal{R}(x) = \emptyset$, we require that it is computationally intractable to find a witness w' for $x' = \mathcal{IC}(x, \text{CRS})$ (even though it may be that such witnesses exist).

► **Definition 9 (Computational instance compression).** Let \mathcal{R} and \mathcal{R}' be NP-relations. Denote by n the instance length for \mathcal{R} , and by m the corresponding witness length. A computational instance compression (CIC) scheme from \mathcal{R} to \mathcal{R}' , with adversary size $T : \mathbb{N} \rightarrow \mathbb{N}$ and soundness error $s : \mathbb{N} \rightarrow [0, 1]$, is a triplet of $\text{poly}(n, m, \lambda)$ -time algorithms $(\mathcal{G}, \mathcal{IC}, \mathcal{WT})$, for probabilistic \mathcal{G} and deterministic \mathcal{IC} and \mathcal{WT} , such that:

- **Completeness:** For $x \in \{0, 1\}^n$ and $w \in \{0, 1\}^m$ such that $(x, w) \in \mathcal{R}$, and every λ , it holds that

$$\Pr_{\text{CRS} \leftarrow \mathcal{G}(1^n, 1^m, 1^\lambda)} \left[(\mathcal{IC}(x, \text{CRS}), \mathcal{WT}(x, w, \text{CRS})) \in \mathcal{R}' \right] = 1.$$

- **Computational soundness:** For every $x \in \{0, 1\}^n$ with $\mathcal{R}(x) = \emptyset$, every $\lambda \in \mathbb{N}$, and every adversary \mathcal{A} of size $\leq T(\lambda)$ it holds that

$$\Pr_{\text{CRS} \leftarrow \mathcal{G}(1^n, 1^m, 1^\lambda)} \left[(\mathcal{IC}(x, \text{CRS}), w^*) \in \mathcal{R}' \right] \leq s(\lambda),$$

where $w^* = \mathcal{A}(x, \text{CRS})$.

By default, we assume a CIC scheme is secure against adversaries of size $T = \text{poly}(\lambda)$, with $s = \text{negl}(\lambda)$.

Discussion. We briefly discuss some important points regarding the definition of CICs:

1. The first point that we highlight, is that for x a NO instance, it could very well be the case that x' is a YES instance (indeed, this is virtually a certainty in our construction). Whereas standard instance compression requires that x' be a NO instance, the CIC soundness requirement states that if x' is indeed a YES instance, then finding a witness w' for x' is intractable.
2. One may also consider a strengthening of CIC in which the compression algorithm, applied to a NO instance, outputs x' that is *computationally indistinguishable* from a false instance. We refer to this notion as a *strong* CIC and note that it indeed implies the weaker notion, since if x' is indistinguishable from a NO instance, then it is intractable to find a witness w' for x' . We leave the study of strong CICs to future work.
3. The last point is regarding the importance of the witness transform algorithm \mathcal{WT} . One might wonder whether the requirement that such an efficient transformation exists is significant. In particular, one can consider a weaker notion of CIC in which it is not required that a witness w' for x' can be efficiently found from (x, w) , i.e., forgoing the use of \mathcal{WT} .

This weaker definition seems substantially less useful to us. In particular, it does not suffice for a main application of instance compression - storing instances more efficiently, in order to reveal their solution at a later time. Using classical instance compression, finding a witness w' for x' proves that there exists a witness w for x . As for CIC, without \mathcal{WT} , it's not clear how to find a witness w' for x' , even given a witness w for x .

Moreover, we note that this weaker definition can be trivially realized for languages \mathcal{L}' that are hard on average as follows: the instance compressor ignores x and simply outputs a “hard” false instance x' (i.e., a YES instance x' that is computationally indistinguishable from a NO instance x''). This trivial compressor satisfies the property that (1) a YES instance is mapped to a YES instance, whereas (2) for a NO instance x it is computationally intractable to find a witness w' for x' (as this would distinguish x' from x'').

Our second main result is a construction of CICs from (sub-exponential) LWE.

- **Theorem 10.** *Assume the sub-exponential hardness of LWE. Then, every NP relation \mathcal{R} , for which membership can be decided in logspace-uniform NC, is computationally instance compressible to SAT.*

The proof of Theorem 10 follows in a relatively straightforward manner from Theorem 8, see the full version [11] for details.

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