Assessing Epidemic Spreading Potential with Encounter Network

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Abstract
Densely populated urban public transportation systems can provide inducive environments for transmitting viruses via close human contact or touching contaminated surfaces. In network analysis, Betweenness Centrality (BC) has been used as the primary metric to measure a node's communication with others. This research extends from the concept of BC and develops new measures to assess the risk of transmitting disease through public transportation links. Three new concepts are introduced: source Total Betweenness centrality (TBC), target TBC, and Encounter Network. From a network node (source node), the set of shortest paths from that node to all other nodes composes a sub-graph (tree). The source TBC of this node is defined as the sum of BC of all edges of this tree. Similarly, using the shortest path tree consists of the set of the shortest paths from all nodes to the node as the destination, the target TBC of the node is defined as the sum of BC of all edges of this tree. Both TBC can be weighted by edge characteristics such as travel time or trip volume. Another new concept, Encounter Network, is constructed as the intersection between all source-target pairs of the public transportation network. We use the source TBC of a node to evaluate the relative risk of transmitting the disease from that node to other nodes. In contrast, the target TBC of a node can be used to assess the relative risk of being infected by a virus transmitted from other nodes to that node. A preliminary case study is conducted to illustrate the process and results.

1 Introduction
Public transportation plays an essential role in many cities to achieve equitable and sustainable goals in urban systems [11]. Public transportation has been recommended in recent decades to reduce car dependency and externalities like traffic congestion and air pollution [2]. Despite its many positive contributions, mass transit network also provides a conducive environment for human contact in proximity which may lead to other effects. For instance, infectious diseases can be spread by human contact, especially in an enclosed space. Human contact between passengers in mass transit systems can easily facilitate the spreading of infectious diseases [7]. The crowded indoor environment in trains and buses intensifies the transmission of pathogens from infected passengers to others [10, 14]. To identify the network components where high-intensity of involuntary human contact and transmission may occur, this research proposes a new concept, encounter network (EN), a subgraph of a transit network, as well as related measures and algorithms to derive an EN from the transit network.
Network science techniques, such as connectivity and centrality measures, have been widely used to study public transportation systems. The approach is useful for simplifying the transportation network by studying the network’s topological properties [3], and it is also effective in studying and assessing the changes and modifications in the network and operational incidents [3, 6]. This research develops a new type of betweenness centrality measure to assess the transmission of infectious diseases in public transportation networks. Three concepts are introduced to study the networks where transmission of disease occurs at their edges: source-node Total Betweenness Centrality (TBC), target-node TBC, and network Encounter Matrix (EM). For every node of the network, the shortest paths tree or sub-network consists of all shortest path(s) from that node to all other network nodes. The source node TBC is defined as the sum of the BC of the edges that belong to this sub-graph. By the same token, the target node TBC of a node could be defined based on the sub-graph shortest paths from all network nodes to that node. Based on the TBC measure, we can determine the stations of those passengers who are exposed to more encounters with other passengers and consequently are more susceptible to transmission of infectious diseases. The EM is defined between each pair of source-target nodes (stations) to measure encounter opportunities in the network, which is achieved by extracting the intersection of shortest paths sub-graphs from (or to) those stations. Comparing the TBC value, the higher value of the source node TBC reveals a greater spreading influence, and the higher target node TBC shows a higher risk of infection.

Since the spread of viruses can occur at the network’s nodes and edges, we develop the encounter network where every source-target node pair of the original network is a node in the encounter network. Two nodes are adjusted if and only if the corresponding shortest paths between them in the original node pairs intersect at least on one edge. The proposed method is implemented and tested on the Sioux Falls network.

## 2 Total Betweenness Centrality and Encounter Network

### 2.1 Total Betweenness Centrality

The complex networks theory has excellent applicability in describing different phenomena and has received much attention in recent years [5]. In this context, various centrality measures for quantifying the network structure have been developed and discussed [1]. The node (edge) betweenness centrality measures the intermediary of a node (edge) and the shortest path between all node pairs, thus characterizing the importance of a node or link in flow organization in the network [1].

Total betweenness centrality (TBC) is a related concept that has been introduced in Network Science [4, 8]. In the prior work, let’s denote $W$ as the subset of a network $V$, the TBC of the subset $W \subseteq V$ is defined as the sum of the betweenness centrality values of its nodes, i.e., $C(W) = \Sigma_{i \in W} C(i)$. However, in the case of transmission of diseases, this measure may not be very useful. Human encounters of indefinite length in the same enclosed space on network edges might impose a higher risk than brief passing at nodes. An edge with a higher BC means more chances of encountering travelers from many different routes and, consequently, a higher risk of transmitting disease between people. Thus, this research modifies the traditional definition of TBC by considering the BC scores of edges. To reckon with the directionality of transportation links, we distinguish between source TBC and target TBC as defined below.
Definition 1. Source TBC: In a directed network V, for a given source node r, the source TBC is defined as the sum of BC values of all edges of the subgraph, $\Psi^s(r)$, that consists of shortest paths from r to all other nodes in V. It can be expressed mathematically as follows.

