Recoverable Robust Periodic Timetabling

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Abstract

We apply the concept of *recoverable robustness* to periodic timetabling, resulting in the *Recoverable Robust Periodic Timetabling Problem (RRPT)*, which integrates periodic timetabling and delay management. Although the computed timetable is periodic, the model is able to take the aperiodicity of the delays into account. This is an important step in finding a good trade-off between short travel times and delay resistance. We present three equivalent formulations for this problem, differing in the way the timetabling subproblem is handled, and compare them in a first experimental study. We also show that our model yields solutions of high quality.

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1 Introduction

An important aspect of optimising public transport is finding a good periodic timetable. From the passengers' point of view, short travel times are desirable, which can be achieved by making the timetable as tight as possible. This problem is known as the *Periodic Event Scheduling Problem (PESP)*, first introduced by Serafini1989, and is well researched. Tight timetables minimise travel times, but are prone to delays which are inevitable in reality and highly dissatisfactory for the passengers. Hence, apart from short travel times, a good timetable should also have some degree of delay resistance. Many concepts and ideas on how to increase the robustness of a timetable against delays exist, see [17]. However, none of these approaches uses the promising concept of *recoverable robustness* introduced by [15]. The aim is to find a periodic timetable with small travel times such that in every delay scenario from a given uncertainty set it is possible to find a disposition timetable which fulfils some quality criteria. To this end, we have to integrate timetabling and delay management. Delay Management was introduced in [26] and has been treated in many papers, see [11, 3] for surveys. Timetables are determined in a periodic network, but delay management is done in an aperiodic network, since in general delays do not occur periodically. In order to integrate delay management into timetabling, we hence have to find a way to bridge this gap. One possibility to do this is to model periodic timetabling also in the aperiodic network, which was done in [9].
In this paper, we introduce the *Recoverable Robust Periodic Timetabling Problem (RRPT)*, which is the first to integrate periodic timetabling and (aperiodic) delay management. We present and analyse three equivalent MIP formulations.

The PESP was introduced by [28] and has received a lot of attention in the literature, see [20, 19, 22, 14] for some early works. Due to its high relevance and complexity it still keeps researchers occupied today in order to find heuristic approaches, see e.g. [1].

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Robustness has been considered for timetabling. Stochastic optimisation models were presented in [13, 12]. Different robustness concepts are used in the literature on robust timetabling, including light robustness ([4]), recoverable robustness ([16]), recover-to-optimality ([7, 6]) and adjustable robustness ([23, 21]). For surveys on robust timetabling we refer to [2] and [17]. So far, robust optimisation models in the literature either considered aperiodic timetabling, i.e. timetables which are not required to repeat in a regular pattern, or periodic timetabling where also the delays are periodic. To the best of our knowledge robust periodic timetabling with aperiodic delays has not been treated in the literature so far.

The remainder of this paper is structured as follows: Basic notions of timetabling and delay management are given in Section 2. In Section 3 we revisit the model *Periodic Timetabling in Aperiodic Network (PTTA)* from [9], which computes a periodic timetable in an aperiodic network. This is then used the formulate the problem RRPT in Section 4, for which we derive three equivalent formulations. We compare these formulations experimentally in Section 5 and conclude in Section 6.

2 Preliminaries

Event-Activity-Networks. An event-activity-network is a graph $\underline{\mathcal{N}} = (\underline{\mathcal{E}}, \underline{\mathcal{A}})$. Its nodes (so-called *events*) represent the departure or arrival of a traffic line at some station and its arcs (so-called *activities*) represent relations between the events. We distinguish different types of activities. *Driving activities* $\underline{\mathcal{A}}_{\text{drive}}$ model a train line driving from one station to another, while *waiting activities* $\underline{\mathcal{A}}_{\text{wait}}$ represent a line waiting at a station. Since these types of activities behave similarly, we denote $\underline{\mathcal{A}}_{\text{train}} = \underline{\mathcal{A}}_{\text{drive}} \cup \underline{\mathcal{A}}_{\text{wait}}$. Passengers can transfer between different lines, which is included by the *transfer activities* $\underline{\mathcal{A}}_{\text{trans}}$. *Headway activities* $\underline{\mathcal{A}}_{\text{head}}$ are used to model safety regulations requiring a minimal distance between two consecutive departures or arrivals, or safety restrictions on single-track lines. They come in pairs, since it is not clear beforehand in which order two departures will take place, see [27] for details.

Periodic Timetabling. The standard model used for periodic timetabling is the *Periodic Event Scheduling Problem (PESP)* introduced by [28]. Given an EAN $\underline{\mathcal{N}} = (\underline{\mathcal{E}}, \underline{\mathcal{A}})$, we want to find a periodic timetable with period T, which is a mapping $\tilde{\pi} \colon \underline{\mathcal{E}} \to \{0, \dots, T-1\}$ assigning a time to every event. To simplify notation we set $\tilde{\pi}_i := \tilde{\pi}(i)$ for $i \in \underline{\mathcal{E}}$. For every activity $a \in \underline{\mathcal{A}}$ a lower bound $L_a \in \mathbb{N}$ and an upper bound $U_a \in \mathbb{N}$ are given. L_a is the minimal time necessary to perform the activity a, while U_a is the maximal time allowed for a. A timetable is *feasible* if it respects the bounds on the activities, i.e. for every activity $a = (i, j) \in \underline{\mathcal{A}}$ we require $\tilde{\pi}_j - \tilde{\pi}_i + z_a T \in [L_a, U_a]$ for some $z_a \in \mathbb{Z}$. The modulo parameter z_a takes the periodicity into account.
The PESP asks for a feasible timetable. In timetabling we additionally want to minimise the total travel time summed over all passengers. For $a \in \underline{\mathcal{A}}$ let $w_a \in \mathbb{N}$ be the number of passengers using activity a. The following is the basic IP formulation for PESP:

$$\min \sum_{a=(i,j)\in\underline{\mathcal{A}}} w_a \cdot (\tilde{\pi}_j - \tilde{\pi}_i + z_a T) \tag{PESP}$$

$$\text{s.t.} \quad \tilde{\pi}_j - \tilde{\pi}_i + z_a T \le U_a \qquad a = (i,j) \in \underline{\mathcal{A}} \tag{1}$$

$$\tilde{\pi}_j - \tilde{\pi}_i + z_a T \ge L_a \qquad a = (i,j) \in \underline{\mathcal{A}} \tag{2}$$

$$\tilde{\pi}_i \in \{0, \dots, T-1\} \qquad i \in \underline{\mathcal{E}} \tag{3}$$

$$z_a \in \mathbb{Z} \qquad a \in \underline{\mathcal{A}}. \tag{4}$$

Details can be found in the literature on PESP, a good introduction is given in [14, 19]. Instead of using node potentials, another approach, see [18], is to use tensions, i.e. instead of assigning a time π_i to every event $i \in \underline{\mathcal{E}}$ we assign a duration ξ_a to every activity $a \in \underline{\mathcal{A}}$. For this purpose, we choose an arbitrary spanning tree $\mathcal{T}$ and define its network matrix Γ by

