Non-Linear Charge Functions for Electric Vehicle Scheduling with Dynamic Recharge Rates

Fabian Löbel[1]
Zuse Institute Berlin, Germany

Ralf Borndörfer
Zuse Institute Berlin, Germany

Steffen Weider
LBW Optimization GmbH, Berlin, Germany

Abstract

The ongoing electrification of logistics systems and vehicle fleets increases the complexity of associated vehicle routing or scheduling problems. Battery-powered vehicles have to be scheduled to recharge in-service, and the relationship between charging time and replenished driving range is non-linear. In order to access the powerful toolkit offered by mixed-integer and linear programming techniques, this battery behavior has to be linearized. Moreover, as electric fleets grow, power draw peaks have to be avoided to save on electricity costs or to adhere to hard grid capacity limits, such that it becomes desirable to keep recharge rates dynamic. We suggest a novel linearization approach of battery charging behavior for vehicle scheduling problems, in which the recharge rates are optimization variables and not model parameters.

2012 ACM Subject Classification Mathematics of computing → Integer programming; Mathematics of computing → Linear programming; Applied computing → Transportation

Keywords and phrases Electric Vehicle Scheduling, Battery Powered Vehicles, Charging Process, Non-linear Charging, Recharge Modeling, Dynamic Recharge Rate

Digital Object Identifier 10.4230/OASIcs.ATMOS.2023.15

Category Short Paper

Funding This work has been conducted within the Research Campus MODAL funded by the German Federal Ministry of Education and Research (BMBF) (fund number 05M20ZBM).

1 Introduction and Problem Overview

The *Electric Vehicle Scheduling Problem (EVSP)* extends the classic *Vehicle Scheduling Problem* to include the scheduling of recharge events such that vehicle batteries are never fully depleted in-service. European operators prefer to recharge their vehicles at depots and fast chargers at selected locations to minimize infrastructure acquisition costs (cf. [3]). The amount of replenished charge depends non-linearly on the charging time and the initial *state of charge* (*soc*). Moreover, as electric fleet sizes continue to grow, operators have begun adopting active charge management tools which may not always recharge at full capacity. Power grid limitations at the depot can bound the total admissible electricity usage over time, pricing schemes may incentivize smoothing out peak loads, and there are even case studies with bi-directional chargers where electric bus batteries fed charge back into the grid (cf. [5]), effectively applying negative charge rates. Therefore, on top of the non-linear charging behavior, vehicle scheduling procedures have to take dynamic recharge rates into account instead of a priori fixing them to the highest available rate.

[1] Corresponding author.

23rd Symposium on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems (ATMOS 2023).
Editors: Daniele Frigioni and Philine Schiewe; Article No. 15; pp. 15:1–15:6
OpenAccess Series in Informatics
OASICS Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

To the best of our knowledge, [6] is the only paper on electric vehicle scheduling to explicitly treat the recharge rate as a decision variable to bound the total simultaneous power draw. However, like in the majority of the available literature (cf. surveys [2, 10]), a linear charging behavior is assumed, which may cause solutions to be infeasible in practice or to underestimate driving ranges depending on the exact data used [8].

We have identified three approaches in the EVSP literature to take non-linear battery behavior into account. [11] and [4] suggest an energy (state) expansion in analogy to the well-known time expansion. While not mentioned by either authors, one could incorporate recharge rate decisions by allowing connections between different charge states at recharge facilities. Naturally, this comes at the cost of an exponential increase in problem size.

A branch-and-check procedure is proposed in [1] for an EVSP application of tow trains at a factory which seem return to their charging facility after every tow duty. Once a master problem finds an integer solution, charge states are explicitly computed from exact charge functions along vehicle courses and infeasible solutions are cut off via subtour elimination constraints. It is unclear how this approach extends to general EVSP settings where vehicles have to be explicitly scheduled for detours to reach charging facilities.

Originally proposed in [7], and adopted by a growing number of publications, is a piecewise linear approximation of a function that maps the time spent charging an empty battery to the resulting *soc*. This approach has the advantage that it is easy to incorporate into MILPs by standard techniques, but it does not allow dynamic recharge rates.

In this paper we develop a linearization technique for battery charging behavior with dynamic recharge rates from a general battery model.

2 Recharging a Battery

According to the literature, e.g., [9], batteries under load have a *terminal voltage*

$$V_{term} = V_{OC}(y) - R \cdot I \tag{1}$$

depending on the *soc* y and the current I. The *open circuit voltage* V_{OC} and R are properties of individual batteries. Note that charging currents are negative by convention, so $V_{term} \geq V_{OC}$. In general, V_{OC} is a monotonically increasing non-linear function of the *soc*.

