Emptiness Of Alternating Tree Automata Using Games With Imperfect Information*

Nathanaël Fijalkow[1,2], Sophie Pinchinat[3], and Olivier Serre[1]

1 **LIAFA, Université Paris Diderot – Paris 7 & CNRS, France**
2 **University of Warsaw, Poland**
3 **IRISA (INRIA & Université de Rennes 1), France**

Abstract

We consider the emptiness problem for alternating tree automata, with two acceptance semantics: classical (all branches are accepted) and qualitative (almost all branches are accepted). For the classical semantics, the usual technique to tackle this problem relies on a Simulation Theorem which constructs an equivalent non-deterministic automaton from the original alternating one, and then checks emptiness by a reduction to a two-player perfect information game. However, for the qualitative semantics, no simulation of alternation by means of non-determinism is known.

We give an alternative technique to decide the emptiness problem of alternating tree automata, that does not rely on a Simulation Theorem. Indeed, we directly reduce the emptiness problem to solving an *imperfect information* two-player parity game. Our new approach can successfully be applied to both semantics, and yields decidability results with optimal complexity; for the qualitative semantics, the key ingredient in the proof is a positionality result for stochastic games played over infinite graphs.

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1 Introduction

Tree automata [24, 14] are a powerful tool to handle sets of infinite trees which are widely needed in verification, since they provide a natural representation of branching-time system executions. It is well known that by equipping tree automata with the parity condition, one captures all ω-regular tree languages [16]. Additionally, tree automata may use *alternation* [8], which makes their complementation a simple task. In particular, combining alternation with the parity condition yields the automata-theoretic counterpart of the propositional μ-calculus, where the translation from one to the other can be done in linear time [1, 16]. Hence, the model-checking and the satisfiability/validity of logical formulas amount to respectively verifying membership and non-emptiness/universality on their corresponding tree automata.

The membership problem for alternating tree automata has a fairly simple algorithm: one compiles the input tree and the automaton into a polynomial size perfect information parity game and solves it. On the contrary, the usual roadmap to check emptiness of an alternating tree automaton is more involved. First one builds an equivalent non-deterministic automaton thanks to the Simulation Theorem [21], and then one checks emptiness of this latter automaton

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by solving an associated two-player perfect information game. This yields an exponential time algorithm, which is optimal as the emptiness problem is EXPTIME-complete.

The first contribution of this paper is an alternative technique to solve the emptiness problem of alternating tree automata, by directly reducing it to two-player games with *imperfect information*.

This builds on a long tradition initiated by Reif in [23], that advocates the use of games with imperfect information to solve algorithmic problems for automata; in his seminal paper, Reif introduced the notion of *blindfold games* and used them to check the universality of non-deterministic automata over finite words. This approach has been later extended in [12, 25] and combined with antichains representations, to check universality and inclusion of non-deterministic automata, as well as emptiness for alternating automata. This was backed with solid experimental results (see *e.g.* the tool Alaska [26]), where the emptiness of alternating Büchi word automata was considered, building on the Miyano-Hayashi construction [20]. To the best of our knowledge, antichains approaches have not yet been extended to alternating parity tree automata. However, solving the emptiness problem for alternating parity tree automata through games of imperfect information has been considered by Puchala in his PhD [22], where he provides a reduction of the emptiness problem for alternating parity automata to solving a *three-player* game with imperfect information, but no algorithm to solve the latter.

We first illustrate our technique in the classical case of alternating parity tree automata, reducing the emptiness problem to two-player parity games with imperfect information. This does not lead to a gain in complexity due to intrinsic hardness, but unravels the two key ingredients: the first one is the positional determinacy of parity games, to prove the correctness of the reduction, and the second is the determinisation property of ω-word automata, to solve the obtained two-player imperfect information game. We compare this with the classical approach for alternating parity tree automata: the Simulation Theorem [21] also combines the above two key ingredients.

Our technique is of interest for at least two reasons: (1) it pushes the algorithmic difficulty to the game solving part, for which antichains representations have recently been developed [4], hence could lead to efficient algorithms, and (2) a “Simulation Theorem”-free technique is required for classes of tree automata for which no (effective) Simulation Theorem exists.