$$\Gamma_{a',a} = \begin{cases} 1 & a \in C^+_{a'}, \\ -1 & a \in C^-_{a'}, \\ 0 & a \notin C_{a'} \end{cases}$$

for $a \in \underline{\mathcal{A}}, a' \in \underline{\mathcal{A}} \backslash \mathcal{T}$, where $C^+_{a'}$ and $C^-_{a'}$ are the arcs of the unique cycle in $\mathcal{T} \cup \{a'\}$ in forward respectively backwards direction. This yields the cycle-base formulation of PESP, which is equivalent to the standard formulation, but needs significantly less computing time:

$$\begin{aligned} \min \quad & w^T \xi & & \text{(PESP-cb)} \\ \text{s.t.} \quad & \Gamma \xi = Tq & & (5) \\ & L \leq \xi \leq U & & (6) \\ & \xi_a \in \mathbb{Z} & a \in \underline{\mathcal{A}} & \quad (7) \\ & q_a \in \mathbb{Z} & a \in \underline{\mathcal{A}} \backslash \mathcal{T}. & \quad (8) \end{aligned}$$

Delay Management. Given a timetable, the periodic EAN can be rolled out to obtain a corresponding aperiodic network for some planning horizon $I = [0, K \cdot T]$ with $K \in \mathbb{N}$. Every event $i \in \underline{\mathcal{E}}$ has K corresponding events $i_1, \ldots, i_K$ at times $\pi_{i_s} = \tilde{\pi}_i + (s-1)T$ for $s \in \{1, \ldots, K\}$ in the rolled out network. We denote it by $\mathcal{N} = (\mathcal{E}, \mathcal{A})$ to distinguish it from the periodic network.
During operation of a timetable, it can happen that some source delays occur, which require to adapt the timetable to the current situation. If an event $i \in \mathcal{E}$ has a source delay of d_i, it cannot take place before $\pi_i + d_i$. If an activity $a \in \mathcal{A}$ has a source delay of d_a, the minimal duration for this activity increases to $L_a + d_a$. The task of delay management is to find a disposition timetable x assigning a new time x_i to every event $i \in \mathcal{E}$ respecting the source delays d. Additionally, for every transfer $a \in \mathcal{A}_{\text{trans}}$ it has to be decided if the transfer should be maintained or if it is to be cancelled, which is modelled by a binary variable y_a. To avoid conflicts between trains, also the headway activities have to be treated with care. Hence, we have binary variables p_{ij}, p_{ji} for all pairs $(i,j),(j,i) \in \mathcal{A}_{\text{head}}$ of headway activities, determining which of the events i and j takes place first. The objective is to minimise the delay of the passengers. If passengers miss a transfer, we use the common assumption that they take the next trip T minutes later and that this trip does not have a delay. Let w_i be the number of passengers leaving the transport system at event $i \in \mathcal{E}$. This yields the following IP formulation (for an appropriately large constant M'):

$$\begin{aligned} \min \quad & \sum_{i \in \mathcal{E}} w_i (x_i - \pi_i) + T \sum_{a \in \mathcal{A}_{\text{trans}}} w_a y_a & & \text{(DM)} \\ \text{s.t.} \quad & x_i \geq \pi_i + d_i & i \in \mathcal{E} & \quad (9) \\ & x_j - x_i \geq L_a + d_a & a = (i,j) \in \mathcal{A}_{\text{train}} & \quad (10) \\ & M' y_a + x_j - x_i \geq L_a & a = (i,j) \in \mathcal{A}_{\text{trans}} & \quad (11) \\ & M'(1 - p_{ij}) + x_j - x_i \geq L_a & a = (i,j) \in \mathcal{A}_{\text{head}} & \quad (12) \\ & p_{ij} + p_{ji} = 1 & (i,j),(j,i) \in \mathcal{A}_{\text{head}} & \quad (13) \\ & x_i \in \mathbb{N} & i \in \mathcal{E} & \quad (14) \end{aligned}$$

$$y_a \in \{0,1\} \qquad a \in \mathcal{A}_{\text{trans}} \tag{15}$$
$$p_{ij} \in \{0,1\} \qquad (i,j) \in \mathcal{A}_{\text{head}}. \tag{16}$$

Recoverable Robustness. The concept of recoverable robustness has been introduced in [15]. The idea is to find solutions to an optimisation problem that can be *recovered* by limited effort for a given set of scenarios. In the context of timetabling, this corresponds to finding a timetable and a disposition timetable for every given delay scenario, such that the delay of the disposition timetable compared to the planned timetable is limited. Specifically, there are two types of recovery actions. The main action is cancelling transfers. Furthermore, the times of the events have to be adapted to the delays.

3 Periodic Timetabling in an Aperiodic Network

In the context of timetabling, finding a recoverable robust timetable boils down to integrating timetabling and delay management. The challenge is that these two problems are usually considered in two different networks: while the PESP uses the periodic network, delay management is done in the rolled out aperiodic network as explained above. A first idea to integrate these problems is to solve the periodic timetabling problem also in the rolled out network, so we can then solve both problems in the same network. For this purpose, *Periodic Timetabling in an Aperiodic Network (PTTA)* was introduced in [9]. We briefly describe the resulting model.
One obstacle when computing a timetable in the rolled out network is that usually the timetable is already given and used as input for rolling out the network, since it influences which of the events are connected to each other by an activity. An example for this can be found in Figure 4 in the appendix. Hence, we adapt the roll-out procedure as follows:

- We set $b_a := \lceil \frac{U_a}{T} \rceil$ for $a \in \underline{\mathcal{A}}$, $b := \max_{a \in \underline{\mathcal{A}}} b_a$.
- For every periodic event $i \in \underline{\mathcal{E}}$ and $1 \le s \le K + b$ create an aperiodic event i_s. Let $\mathcal{E}(i) := \{i_s : 1 \le s \le K + b\}$ be the set of all aperiodic events corresponding to i. The set of all events is $\mathcal{E} := \cup_{i \in \underline{\mathcal{E}}} \mathcal{E}(i)$.
- For every periodic activity $a = (i,j) \in \underline{\mathcal{A}} \backslash \underline{\mathcal{A}}_{\text{head}}$, for exactly one arc $a = (i,j)$ of every pair of headway activities and for every $1 \le s \le K, s \le t \le K + b_a$ create a *potential (aperiodic) activity* a_{st} with $L_{a_{st}} = L_a$, $U_{a_{st}} = U_a$ and $w_{a_{st}} = w_a$. Let $\mathcal{A}(a) := \{a_{st} = (i_s, j_t) : 1 \le s \le K, s \le t \le s + b_a\}$ be the set of potential activities corresponding to a. The set of all potential activities is $\mathcal{A} := \bigcup_{a \in \underline{\mathcal{A}}} \mathcal{A}(a)$. Analogous to $\underline{\mathcal{A}}$, we also partition $\mathcal{A}$ into subsets $\mathcal{A}_{\text{drive}}, \mathcal{A}_{\text{wait}}, \mathcal{A}_{\text{train}}, \mathcal{A}_{\text{trans}}$ and $\mathcal{A}_{\text{head}}$ for different types of activities.