Chargers must keep battery voltage and charging current within safety limits V_{max} and $-I_{max}$, moreover, operating near those limits accelerates battery aging. Consequently, vehicle batteries are usually replenished following a *Constant Current - Constant Voltage* (*CC-CV*) charging scheme. Initially, a roughly constant current is applied causing the *soc* to increase nearly linearly accompanied by a rising terminal voltage. Once V_{term} hits some threshold, at most V_{max}, the charger switches to a constant voltage phase, limiting the maximum incoming current as given in (1) by fixing V_{term} to the threshold value. Note that a high initial current will force this cut-off to happen earlier, causing more time to be spend in the slower CV phase. The battery is defined to be full once the maximum charging current permitted by the battery voltage drops below some minimum dictated by the battery and charger combination.

In the EVSP literature on non-linear charging, this behavior is usually modeled using a *charge curve* that maps the time t spent charging an initially empty battery at presumably full power to the final *soc*. In accordance with the CC-CV charging scheme, the charge curve is initially linear until some time t_V and then monotonically and concavely grows towards the maximum *soc* as the charge rate decreases during the CV phase.

Since we wish to keep the charge rate dynamic, we think of charge curves as solutions of differential inequalities.

▶ **Definition 1.** *A* charging power profile *is a function* $f : [0,1] \to [0,1]$ *that maps the (relative) battery* soc *to the (relative) maximal charge rate, and that is of the form*

$$f(y) = \begin{cases} f_{CC}, & y < y_V \\ f_{CV}(y), & y \geq y_V, \end{cases} \tag{2}$$

where f_{CV} *is differentiable, monotonically non-increasing, and satisfies* $f_{CV}(y_V) = f_{CC}$*,* $f_{CV}(1) = 0$*.*

▶ **Example 2.** Equation (1) for the terminal voltage yields the power profile

$$f(y) = \min\left(\frac{-R \cdot I_{max}}{\mathcal{K}}, \frac{V_{max} - V_{OC}(y)}{\mathcal{K}}\right) \tag{3}$$

where $\mathcal{K}$ scales the *soc* to be within $[0,1]$. Note that f need not be differentiable in y_V.

▶ **Definition 3.** *Given a charging power profile* f*, a* charge curve *mapping time to* soc *is a differentiable function* $\xi : [0,\infty) \to [0,1]$ *satisfying*

(i) $\xi(0) = 0$*,*
(ii) *there exists* $t_{full} > 0$ *such that* $\xi(t) = 1$ *for* $t \geq t_{full}$*,*
(iii) $0 < \xi'(t) \leq f(\xi(t))$ *for all* $t \in [0, t_{full})$*.*

▶ **Observation 4.** *Charge curves are bijective as functions from* $[0, t_{full}]$ *to* $[0,1]$*.*

▶ **Definition 5.** *The* maximum power charge curve ζ *is the charge curve satisfying condition* (iii) *at equality, i.e,* ζ *is the unique solution to the autonomous non-linear ordinary differential equation* $\zeta' = f(\zeta)$ *with boundary conditions* (i) *and* (ii)*.*

In general, there is no closed form for ζ on the CV segment and it has to be determined empirically or computed numerically from a given f. We want to emphasize the distinct interpretations of f, ξ and ξ': f yields the maximum permissible rate the charger may apply to the battery at its current *soc*. It is a property of the battery and charger combination and needs to be fixed a priori as part of the model. The charge curve derivative ξ' gives the actually applied charge rate at time t of a particular charging process. By condition (iii), throttling the rate is explicitly allowed. Subsequently, ξ gives the resulting *soc* of charging an initially empty battery for t time units at the rate its derivative specifies. See Figure 1 for an example of these three functions with $\xi = \zeta$.

The current state-of-the-art in the literature of using a piecewise linear interpolation of ζ does not permit charge rate throttling. Conceivable generalizations to incorporate dynamic recharge rates into such a model, i.e., treating the choice of ξ as a decision variable for every

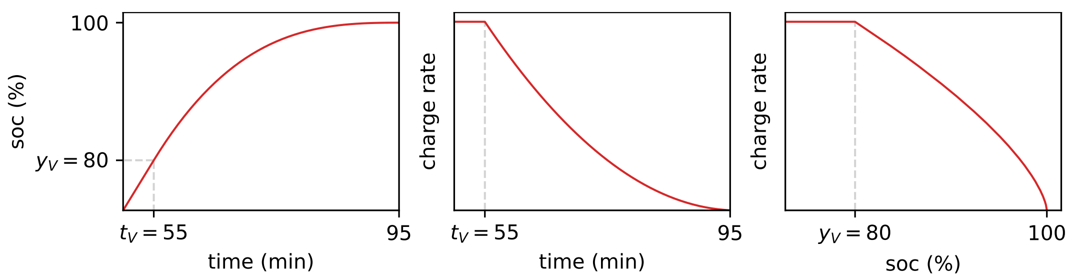

Figure 1 CV segment of an example ζ, ζ', and the underlying profile f (left to right) modeled after real electric bus fast charging data. The linear CC segment is mostly cropped out.

