Brief Announcement: Distributed Algorithms for Minimum Dominating Set Problem and Beyond, a New Approach

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Abstract

In this paper, we study the minimum dominating set (MDS) problem and the minimum total dominating set (MTDS) problem. We propose a new idea to compute approximate MDS and MTDS. This new approach can be implemented in a distributed model or parallel model. We also show how to use this new approach in other related problems such as set cover problem and $k$-distance dominating set problem.

2012 ACM Subject Classification Mathematics of computing → Graph algorithms; Theory of computation → Distributed algorithms

Keywords and phrases Minimum dominating set problem, set cover problem, $k$-distance dominating set problem, distributed algorithms

Digital Object Identifier 10.4230/LIPIcs.DISC.2022.40

Introduction

Let $G = (V,E)$ be an undirected graph with the vertex set $V$ and the edge set $E$ and without isolated vertex. We denote the set of adjacent vertices to a vertex $v$, neighbors of $v$, by $N(v)$. A set $S \subseteq V$ is a dominating set of $G$ if each node $v \in V$ is either in $S$ or has a neighbor in $S$. Also, $S \subseteq V$ is a total dominating set of $G$ if each node $v \in V$ has a neighbor in $S$. Let $\gamma(G)$ and $\gamma_t(G)$ be the size of a minimum dominating set (MDS) and a minimum total dominating set (MTDS) of $G$, respectively. It is easy to prove that $\gamma(G) \leq \gamma_t(G) \leq 2\gamma(G)$.

Also, a subset of vertices such that each edge of the graph $G$ is incident to at least one vertex of the subset is a vertex cover. A minimum vertex cover (MVC) of $G$ is a vertex cover with the smallest possible number of vertices. The size of MVC is denoted by $\beta(G)$. A subset of the vertices such that no two vertices in the subset represent an edge of $G$ is an independent set of $G$. An independent set with the largest possible number of vertices is called a maximum independent set (MIS). The size of MIS is denoted by $\alpha(G)$.

An interesting problem is computing the MDS in the distributed model, which we will consider in this paper. In a distributed model the network is abstracted as a simple $n$-node undirected graph $G = (V,E)$. There is a processor on each node $v \in V$, with a unique $\Theta(\log n)$-bit identifier $ID(v)$, who initially knows only its neighbors in $G$. Communication happens in synchronous rounds. Per round, each node can send one, possibly different, $O(\log n)$-bit message to each of its neighbors. Ultimately, each node should know its own...
part of the output. When computing the dominating set, each node knows whether it is in the dominating set or has a neighbor in the dominating set [2]. When the size of the messages is restricted to be $O(\log n)$, then the algorithm is CONGEST.

## 2 Theoretical result and the algorithm

For a given graph $G$, with no isolated vertex, we construct a graph $G'$ with the same set of vertices as in $G$ as follows. For each vertex $v$ of degree at least two, we choose two of its neighbors arbitrarily and add an edge between them in $G'$. If $v$ is of degree one, we add a loop edge on its neighbor. We call this edge the corresponding edge of $v$ in $G'$ and denote it by $e_v$. Note that if the graph $G$ has a cycle of length 4, with vertices $a, b, c, d$ then the edge $bd$ can be the corresponding edge of both $a$ and $c$ in $G'$. Let $\alpha(G)$ and $\beta(G)$ be the size of a maximum independent set and the size of a minimum vertex cover of $G$, respectively (as defined previously). Then, we have the following theorem.

\begin{itemize}
  \item \textbf{Theorem 1.} $\gamma_t(G) \leq n - \alpha(G') = \beta(G').$
\end{itemize}

\textbf{Proof.} Suppose that $D$ is a maximum independent set of $G'$, so $|D| = \alpha(G')$. We show that $V \setminus D$ is a total dominating set for $G$. For each vertex $v$, we choose two of its neighbors, for example, $u$ and $w$, and add an edge between them in $G'$. Since there is an edge between $u$ and $w$, at most, one of them can be in $D$, which means at least one of them is in $V \setminus D$. The same argument applies when an edge is a loop. Thus, for each vertex $v$, at least one of its neighbors in $G$ is in $V \setminus D$, so $V \setminus D$ is a total dominating set for $G$. The size of $|V \setminus D|$ equals $n - \alpha(G')$ and we have $\gamma_t(G) \leq n - \alpha(G') = \beta(G').$ \hfill \qed

Note that the graph $G'$ can be constructed is a constant number of rounds in the CONGEST model. According to our time and space constraints, we can use the known distributed algorithms for computing a vertex cover for $G'$ (See [3, 4]).

## 3 Extension to the other problems

### Set cover problem

In the set cover problem we are given a set $A = \{a_1, a_2, \ldots, a_n\}$ of $n$ elements and $m$ subsets, $A_1, A_2, \ldots, A_m$ of $A$. The goal is to choose the minimum number of subsets that cover all the elements of $A$.

Our algorithm is as follows. Each element $a_i$ chooses a subset $A_j$ with maximum size such that $a_i \in A_j$. Let $x_i$ be the number of times that $A_i$ is chosen by the elements of $A$. We construct a graph $G'$ that its vertices are the subsets $A_1, A_2, \ldots, A_m$. For each $a \in A$ we choose two subsets $A_i$ and $A_j$ with maximum values of $x_i$’s such that $a \in A_i$ and $a \in A_j$ and add an edge between them. Similar to the proof of Theorem 1, it can be shown that a vertex cover for $G'$ is a set cover for $A$.

### $k$-distance dominating set

An extension of the MDS problem is the minimum $k$-distance dominating set problem where the goal is to choose a subset $S \subseteq V$ with the minimum cardinality such that for every vertex $v \in V \setminus S$, there is a vertex $u \in S$ where the shortest path between them at most $k$. The minimum total $k$-distance dominating set is defined similarly. A $k$-observer $Ob$ of a network $N$ is a set of nodes in $N$ such that each message, that travels at least $k$ hops in $N$, is handled (and so observed) by at least one node in $Ob$. A $k$-observer $Ob$ of a network $N$ is minimum
iff the number of nodes in $Ob$ is less than or equal to the number of nodes in every $k$-observer of $N$ (See [1]). This problem is equivalent to the $k$-distance dominating set problem. In this problem for each node $v$, the neighbors of $v$, is the set of nodes whose distance from $v$ is less than $k + 1$. Then we apply the proposed algorithms as before.

Note that computing a minimum $k$-distance dominating set for a graph $G$ is equivalent to computing a minimum dominating set for $G^k$, where $G^k$ is a graph with the same vertex set as $G$ and we put an edge between two vertices in $G^k$ if the distance between them in $G$ is less than $k + 1$.

References