Note that additional b periods are added at the end of the planning horizon to ensure that we can define activities that start in I but end outside of I.

The rolled out network contains not only the actual activities, but also potential activities. Thus, when fixing the timetable we have to simultaneously solve an assignment problem: for each periodic activity we have to choose exactly one of the corresponding arcs in every considered period. In order to do so we introduce a binary variable u_a for every $a \in \mathcal{A}$ which is set to 1 if and only if a is chosen. The variable F_a gives the duration of the activity $a \in \mathcal{A}$ in the case that $u_a = 1$. Due to the periodicity of the timetable, it is not needed for all activities, but only for those in the first period. This yields the following MIP formulation. Recall that $(\underline{\mathcal{E}}, \underline{\mathcal{A}})$ is the periodic and $(\mathcal{E}, \mathcal{A})$ the rolled out network.

$$\min \sum_{a=(i_1,j_t) \in \mathcal{A}} w_a F_a \cdot K \tag{PTTA}$$

$$
\begin{aligned}
\text{s.t. } & \pi_{j_t} - \pi_{i_s} + M(u_a - 1) \le U_a && a = (i_s, j_t) \in \mathcal{A} && (17)\\
& \pi_{j_t} - \pi_{i_s} + M(1 - u_a) \ge L_a && a = (i_s, j_t) \in \mathcal{A} && (18)\\
& \pi_{i_s} - \pi_{i_{s-1}} = T && i_s \in \mathcal{E}, 2 \le s \le K + b && (19)\\
& \sum_{t: a' = (i_s, j_t) \in \mathcal{A}} u_{a'} = 1 && a = (i, j) \in \underline{\mathcal{A}}, 1 \le s \le \ K && (20)\\
& \pi_{i_1} \le T - 1 && i \in \underline{\mathcal{E}} && (21)\\
& F_a \ge M(u_a - 1) + \pi_{j_t} - \pi_{i_1} && a = (i_1, j_t) \in \mathcal{A} && (22)\\
& \pi_i \in \mathbb{N} && i \in \mathcal{E} && (23)\\
& u_a \in \{0, 1\} && a \in \mathcal{A}. && (24)\\
& F_a \in \mathbb{N} && a = (i_1, j_t) \in \mathcal{A}. && (25)
\end{aligned}
$$

The objective function minimises the total travel time over all passengers. Note that due to the periodicity of input data and timetable it is sufficient to consider only the first period here. In the case that an activity a is chosen, i.e. $u_a = 1$, Constraints (17) and (18) ensure that the upper and lower bounds for this activity are respected. If a is not selected, the constraints become redundant for appropriately chosen M. Constraints (19) are called *periodicity constraints* and ensure that the timetable has period T. For every periodic activity the assignment constraint (20) chooses exactly one of the corresponding aperiodic activities in every period in such a way that it fits to the timetable constraints (17) and (18). Constraints (21) enforce that the first event takes place in the first period we consider. Constraints (22) set the auxiliary variables needed for the objective function correctly.

▶ **Lemma 1** ([9], Lemma 7). *PTTA and PESP are equivalent. More precisely: Let $(\tilde{\pi}, z)$ be a solution to PESP with objective value $\tilde{f}$. We set $\pi_{i_s} = \pi_{i_1} + (s-1)T$. Furthermore, for $a' = (i_s, j_t) \in \mathcal{A}(a)$ we choose*

$$u_{a'} = \begin{cases} 1 & \text{if } t = z_a + s, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and for } a' = (i_1, j_t) \in \mathcal{A}(a),\ F_{a'} = \begin{cases} \pi_{j_t} - \pi_{i_1} & \text{if } u_{a'} = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Then (π, u, F) is a feasible solution to PTTA and the corresponding objective value is $f = K\tilde{f}$.
The other direction also holds, for details see [9].

4 Recoverable Robust Models

We now formulate the *Recoverable Robust Periodic Timetabling Problem.*

Let $\mathcal{U}$ be a set of scenarios, where each scenario $r \in \mathcal{U}$ consists of some source delays $d_i^r \in \mathbb{N}$ for events $i \in \mathcal{E}$ and $d_a^r \in \mathbb{N}$ for $a \in \mathcal{A}_{\text{train}}$.

▶ **Definition 2.** *Let a timetable π be given. For delay scenario $r \in \mathcal{U}$ let x^r be an optimal disposition timetable and y^r wait-/no-wait decisions. Let*

$$Z_1^r(\pi) := \sum_{i \in \mathcal{E}} w_i (x_i^r - \pi_i) \text{ and } Z_2^r(\pi) := \sum_{a \in \mathcal{A}_{\text{trans}}} w_a y_a^r$$

be the weighted event delay and the number of missed transfers in scenario r, respectively. We denote the worst-case delay of π with respect to $\mathcal{U}$ by

$$f^{\text{del}}(\pi) := \max_{r \in \mathcal{U}} Z_1^r(\pi) + T Z_2^r(\pi).$$

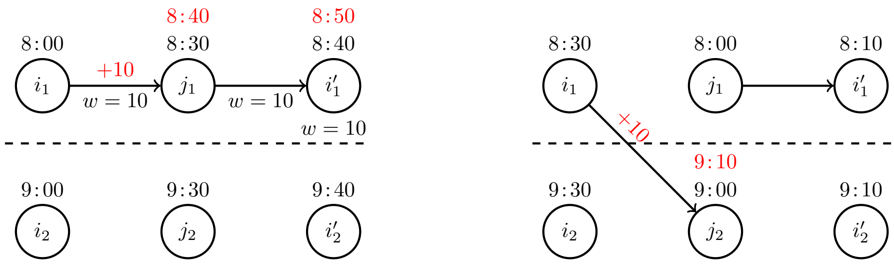


Figure 1 The delay of passengers leaving the planning horizon is not counted correctly.

We are interested in finding a recoverable robust timetable. This means we want to be able to recover our timetable in every given scenario. Recovering a timetable is done by applying delay management. Hence, our goal can be formulated as follows:

Recoverable Robust Periodic Timetabling (RRPT)

Input: Periodic EAN $\mathcal{N} = (\underline{\mathcal{E}}, \underline{\mathcal{A}})$ with period T, interval I, set $\mathcal{U}$ of sets of source delays within I.

Task: Find a periodic timetable π and disposition timetables x^r with wait-/no-wait decisions y^r for every $r \in \mathcal{U}$ such that the *real travel time* $f^{\text{real}}(\pi) := f^{\text{nom}}(\pi) + f^{\text{del}}(\pi)$ is minimal, where $f^{\text{nom}}(\pi)$ is the nominal travel time of π.

To derive an MIP formulation for this problem we now can use the preparatory work from [9]: Since we have formulated the timetabling problem, which is a subproblem of RRPT, already in the aperiodic network, we can now simply add the delay management constraints (9)-(16) for every scenario $r \in \mathcal{U}$ to PTTA. Of course we only have constraints for those arcs a which are actually chosen in the assignment subproblem of PTTA, i.e. those with $u_a = 1$. Hence, we have to add the delay propagation constraints as big-M-constraints.