recharge event, seem to be cumbersome. Moreover, if a piecewise linear approximation of ζ is used, its derivative is piecewise constant and the model therefore can only consider a discrete set of charge rates, which is inadequate for the CV phase. Furthermore, since the piecewise constant derivative periodically overestimates the actual charge rate, approximate models may underestimate recharge durations and overestimate final charge states.

3 Recharge Modeling with Dynamic Rates

Consider a recharge event and let $[t_s, t_e]$ be its time interval and y_s the initial *soc* at t_s. Let Δ be a function operator working on charge curves as

$$\Delta\xi(y,t) = \xi(\xi^{-1}(y) + t) - y; \tag{4}$$

which is well-defined by Observation 4 if we choose $\xi^{-1}(1) = t_{full}$. $\Delta\xi$ gives the difference between the final charge state reached from an initial charge state y of a charging process of duration t. Consequently, the final *soc* at t_e is then $\Delta\xi(y_s, t_e - t_s) + y_s$.

$\Delta\xi$ can be evaluated iteratively.

▶ **Lemma 6.** *For a charge curve ξ, let $y_0 = y_s$ and $\theta_i > 0$, $y_i = y_{i-1} + \Delta\xi(y_{i-1}, \theta_i)$ for $i = 1, \dots, k$. Then $\Delta\xi(y_s, \sum_{i=1}^{k} \theta_i) = y_k - y_s$.*

Proof. By induction: For $k = 1$ the claim is trivially true, so let $k > 1$ and exercise

$$y_k - y_s = y_{k-1} + \Delta\xi(y_{k-1}, \theta_k) - y_s = \Delta\xi(y_s, \sum_{i=1}^{k-1} \theta_i) + \Delta\xi(\Delta\xi(y_s, \sum_{i=1}^{k-1} \theta_i) + y_s, \theta_k)$$

$$= \xi(\xi^{-1}(\xi(\xi^{-1}(y_s) + \sum_{i=1}^{k-1} \theta_i)) + \theta_k) - y_s = \xi(\xi^{-1}(y_s) + \sum_{i=1}^{k} \theta_i) - y_s = \Delta\xi(y_s, \sum_{i=1}^{k} \theta_i), \tag{5}$$

where the second line is the result of applying (4) to the outer and then inner occurrence of $\Delta\xi$ in the rightmost term on the first line. ◀

Moreover, if the time steps θ_i admit an equidistant discretization θ, the *soc* y_i at the end of any time step is $y_{i-1} + \Delta\xi(y_{i-1}, \theta)$ from the immediately preceding charge state. By fixing the step size we obtain a unary recharge function depending solely on the initial *soc* and we may write $\Delta\xi(y) = \Delta\xi(y, \theta)$.

▶ **Lemma 7.** *Let ζ be a maximum power charge curve w.r.t. a charging power profile f. Then for fixed time step $\theta > 0$, $\Delta\xi(y) \leq \Delta\zeta(y)$ for every $y \in [0, 1]$ and charge curve ξ of f.*

Proof. Monotonicity of f and the mean value theorem assert the claim by contradiction. ◀

▶ **Corollary 8.** *$\xi \leq \zeta$ for every charge curve ξ of f.*

This formally asserts the intuitive observation that maximizing the charge rate does maximize the obtained charge state. More importantly, Lemma 7 guarantees that the computation scheme justified with Lemma 6 can be modified by introducing *charge increment variables* φ_i such that $y_i = y_{i-1} + \varphi_i$ and $\varphi_i \leq \Delta\zeta(y_{i-1})$. Any sequence of positive φ_i can be associated with a sequence of $\Delta\xi(y_i)$ for some charge curve ξ and vice versa. Additionally, we may allow φ_i to become zero to temporarily suspend charging or possibly even negative if charge is fed back into the grid, although we will assume $\varphi_i \geq 0$ for the remainder of this paper. Furthermore, we can incorporate φ_i into the objective function to consider time or peak-dependent pricing, and we can add φ_i to additional constraints limiting the total power draw per time step.

