Online Algorithms with Predictions

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Abstract

We give an introduction to online algorithms with predictions, from an algorithms researcher's perspective, concentrating on minimization problems.

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1 Extended Abstract

We begin with an introduction to online algorithms with predictions, where the online algorithm is given additional information, some predictions, which should, presumably, improve its performance. The seminal papers in this area, by Lykouris and Vassilvitskii [9, 10] and Purohit, Svitkina and Kumar [11], appeared at conferences in 2018 and have since inspired many other researchers to work in this area. A current list of related publications can be found on a dedicated website [1], which listed 145 articles as of July 10, 2023.

Online algorithms are those that as input receive a sequence of requests, each of which must be handled by the algorithm making an irrevocable decision, before the next request arrives. The research area, online algorithms with predictions, is related to an older line of research within online algorithms, advice complexity [5, 7, 4, 6], where the online algorithm is given "advice" which is assumed to be correct. In contrast, in the area of online algorithms with predictions, the predictions given to these online algorithms may come from machine learning and generally contain errors. In both of these models, the online algorithm receives extra information and the performance of an algorithm is measured using the competitive ratio (asymptotically, the worst-case ratio over all possible input sequences of the cost obtained by the algorithm compared to the cost of OPT, the optimal offline algorithm, on the same input). In advice complexity, the goal is to achieve a good competitive ratio with as few bits of advice as possible. In contrast, the goal in online algorithms with predictions is to achieve a good competitive ratio, despite errors in the predictions.

The amount of error in the predictions, $\hat{p}$, for an input sequence I with correct values, p, is given by some error measure, $\eta(I, \hat{p}, p)$. This is typically normalized as $\eta(I, \hat{p}, p)/\,\mathrm{OPT}(I)$, where $\mathrm{OPT}(I)$ is the cost achieved by an optimal offline algorithm on input sequence I. An online algorithm with predictions should have a competitive ratio that degrades gracefully with increasing error (smoothness), performing near optimally if there is no error (consistency), but not performing too poorly, even if the predictions are terrible (robustness).

Results for two different minimization problems are presented, demonstrating

- the relevance of advice complexity for the paging problem with predictions [2] and
- the relevance of random order analysis [8] for a problem where, according to competitive analysis, no algorithm can be better than the trivial Follow-the-Predictions algorithm [3].

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