Another problem we have to deal with are the passengers leaving our planning horizon I, as the following example demonstrates.

▶ **Example 3.** We consider a part of a rolled out EAN as depicted in Figure 1 with only a single delay scenario for two different timetables. In the first one, 10 passengers arrive at i'_1 with 10 minutes delay, so we have $Z_1(\pi^1) = 10$. However, if we shift the timetable by 30 minutes as seen in the right subfigure, different arcs are chosen, so we have the arc (i_1, j_2) leaving the planning horizon. In this case there is no delay at the event i'_1. Since the event j_2, which is delayed in this case, has weight zero, the delay is $Z_1(\pi^2) = 0$. The reason for this is that the passengers' delay is counted when they arrive at their final destination. With the shifted timetable, the arrival is outside of our planning horizon, so no delay is recognised by our objective function. To prevent this, we count the last known delay of those passengers leaving the planning horizon: in this case this are the 10 minutes delay at the event j_2, which we weight with the number of passengers using the arc (i_1, j_2).

To handle this problem, we adapt the definition of Z_1^r (and hence also that of f^{del} and f^{real}).

▶ **Definition 4.** *We denote the rolled out driving and waiting activities leaving the planning horizon I by $\mathcal{A}_{\text{out}} := \{a = (i_s, j_t) \in \mathcal{A}_{\text{train}} \cup \mathcal{A}_{\text{trans}} : t > K\}$ and adapt the definition of the weighted event delay: $Z_1^r(\pi) := \sum_{i_s \in \mathcal{E}: s \leq K} w_{i_s}(x^r_{i_s} - \pi_{i_s}) + \sum_{a=(i,j) \in \mathcal{A}_{\text{out}}} w_a(x^r_j - \pi_j)$.*

While in periodic timetabling headway activities can be treated in the same way as the other activities, this is not the case for aperiodic timetabling and delay management. To be able to change the order of trains in case of delays, we need precedence constraints between all pairs of events using the same piece of infrastructure. Additionally to the headways $\underline{\mathcal{A}}_{\text{head}}$ we now also need to respect headways between repetitions of the same periodic event: If the event i_s has a big delay, there can be a conflict with the next event i_{s+1}. Therefore, we define

$$\mathcal{A}'_{\text{head}} := \{(i_s, j_t) : (i,j) \in \underline{\mathcal{A}}_{\text{head}}, 1 \leq s,t \leq K\} \cup \{(i_s, i_t) : i \in \underline{\mathcal{E}}, 1 \leq s,t \leq K\}.$$

Note that $\mathcal{A}'_{\text{head}}$ is not a subset of $\mathcal{A}$, since it also contains arcs of the form (i_s, j_t) for $t < s$ and $t > s + b$. This is due to the fact that delays can change the order of the events.

In Section 4.1 we present a formulation of RRPT which uses PTTA in the rolled out network. Section 4.2 presents two formulations in the periodic network $(\underline{\mathcal{E}}, \underline{\mathcal{A}})$.

4.1 Formulation using PTTA

We formulate RRPT as MIP in an aperiodic network.

$$\min \quad f^{\text{real}} = \sum_{a=(i_1,j_t)\in\mathcal{A}_{\text{train}}\cup\mathcal{A}_{\text{trans}}} w_a F_a \cdot K + Z \tag{RRPT-a}$$

$$\begin{aligned}
\text{s.t.} \quad & \pi_j - \pi_i + M(u_a - 1) \leq U_a && a = (i,j) \in \mathcal{A} && (26)\\
& \pi_j - \pi_i + M(1 - u_a) \geq L_a && a = (i,j) \in \mathcal{A} && (27)\\
& \pi_{i_s} - \pi_{i_{s-1}} = T && i_s \in \mathcal{E}, 2 \leq s \leq K + b && (28)\\
& \sum_{t: a'=(i_s,j_t)\in\mathcal{A}} u_{a'} = 1 && (i,j) \in \underline{\mathcal{A}}, 1 \leq s \leq K && (29)\\
& F_a \geq M(u_a - 1) + \pi_{j_t} - \pi_{i_1} && a = (i_1, j_t) \in \mathcal{A}_{\text{train}} \cup \mathcal{A}_{\text{trans}} && (30)\\
& \pi_{i_1} \leq T - 1 && i \in \underline{\mathcal{E}} && (31)\\
& x_i^r \geq \pi_i + d_i^r && i \in \mathcal{E}, r \in \mathcal{U} && (32)\\
& M'(1 - u_a) + x_j^r - x_i^r \geq L_a + d_a^r && a = (i,j) \in \mathcal{A}_{\text{train}}, r \in \mathcal{U} && (33)\\
& M'(1 - u_a) + M' y_a^r + x_j^r - x_i^r \geq L_a && a = (i,j) \in \mathcal{A}_{\text{trans}}, r \in \mathcal{U} && (34)\\
& M'(1 - p_{ij}^r) + x_j^r - x_i^r \geq L_a && a = (i,j) \in \mathcal{A}'_{\text{head}}, r \in \mathcal{U} && (35)\\
& p_{ij}^r + p_{ji}^r = 1 && (i,j),(j,i) \in \mathcal{A}'_{\text{head}}, r \in \mathcal{U} && (36)\\
& \sum_{a\in\mathcal{A}_{\text{trans}}} w_a y_a^r \leq Z_2^r && r \in \mathcal{U} && (37)\\
& \sum_{i_s\in\mathcal{E}: s\leq K} w_{i_s}(x_{i_s}^r - \pi_{i_s}) + \sum_{a\in\mathcal{A}_{\text{out}}} w_a H_a^r \leq Z_1^r && r \in \mathcal{U} && (38)\\
& Z_1^r + T Z_2^r \leq Z && r \in \mathcal{U} && (39)\\
& H_a^r \geq M''(u_a - 1) + x_j^r - \pi_j && a = (i,j) \in \mathcal{A}_{\text{out}}, r \in \mathcal{U} && (40)\\
& \pi_i \in \mathbb{N} && i \in \mathcal{E} && (41)\\
& F_a \geq 0 && a = (i_1, j_t) \in \mathcal{A}_{\text{train}} \cup \mathcal{A}_{\text{trans}} && (42)\\
& u_a \in \{0,1\} && a \in \mathcal{A} && (43)\\
& x_i^r \in \mathbb{N} && i \in \mathcal{E}, r \in \mathcal{U} && (44)\\
& y_a^r \in \{0,1\} && a \in \mathcal{A}_{\text{trans}}, r \in \mathcal{U} && (45)\\
& p_{ij}^r \in \{0,1\} && (i,j) \in \mathcal{A}'_{\text{head}}, r \in \mathcal{U} && (46)\\
& H_a^r \geq 0 && a \in \mathcal{A}_{\text{out}}, r \in \mathcal{U} && (47)
\end{aligned}$$