▶ **Definition 9.** *For a maximum power charge curve ζ, let $\Phi := \{(y, \varphi) \in [0,1]^2 \mid \varphi \leq \Delta\zeta(y)\}$ be the* charge increment variable domain*.*

▶ **Observation 10.** *Since $\Delta\zeta(y) \leq 1 - y$ and $0 \leq \Delta\zeta(0) \leq \theta f_{CC}$, Φ is the intersection of the triangle spanned by the unit vectors in the upper right quadrant of the two-dimensional plane and the area below the graph of $\Delta\zeta$, see Figure 2 for an illustration.*

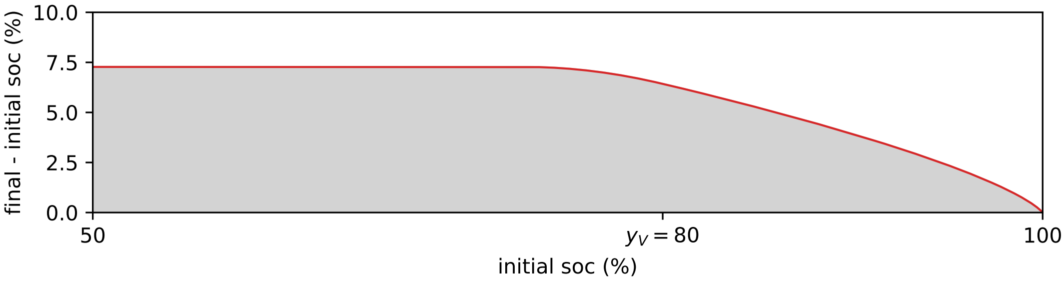

Figure 2 The shaded area is Φ on $[0.5, 1]$ for the charge curve presented in Figure 1 with $\theta = 5$min.

In order to obtain a linearization of the charging process, we need to approximate Φ by a polygon. In particular, we have to approximate the function graph of $\Delta\zeta$ with a concave piecewise linear function so that we can replace $\varphi_i \leq \Delta\zeta(y_{i-1})$ by linear inequalities. The left- and rightmost linear segment of such an approximation might be $\varphi_i \leq \theta f_{CC}$ and $\varphi_i \leq 1 - y_{i-1}$. Fitting the rest of the boundary in general requires finding an acceptable trade-off between approximation accuracy and possible charge state overestimation.

▶ **Theorem 11.** *Let ζ be a maximum power charge curve w.r.t. a charging power profile f. Furthermore, let f_{CV} be concave. Then the corresponding charge increment variable domain Φ is convex for any time step size $\theta > 0$.*

Proof. By Observation 10 it suffices to show that $\Delta\zeta$ is concave or equivalently, $\frac{\partial^2}{\partial y^2}\Delta\zeta \leq 0$. Computing the second derivative (note that ζ' is differentiable almost everywhere) we see

$$\frac{\partial^2}{\partial y^2}\left(\zeta(\zeta^{-1}(y) + \theta) - y\right) = \frac{\zeta'(\zeta^{-1}(y))\zeta''(\zeta^{-1}(y) + \theta) - \zeta''(\zeta^{-1}(y))\zeta'(\zeta^{-1}(y) + \theta)}{\zeta'(\zeta^{-1}(y))^3} \tag{6}$$

and plugging in $\zeta'(t) = f(\zeta(t))$ and $\zeta''(t) = f'(\zeta(t))f(\zeta(t))$ we obtain

$$\frac{\partial^2}{\partial y^2}\Delta\zeta(y) = \left(f'(\zeta(\zeta^{-1}(y) + \theta)) - f'(y)\right)\frac{f(\zeta(\zeta^{-1}(y) + \theta))}{f(y)^2}. \tag{7}$$

Since f is non-negative, the sign of $\frac{\partial^2}{\partial y^2}\Delta\zeta$ is entirely dictated by $f'(\zeta(\zeta^{-1}(y) + \theta)) - f'(y)$. Note that $y = \zeta(\zeta^{-1}(y)) \leq \zeta(\zeta^{-1}(y) + \theta)$ because ζ is monotonically increasing. Moreover, since we have $f' \equiv 0$ approaching y_V from the left and $f'_{CV} \leq 0$ by Definition 1, concavity of f_{CV} extends to the entirety of f. Thus, f' is monotonously non-increasing on its entire domain and we obtain $f'(y) \geq f'(\zeta(\zeta^{-1}(y) + \theta))$. Therefore, $\frac{\partial^2}{\partial y^2}\Delta\zeta \leq 0$ and Φ is convex. ◀

Hence, a straightforward piecewise linear interpolation yielding an inequality of the form $\varphi_i \leq m y_{i-1} + b$ per linear segment will do if f is concave. These inequalities can be incorporated directly into any mixed-integer linear program for the EVSP. By Lemma 7 and Theorem 11, computing the *soc* via the φ_i along a recharge event is then guaranteed to never overestimate the final *soc* and the approximation error at the boundary is well understood numerically.