$$TC^s(r) = \sum_{k \in \Psi^s(r)} C(k) = \sum_{k \in \Psi^s(r)} \sum_{s \neq t} \frac{\sigma_{st}(k)}{\sigma_{st}}$$  (1)

Definition 2. Target TBC: In a directed network V, for a given target node r, the target TBC is defined as the sum of BC values of all edges of the subgraph, $\Psi^t(r)$, that consists of shortest paths from all other nodes to r. It can be expressed mathematically as follows.

$$TC^t(r) = \sum_{k \in \Psi^t(r)} C(k) = \sum_{k \in \Psi^t(r)} \sum_{s \neq t} \frac{\sigma_{st}(k)}{\sigma_{st}}$$  (2)

An illustration is provided in Figure 1 using the Sioux Falls network. The directed network consists of 24 nodes and 76 edges. In Figure 1(a), the sub-graph $\Psi^s(r)$ is in bold red color. In Figure 1-b, the sub-graph $\Psi^t(r)$ is shown in bold blue color. The two sub-graphs display all the edges where travelers from Source Node 1 might encounter travelers going to Target Node 18.

The volume and duration of encounters on each network edge could be different, subject to the travel time and trip volume on each. The following equations formalize the calculation of TBC values with selected weight:

$$TC^w_e(r) = \sum_{k \in \Psi^t(r)} w_k C(k) = \sum_{k \in \Psi^t(r)} \sum_{s \neq t} w_k \frac{\sigma_{st}(k)}{\sigma_{st}}$$  (3)

$$TC^d_e(r) = \sum_{k \in \Psi^t(r)} w_k C(k) = \sum_{k \in \Psi^t(r)} \sum_{s \neq t} w_k \frac{\sigma_{st}(k)}{\sigma_{st}}$$  (4)

2.2 Encounter Network

Definition 3. Encounter subgraph: the intersection sub-graph of two directed shortest-path trees, either from the source node or to a target node, is defined as the encounter subgraph.

An encounter network is constructed with the Encounter subgraph of every pair of nodes in the transit network. Consider $G_o(N_o, \epsilon_o, W_o)$, the original weighted directed graph with total N node number, where $N_o = \{i_1, i_2, ..., i_N\}$ is the node set, $\epsilon_o$ is the edge set, and $W_o$ is the weight set. The encounter network consists of all source-target node pairs, where each source-target pair represent one node in the encounter network. The total nodes of the encounter network would be $N \cdot (N - 1)$, and the network could be represented with $G_e(N_e, \epsilon_e, W_e)$, where $N_e = \{I_1, I_2, ..., I_N(N - 1)\}$ is the node-set, $\epsilon_e$ and $W_e$ are the edge and the weight sets, respectively. In the encounter network, two nodes are neighbors if and only if the shortest path(s) between the original source-target node pairs pass through at least one shared edge of the original network. Denote two arbitrary nodes in the encounter network as K and J, related to the node pairs $\{k - k'\}$ and $\{j - j'\}$, respectively. The edge between these two nodes in the encounter network is denoted as $E_{K,J}$, and is defined as:

$$E_{K,J} = \begin{cases} 0 & \text{if } \sigma_{kk'} \cap \sigma_{jj'} = \emptyset \\ 1 & \text{if } \sigma_{kk'} \cap \sigma_{jj'} \neq \emptyset \end{cases}$$  (5)

The encounter network can also be either unweighted or weighted. Depending on whether the original network is unweighted or weighted. The weight set of the network can take two different values.
3 Preliminary results of model testing and validation

Encounter network and the TBC measures provide a structure to study epidemic spreading in the networks where the transmission occurs on network edges. To test the feasibility, we adopt the SIR epidemic model to mimic the network’s spreading process. Since all contacts are not equally facilitating contagion [12], to make the simulation more compatible with the natural spreading process, the network’s weight structure (e.g., passenger volume, etc) is considered to investigate the network nodes’ spreading capability. Previous studies have assumed different transmission forms such as linear [9] or nonlinear [13] to model the infection rate based on the edges’ weight. Here, a linear transmission rate is applied to calculate the probability of infection of a susceptible node by an infected node. The work is ongoing and reported here are the calculated source and target TBC values for each node, as shown in Figure 2. The figure shows the source and target TBCs of the Sioux Falls network for both unweighted and weighted based on the edges’ travel time.
4 Conclusions and Discussions

Recent infectious disease outbreaks have demonstrated the vulnerability of human communities. Human encounters are a primary medium for the spreading of contagious diseases. In urban areas, public transportation systems can transfer infected people in the network and provide a conducive environment for transmitting disease through direct (Person-to-person contact) and indirect (airborne transmission or touching contaminated objects) contact between passengers. This research extended from the betweenness centrality measures and defined new TBC measures and a new concept of encounter network. They can be used to represent and model encounter opportunities on a network.

The work presents a novel method to identify more influential nodes/edges that are more likely to be the source of spreading and higher-risk nodes/edges that are more likely to receive infection. Moreover, beyond the application to infectious diseases, encounter network TBC measures can be used to study the communication characteristics of transportation networks and other complex networks like social networks.
References