$$Z_1^r, Z_2^r \geq 0 \qquad r \in \mathcal{U} \tag{48}$$

$$Z \geq 0. \tag{49}$$

The objective function is the sum of the nominal travel time (i.e. the objective function of PTTA) and the worst-case delay. Constraints (26) to (31) are the same as in PTTA. The subsequent constraints are the constraints from DM adapted to our needs: (32) ensure that for every delay scenario and every event the time in the disposition timetable is not earlier than in the original timetable. Constraints (33) make sure that the delays are propagated along the driving and waiting activities for those arcs a fulfilling $u_a = 1$. Similarly, the delay propagation along maintained transfers is ensured by (34). The delay propagation along headway constraints is handled by (35). For this we need to determine for $(i,j),(j,i) \in \mathcal{A}_{\text{head}}$ in which order the events i and j take place. This is done by binary variables p_{ij}^r and (36). The number of missed transfers and the weighted event delay for every scenario are counted by (37) and (38), respectively, and the worst-case delay Z is determined in (39). Note that for the weighted event delay we count the weighted delay of every event within the planning horizon (i.e. those i_s with $s \leq K$) as well as the weighted delay of the arcs $\mathcal{A}_{\text{out}}$ leaving the planning horizon. For the latter we introduce a binary variable H_a^r which determines the delay at event j with $a = (i,j) \in \mathcal{A}_{\text{out}}$. To ensure that only those arcs with $u_a = 1$ are respected here, we need big-M-constraints given in (40).

Lemma 5 makes sure that we can find constants which are sufficiently large. The proof can be found in the appendix.

▶ **Lemma 5.** *There exist finite values for M' and M'' which are sufficiently large.*

Note that due to the periodicity of the timetable, also the assignment variables u are periodic (as shown in [9]), meaning that the values of those variables corresponding to activities in the first period determine the values for the later periods. Hence, we can obtain a reduced version with less variables. However, to simplify notation we use the full version.

4.2 Formulations using PESP

So far we have used the PTTA constraints and added delay management constraints to obtain a formulation for RRPT. An alternative approach is to use the PESP constraints and our knowledge from the development of the model PTTA to retrieve the assignment variables u from the PESP variables. We present two formulations:

The *event-based formulation* uses PESP, while the *cycle-base formulation* uses PESP-cb.

4.2.1 Event-based formulation

As shown in Lemma 1, if we have a feasible solution to PESP with $(\tilde{\pi}, z)$, setting

$$u_{a'} = \begin{cases} 1 & \text{if } t = z_a + s, \\ 0 & \text{otherwise}, \end{cases} \tag{50}$$

for $a' = (i_s, j_t), a' \in \mathcal{A}(a)$, yields a feasible PTTA solution. To formulate this as linear constraints, we again need big-M-constraints. Fortunately, for the big-M we can choose b, which is usually quite small (≈ 2). This yields the following formulation:

$$\min \quad f^{\text{real}} = \sum_{a=(i,j)\in\underline{\mathcal{A}}} w_a(\tilde{\pi}_j - \tilde{\pi}_i + z_a T) \cdot K + Z \tag{RRPT-pe}$$

$$\text{s.t.} \quad \tilde{\pi}_j - \tilde{\pi}_i + z_a T \leq U_a \qquad a = (i,j) \in \underline{\mathcal{A}} \tag{51}$$

$$\tilde{\pi}_j - \tilde{\pi}_i + z_a T \geq L_a \qquad a = (i,j) \in \underline{\mathcal{A}} \tag{52}$$

$$\pi_{i_s} - \tilde{\pi}_i = (s-1)T \qquad i \in \underline{\mathcal{E}}, 1 \leq s \leq K+b \tag{53}$$

$$b(1 - u_{a'}) + t - s - z_a \geq 0 \qquad a \in \underline{\mathcal{A}}, a' = (i_s, j_t) \in \mathcal{A}(a) \tag{54}$$

$$b(u_{a'} - 1) + t - s - z_a \leq 0 \qquad a \in \underline{\mathcal{A}}, a' = (i_s, j_t) \in \mathcal{A}(a) \tag{55}$$

$$(29), (32) - (40), (43) - (49)$$

$$\tilde{\pi}_i \in \mathbb{N}, 0 \leq \tilde{\pi}_i \leq T - 1 \qquad i \in \underline{\mathcal{E}} \tag{56}$$

$$z_a \in \mathbb{Z} \qquad a \in \underline{\mathcal{A}}. \tag{57}$$

The objective function minimises the real travel time. Constraints (51) and (52) are the regular PESP constraints. Constraints (53) ensure that the times for the rolled out events are set correctly. As stated in (50), the values of the modulo variables already determine the values of the assignment variables. This relation is accounted for in (54) and (55). The other constraints are taken from our previous formulation for RRPT-a.

▶ **Theorem 6.** *RRPT-a and RRPT-pe are equivalent.*

Proof. Let $(\pi, u, F, x, y, Z_1, Z_2, Z, H, p)$ be a solution to RRPT-a. Let $a = (i,j) \in \underline{\mathcal{A}}$. Choose the unique t such that $u_{(i_1,j_t)} = 1$, which exists due to (29), and define $z_a := t - 1$. For $i \in \underline{\mathcal{E}}$ set $\tilde{\pi}_i := \pi_{i_1}$. Note that since $0 \leq \pi_{i_1} \leq T-1$ it follows $0 \leq \tilde{\pi}_i \leq T-1$. We now show that $(\tilde{\pi}, z, \pi, u, x, y, Z_1, Z_2, Z, H, p)$ is feasible for RRPT-pe with the same objective value.

- $\tilde{\pi}_j - \tilde{\pi}_i + z_a T = \pi_{j_1} - \pi_{i_1} + (t-1)T = \pi_{j_t} - \pi_{i_1} \in [L_a, U_a]$ by choice of t and (26) and (27), which shows (51) and (52).
- Let $i \in \underline{\mathcal{E}}, 1 \leq s \leq K+b$. We have $\pi_{i_s} \overset{(28)}{=} \pi_{i_1} + (s-1)T = \tilde{\pi}_i + (s-1)T$, so (53) is satisfied.
- Let $a \in \underline{\mathcal{A}}, a' = (i_s, j_t) \in \mathcal{A}(a)$. We know from [9] that $u_{(i_s,j_t)} = u_{(i_1,j_{t-s+1})}$. Hence, if $u_{a'} = 1$, then also $u_{(i_1,j_{t-s+1})} = 1$, so by definition $z_a = t - s$. For $u_{a'} = 0$, note that by construction of $\mathcal{A}$ we have $0 \leq t - s \leq b$. Furthermore, it is well known from the literature on PESP that $0 \leq z_a \leq b$. Hence, it follows that $b + t - s - z_a \geq 0$ and $-b + t - s - z_a \leq 0$. This implies that (54) and (55) are fulfilled.
- All other constraints are clearly fulfilled.