▶ **Proposition 12.** *Let $\Delta\tilde{\zeta}$ be a piecewise linear spline interpolation of $\Delta\zeta$. Then the approximation error is*

$$\left\|\Delta\zeta - \Delta\tilde{\zeta}\right\| \leq \frac{\theta h^2}{8} \left\|f''_{CV}\right\| \tag{8}$$

where h is the width of the largest linear segment.

Proof. It is a well-known result that the linear spline approximation error is bounded by $h^2 \left\|(\Delta\zeta)''\right\|/8$. Using Taylor approximation for some $t^* \in (\zeta^{-1}(y), \zeta^{-1}(y) + \theta)$,

$$f'(\zeta(\zeta^{-1}(y) + \theta)) = f'(y) + \theta \frac{\partial}{\partial t} f'(\zeta(t^*)) = f'(y) + \theta f''(\zeta(t^*)) f(\zeta(t^*)) \tag{9}$$

and plugging into (7) yields $\left\|(\Delta\zeta)''\right\| \leq \theta \left\|f\right\|^2 \left\|f''\right\| / \left\|f\right\|^2 = \theta \left\|f''\right\|$. ◀

The key observation enabling the approach presented in this paper is that while the maximum charge rate as a function of time (given by ζ') is usually convex during the CV phase, there are reasonable battery models where the rate as a function of battery *soc* (given by f) is concave (see Figure 1) and the shape of $\Delta\zeta$ and thus Φ is dictated by the latter.

References

1 H. Diefenbach, S. Emde, and C.H. Glock. Multi-depot electric vehicle scheduling in in-plant production logistics considering non-linear charging models. *European Journal of Operational Research*, 306(2):828–848, 2023. doi:10.1016/j.ejor.2022.06.050.

2 T. Erdelić and T. Carić. A Survey on the Electric Vehicle Routing Problem: Variants and Solution Approaches. *Journal of Advanced Transportation*, 2019:1–48, May 2019. doi:10.1155/2019/5075671.

3 D. Jefferies and D. Göhlich. A Comprehensive TCO Evaluation Method for Electric Bus Systems Based on Discrete-Event Simulation Including Bus Scheduling and Charging Infrastructure Optimisation. *World Electric Vehicle Journal*, 11, August 2020. doi:10.3390/wevj11030056.

4 L. Li, H.K. Lo, and F. Xiao. Mixed bus fleet scheduling under range and refueling constraints. *Transportation Research Part C: Emerging Technologies*, 104:443–462, 2019. doi:10.1016/j.trc.2019.05.009.

5 S. Lösel. Elektrobusse im ländlichen Raum: VLP startet Sektorenkopplung Energiewirtschaft und Verkehr. *Der Nahverkehr Elektrobus-Spezial 2023*, 41:40–43, March 2023. (German).

6 B. Messaoudi and A. Oulamara. Electric Bus Scheduling and Optimal Charging. In Carlos Paternina-Arboleda and Stefan Voß, editors, *Computational Logistics*, pages 233–247. Springer International Publishing, 2019. doi:10.1007/978-3-030-31140-7_15.

7 A. Montoya, C. Guéret, J.E. Mendoza, and J.G. Villegas. The electric vehicle routing problem with nonlinear charging function. *Transportation Research Part B: Methodological*, 103:87–110, 2017. Green Urban Transportation. doi:10.1016/j.trb.2017.02.004.

8 N. Olsen and N. Kliewer. Scheduling Electric Buses in Public Transport: Modeling of the Charging Process and Analysis of Assumptions. *Logistics Research*, 13(4), 2020. doi:10.23773/2020_4.

9 S. Pelletier, O. Jabali, G. Laporte, and M. Veneroni. Battery degradation and behaviour for electric vehicles: Review and numerical analyses of several models. *Transportation Research Part B: Methodological*, 103:158–187, 2017. doi:10.1016/j.trb.2017.01.020.

10 S.S.G. Perumal, R.M. Lusby, and J. Larsen. Electric bus planning & scheduling: A review of related problems and methodologies. *European Journal of Operational Research*, 2022. doi:10.1016/j.ejor.2021.10.058.

11 M.E. van Kooten Niekerk, J.M. van den Akker, and J.A. Hoogeveen. Scheduling electric vehicles. *Public Transport*, 9:155–176, 2017. doi:10.1007/s12469-017-0164-0.