Furthermore, as seen above, $\tilde{\pi}_j - \tilde{\pi}_i + z_a T = \pi_{j_t} - \pi_{i_1} \leq F_a$, with t such that $u_{(i_s,j_t)} = 1$, so the objective value of the constructed solution is not higher than that of the RRPT-a-solution. Let a solution $(\tilde{\pi}, z, \pi, u, x, y, Z_1, Z_2, H, p)$ to RRPT-pe be given. In particular, $(\tilde{\pi}, z)$ is a solution to PESP. By (54) and (55) $u_{(i_s,j_t)} = 1$ is only possible for $t = s + z_a$. Together with (29), even equivalence holds, i.e. $u_{(i_s,j_t)} = 1$ if and only if $t = s + z_a$. If we additionally set

$$F_a = \begin{cases} \pi_{j_t} - \pi_{i_1}, & u_{(i_1,j_t)} = 1 \\ 0, & \text{otherwise} \end{cases}$$

we know from Lemma 1 that (π, u, F) is feasible for PTTA, i.e. (26)-(31) are fulfilled. Since all other constraints are fulfilled as well, $(\pi, u, F, x, y, Z_1, Z_2, Z, H, p)$ is feasible for RRPT-a. Lemma 1 says that $\tilde{f} = K \cdot f$, hence the equality of the objective function values follows. ◀

4.2.2 Cycle-base Formulation

The cycle-base formulation is computationally superior for solving PESP. This motivates to use it also for RRPT. However, since we need the times of the events, and these are not present in the cycle-base formulation, we have to extract them from the tensions. We first need some notation.

▶ **Notation 7.** *Let $\mathcal{T}$ be a spanning tree in $\underline{\mathcal{N}} = (\underline{\mathcal{E}}, \underline{\mathcal{A}})$ and $\hat{\imath} \in \underline{\mathcal{E}}$ some fixed event. For $i \in \mathcal{E}$ let $\mathcal{P}_i$ be the unique path from $\hat{\imath}$ to i in $\mathcal{T}$. The set of arcs in $\mathcal{P}_i$ can be partitioned into the sets $\mathcal{P}_i^+$ and $\mathcal{P}_i^-$ of forward and backward arcs.*

Now, if we have a feasible solution to PESP given by the tensions ξ of the activities, we can use these to obtain the time for every event i by adding respectively subtracting the tensions along the path $\mathcal{P}_i$. Namely, for some $\hat{q}_i \in \mathbb{Z}$ we have:

$$\tilde{\pi}_i = \sum_{a \in \mathcal{P}_i^+} \xi_a - \sum_{a \in \mathcal{P}_i^-} \xi_a + \tilde{\pi}_{\hat{\imath}} + \hat{q}_i T.$$

Let q be the modulo variables of the tension ξ. We can use q and $\hat{q}$ to obtain the modulo parameters z in the formulation RRPT-pe, namely, as we will see in the proof of Lemma 8,

$$z_a = \begin{cases} \hat{q}_i - \hat{q}_j, & a \in \mathcal{T} \\ \hat{q}_i - \hat{q}_j + q_a, & a \notin \mathcal{T}. \end{cases} \tag{58}$$

Then, as before, we also know the values of the assignment variables u, which leads to the following IP formulation:

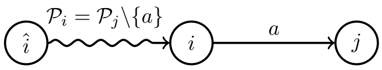
$$\begin{aligned}
\min \quad & f^{\text{real}} = \sum_{a=(i,j) \in \underline{\mathcal{A}}} w_a \xi_a \cdot K + Z && && \text{(RRPT-cb)} \\
\text{s.t.} \quad & \Gamma\xi = Tq && && (59) \\
& \tilde{\pi}_i = \sum_{a \in \mathcal{P}_i^+} \xi_a - \sum_{a \in \mathcal{P}_i^-} \xi_a + \tilde{\pi}_{\hat{\imath}} + \hat{q}_i T && i \in \underline{\mathcal{E}} && (60) \\
& b(1 - u_{a'}) + t - s - \hat{q}_i + \hat{q}_j \geq 0 && a \in \mathcal{T}, a' = (i_s, j_t) \in \mathcal{A}(a) && (61) \\
& b(u_{a'} - 1) + t - s - \hat{q}_i + \hat{q}_j \leq 0 && a \in \mathcal{T}, a' = (i_s, j_t) \in \mathcal{A}(a) && (62) \\
& b(1 - u_{a'}) + t - s - \hat{q}_i + \hat{q}_j - q_a \geq 0 && a \in \underline{\mathcal{A}} \backslash \mathcal{T}, a' = (i_s, j_t) \in \mathcal{A}(a) && (63) \\
& b(u_{a'} - 1) + t - s - \hat{q}_i + \hat{q}_j - q_a \leq 0 && a \in \underline{\mathcal{A}} \backslash \mathcal{T}, a' = (i_s, j_t) \in \mathcal{A}(a) && (64) \\
& (29), (32) - (40), (43) - (49), (53), (56) && && \\
& \xi_a \in \mathbb{N}, L_a \leq \xi_a \leq U_a && a \in \underline{\mathcal{A}} && (65) \\
& q_a \in \mathbb{Z} && a \in \underline{\mathcal{A}} \backslash \mathcal{T} && (66) \\
& \hat{q}_i \in \mathbb{Z} && i \in \underline{\mathcal{E}}. && (67)
\end{aligned}$$

The objective function minimises the real travel time. Constraint (59) ensures that ξ is indeed a periodic tension (as (5) in the PESP cycle base formulation). Constraints (60) construct the event times from the tensions. The correspondence between $u, q, \hat{q}$ is respected in (61) to (64). The other constraints are the same as in the formulation of RRPT-pe.

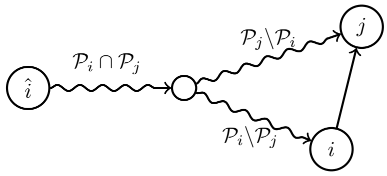
▶ **Theorem 8.** *RRPT-pe and RRPT-cb are equivalent.*

Proof. Let $(\tilde{\pi}, z, \pi, u, x, y, Z_1, Z_2, Z, H, p)$ be a solution to RRPT-pe. In particular, $(\tilde{\pi}, z)$ is a solution to PESP. We construct a solution to RRPT-cb:

- From the literature on PESP we know that by setting $\xi_a := \tilde{\pi}_j - \tilde{\pi}_i + z_a T$ and $q_a := \sum_{a' \in C_a^+} z_{a'} - \sum_{a' \in C_a^-} z_{a'}$ we obtain a periodic tension ξ such that $\Gamma\xi = Tq$, i.e. (59) holds.
- We define $\hat{q}_i := \sum_{a \in \mathcal{P}_i^-} z_a - \sum_{a \in \mathcal{P}_i^+} z_a$. By induction on the length of the unique path $\mathcal{P}_j$ from $\hat{\imath}$ to j in $\mathcal{T}$ we obtain $\tilde{\pi}_j = \tilde{\pi}_{\hat{\imath}} + \sum_{a \in \mathcal{P}_i^+} (\xi_a - z_a T) + \sum_{a \in \mathcal{P}_i^-} (-\xi_a + z_a T) = \tilde{\pi}_{\hat{\imath}} + \sum_{a \in \mathcal{P}_i^+} \xi_a - \sum_{a \in \mathcal{P}_i^-} \xi_a + \hat{q}_i T$, which shows (60).



(a) Case $(i,j) \in \mathcal{T}$



(b) Case $(i,j) \notin \mathcal{T}$

Figure 2 Paths $\mathcal{P}_i$ and $\mathcal{P}_j$ in the proof of Theorem 8.

- For $a = (i,j) \in \mathcal{T}$ it holds $\mathcal{P}_j^+ = \mathcal{P}_i^+ \cup \{a\}$, $\mathcal{P}_j^- = \mathcal{P}_i^-$ (see Figure 2a) and hence $\hat{q}_i - \hat{q}_j = z_a$, so by (54) and (55) also (61) and (62) are fulfilled.
- Furthermore, note that for $a = (i,j) \in \mathcal{A} \backslash \mathcal{T}$ we have $C_a^+ = (\mathcal{P}_i^+ \backslash \mathcal{P}_j) \cup (\mathcal{P}_j^- \backslash \mathcal{P}_i) \cup \{a\}$ and $C_a^- = (\mathcal{P}_j^+ \backslash \mathcal{P}_i) \cup (\mathcal{P}_i^- \backslash \mathcal{P}_j)$ (see Figure 2b). Hence, we get $\hat{q}_i - \hat{q}_j + q_a = (\sum_{a' \in \mathcal{P}_i^-} z_{a'} - \sum_{a' \in \mathcal{P}_i^+} z_{a'}) - (\sum_{a' \in \mathcal{P}_j^-} z_{a'} - \sum_{a' \in \mathcal{P}_j^+} z_{a'}) + (\sum_{a' \in C_a^+} z_{a'} - \sum_{a' \in C_a^-} z_{a'}) = z_a$, so also (63) and (64) are satisfied.
- Constraints (65) to (67) are trivially fulfilled.

On the other hand, let $(\xi, \tilde{\pi}, q, \hat{q}, \pi, u, x, y, Z_1, Z_2, Z, H, p)$ be a solution to (RRPT-cb). We construct a solution to RRPT-pe as follows: For $a = (i,j) \in \underline{\mathcal{A}}$ we define z_a as in (58).

- For $a = (i,j) \in \underline{\mathcal{A}}$ by (60) we have

$$\tilde{\pi}_j - \tilde{\pi}_i = (\sum_{a \in \mathcal{P}_j^+} \xi_a - \sum_{a \in \mathcal{P}_j^-} \xi_a + \tilde{\pi}_{\hat{i}} + \hat{q}_j T) - (\sum_{a \in \mathcal{P}_i^+} \xi_a - \sum_{a \in \mathcal{P}_i^-} \xi_a + \tilde{\pi}_{\hat{i}} + \hat{q}_i T).$$

  - For the case $a \in \mathcal{T}$ this term simplifies to $\xi_a + (\hat{q}_j - \hat{q}_i)T = \xi_a - z_a T$.
  - If $a \notin \mathcal{T}$, this is equal to $\sum_{a' \in C_a^-} \xi_{a'} - (\sum_{a' \in C_a^+} \xi_{a'} - \xi_a) + (\hat{q}_j - \hat{q}_i)T = \xi_a - (\Gamma\xi)_a + (\hat{q}_j - \hat{q}_i)T = \xi_a - (q_a - \hat{q}_j + \hat{q}_i)T = \xi_a - z_a T$.

  Hence, in both cases we have $\tilde{\pi}_j - \tilde{\pi}_i + z_a T = \xi_a \in [L_a, U_a]$, which shows (51) and (52).
- Constraints (54) and (55) are satisfied by definition of z and constraints (61) to (64).

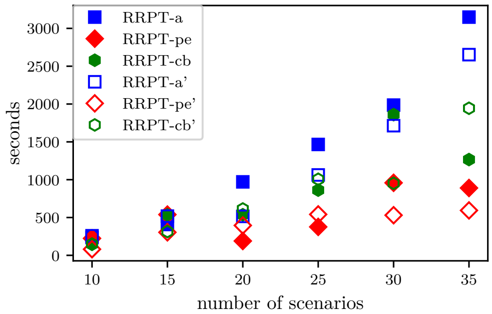
In both constructions the objective function values coincide, which completes the proof. ◀

5 Computational Experiments

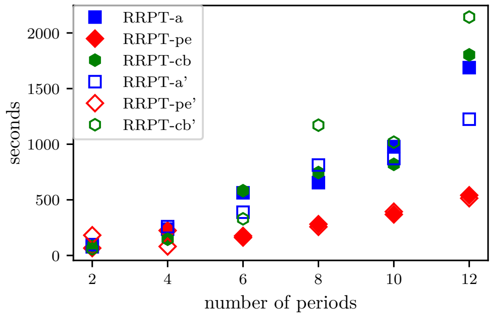
In the previous section, we derived three equivalent formulations for the recoverable robust periodic timetabling problem. An obvious question is which one of these formulations is best. To answer this, we run some experiments and compare their computing times for solving the MIP. However, RRPT is a very hard problem: PESP and delay management both are NP-hard ([28, 5]) and RRPT integrates PESP and several delay management problems. Therefore, we are not able to solve RRPT on any large instances. For the experiments we thus use a rather small network from the LinTim library [25] with 156 periodic events and 188 periodic activities without headway constraints. The period length is 60 minutes. For the delay scenarios we generated uniformly distributed source delays: in every scenario we generated a source delay between 1 and 15 minutes for 1% of all *aperiodic* events and activities.

We implemented the MIP formulations in Python and solved them using Gurobi 8.1.1 [10] on a compute server with 48 cores @2.9 GHz and 196GB RAM. The MIP optimality gap of the solver was set to 0.015%.

We ran two different experiments: one where the number K of periods is fixed to 4 with varying number of scenarios $|\mathcal{U}|$ and one with $|\mathcal{U}| = 10$ and varying number of periods. These numbers are quite small, which is due to the high complexity of the problem. However, the experiments provide some insights on the performance of the different formulations. For



(a) Computing times for $K = 4$ with increasing number of scenarios $|\mathcal{U}|$.



(b) Computing times for $|\mathcal{U}| = 10$ with increasing number of periods K.

Figure 3 Computing times for solving MIP formulations.

Table 1 Nominal travel time and worst-case delay of RRPT compared to sequential approach for $K = 4$.

# scenarios	10	15	20	25	30	35
increase of nominal travel time (%)	0.78	0.27	1.87	1.87	1.87	1.87
decrease of delay (%)	11.80	4.92	24.85	24.85	24.85	24.85

every formulation we also test its reduced version with less variables, which is indicated by an apostrophe behind the name.

The results are shown in Figure 3. For the first experiment we can see that RRPT-a has the highest computing times. This is not surprising, since it is based on PTTA, which is slower than PESP for the pure timetabling problem. The lowest computing times are achieved for RRPT-pe. Since the cycle-base formulation PESP-cb is faster than the standard formulation when only looking for a timetable, this is a bit surprising. However, RRPT-cb not only uses variables for the tensions, but additionally also for the event times, since they are needed for the delay management part. This could be an explanation for the worse performance. The variable reduction does not have an significant effect on the computing time.

For the second experiment, we have similar results. RRPT-pe performs best, while for RRPT-a and RRPT-cb the computing times become much larger with increasing K.

Compared to an sequential approach, i.e. first fixing the timetable and then doing delay management afterwards, we expect that RRPT yields solutions with a higher nominal travel time and lower delays (due to added buffer times on the activities). Indeed, this behaviour can be observed in Tables 1 and 2. For the same instances as in the previous experiment we can see that the nominal travel time increases by up to 3.77%. On the other hand, the worst-case delay decreases by 4.92% to 24.85%, meaning the delay can be reduced significantly by adding only small buffer times.

Table 2 Nominal travel time and worst-case delay of RRPT compared to sequential approach for $|\mathcal{U}| = 10$.

# periods	2	4	6	8	10	12
increase of nominal travel time (%)	3.77	0.78	0.22	0.33	0	0.27
decrease of delay (%)	17.95	11.80	7.56	5.67	9.21	5.17

6 Conclusion

We have introduced the Recoverable Robust Periodic Timetabling Problem, which is the first to apply the concept of recoverable robustness to periodic timetabling with aperiodic source delays. We have developed three equivalent formulations based on different ways to incorporate the timetabling subproblem. We have compared the formulations with respect to their computing time when solving them with a state-of-the-art solver, showing that - as opposed to the pure timetabling problem - a cycle-base approach is not the best choice. By comparing the solutions to those of the standard sequential approach, we have shown that our model manages to find solutions with significantly less delay at the cost of only a small increase in the nominal travel time. Further experiments are subject to ongoing research, in particular on instances with headway constraints.

Due to the high complexity of the problem, the IP formulation is only able to handle rather small instances. Hence, developing heuristic approaches for the problem could be a promising direction for further research. Another interesting question is how the model performs compared to models using other robustness concepts with respect to solution quality. Since RRPT focusses on the real travel time, the obtained timetables should be beneficial for the passengers compared to timetables which were computed using different models, see [8].

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A Proofs

Proof of Lemma 5

Proof. We choose

$$M' := \max_{r\in\mathcal{U}}(\max_{a\in\mathcal{A}}(L_a + d_a^r) + \max_{i\in\mathcal{E}} d_i^r + \sum_{a\in\mathcal{A}} d_a^r) + K \cdot T + \frac{T \cdot |\mathcal{A}_{\text{head}}|}{2}$$

and

$$M'' := \max_{r\in\mathcal{U}}(\max_{i\in\mathcal{E}} d_i^r + \sum_{a\in\mathcal{A}} d_a^r) + \frac{T\cdot|\mathcal{A}_{\text{head}}|}{2}.$$

Let (π, u, F) be a feasible solution to the subproblem PTTA given by constraints (26) to (31), (41), (43), (42). For some fixed $r \in \mathcal{U}$ we consider the constraints (33) and (34) for those $a \in \mathcal{A}$ with $u_a = 1$ and (32), (44), (45). These are the constraints of the delay management problem, for which it is known (see [24]) that there is an optimal solution (x^r, y^r) fulfilling

$$x_i^r - \pi_i \leq \max_{i\in\mathcal{E}} d_i^r + \sum_{a\in\mathcal{A}} d_a^r + \sum_{\substack{a=(i,j)\in\mathcal{A}_{\text{head}}:\\ \pi_i>\pi_j}} (\pi_i - \pi_j + L_a). \tag{68}$$

We consider the last term in (68):

$$\sum_{\substack{a=(i,j)\in\mathcal{A}_{\text{head}}:\\ \pi_i>\pi_j}} (\underbrace{\pi_i - \pi_j}_{\leq T-L_a} + L_a) \leq \sum_{\substack{a=(i,j)\in\mathcal{A}_{\text{head}}:\\ \pi_i>\pi_j}} T \leq \frac{T\cdot|\mathcal{A}_{\text{head}}|}{2}.$$

Hence, we obtain

$$x_i^r - \pi_i \leq \max_{i\in\mathcal{E}} d_i^r + \sum_{a\in\mathcal{A}} d_a^r + \frac{T\cdot|\mathcal{A}_{\text{head}}|}{2}. \tag{69}$$

We set

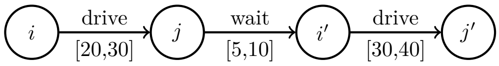
$$H_a^r = \begin{cases} x_j^r - \pi_j & \text{if } u_a = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Note that by choice of M'' (69) implies that this is feasible. Furthermore, we set Z_1^r, Z_2^r and Z according to the left-hand side of (37) to (39). Then (37) to (40), (47) and (48) are also fulfilled. Hence, it remains to show constraints (33) and (34) for those $a \in \mathcal{A}$ with $u_a = 0$ and (35). For $r \in \mathcal{U}$ we have

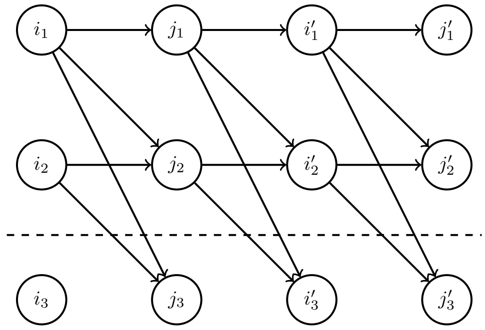
$$\begin{aligned}
&L_a + d_a^r - x_j^r + x_i^r \\
\leq &L_a + d_a^r + x_i^r \\
\leq &L_a + d_a^r + K\cdot T + \max_{i\in\mathcal{E}} d_i^r + \sum_{a'\in\mathcal{A}} d_{a'}^r + \frac{T\cdot|\mathcal{A}_{\text{head}}|}{2} \\
\leq &M'.
\end{aligned}$$

◀

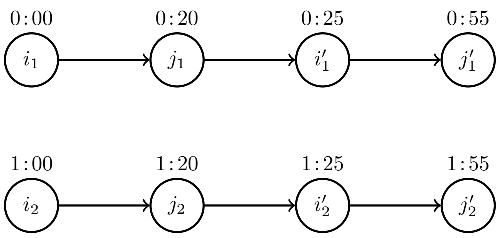
B Figures



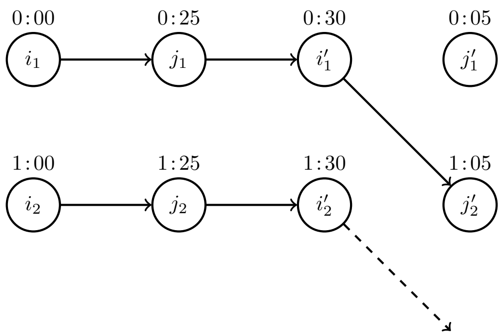
(a) Periodic EAN with $[L_a, U_a]$ given below the arcs.



(b) EAN rolled out with all potential activities for $K = 2, b = 1$.



(c) Rolled out EAN after choosing a feasible timetable and the corresponding activities.



(d) Rolled out EAN with another feasible timetable, which results in different activities.

Figure 4 Rolling out a periodic EAN without knowing the timetable for $T = 60$ and $K = 2$.