The second contribution illustrates this latter situation. Indeed, we consider an alternation extension of the class of qualitative tree automata as introduced in [6]: rather than requiring all branches to be accepting (classical semantics), the qualitative semantics requires *almost all* branches to be accepting. We apply our technique to check emptiness of an alternating qualitative Büchi tree automaton. Furthermore, we observe that the emptiness problem becomes undecidable for the co-Büchi condition, implying that there is no simulation theorem for alternating qualitative tree co-Büchi automata. For our technique to go through, the key ingredient is a positionality result for stochastic Büchi games over *infinite* arenas. To the best of our knowledge, very few positionality results are known in the literature that combine both stochastic aspects and infiniteness of the game arena; notable exceptions are [5, 18].

The paper is organised as follows. Section 2 gives the key definitions of both perfect/imperfect information games and classical/qualitative alternating tree automata. Section 3 introduces our technique by revisiting the emptiness problem for alternating parity tree automata and compares with the usual approach. Our main contribution is developed in Section 4: we prove that the emptiness problem for alternating qualitative Büchi tree automata is EXPTIME-complete. This is divided into two subsections. The main technical contribution is given in Section 4.1, where we establish a positionality result for stochastic games played on infinite finite out-degree chronological arenas. Finally, we use this positionality result in Section 4.2 to apply our technique to qualitative alternating Büchi tree automata.

2 Definitions

Let X be a (possibly infinite) alphabet. We denote by X^* (*resp.* X^ω) the set of finite (*resp.* infinite) words over X and we let ε be the empty word. Let Σ be a *finite* alphabet. An *infinite* Σ*-labelled binary tree* (or simply a *tree* when Σ is clear from the context) is a map $t : \{0,1\}^* \to \Sigma$. In this setting, we shall refer to an element $n \in \{0,1\}^*$ as a *node* and to ε as the *root*. For a node n, we call $t(n)$ the *label* of n in t. A (tree) *language* is a set of infinite Σ-labelled binary trees.

A *graph* is a pair $G = (V, E)$ where V is a set of *vertices* and $E \subseteq V \times V$ is a set of *edges*. For every vertex v we let $E(v) = \{w \mid (v,w) \in E\}$. A *dead-end* is a vertex v such that $E(v) = \emptyset$; in the rest of the paper, we only consider graphs that have no dead-end, hence this is implicit from now on. The *size* of a graph is defined to be $|V| + |E|$.

For a finite set S, a probability distribution on S is a function $\delta : S \to [0,1]$ such that $\sum_{s \in S} \delta(s) = 1$. We denote the set of probability distributions on S by $\mathcal{D}(S)$ and we write $\mathrm{Supp}(\delta) = \{s \in S \mid \delta(s) > 0\}$ for the support set of δ.

2.1 Perfect Information Stochastic Games

A (turn-based) *stochastic arena* is a tuple $\mathcal{G} = (G, V_{\mathrm{E}}, V_{\mathrm{A}}, V_R, \delta, v_0)$ where $G = (V, E)$ is a graph, $V = V_{\mathrm{E}} \uplus V_{\mathrm{A}} \uplus V_R$ is a partition of the vertices among two players, Éloïse and Abélard, and an extra player Random, $\delta : V_R \to \mathcal{D}(V)$ is a map such that $\mathrm{Supp}(\delta(v)) = E(v)$ for all $v \in V_R$, and $v_0 \in V$ is an *initial* vertex. In a vertex $v \in V_{\mathrm{E}}$ (*resp.* $v \in V_{\mathrm{A}}$) Éloïse (*resp.* Abélard) chooses a successor vertex from $E(v)$ and in a random vertex $v \in V_R$, a successor vertex is chosen according to the probability distribution $\delta(v)$.

A (pure) *strategy*[1] for Éloïse is a function $\varphi_{\mathrm{E}} : V^* \cdot V_{\mathrm{E}} \to V$ such that for every $\lambda \cdot v \in V^* \cdot V_{\mathrm{E}}$ one has $\varphi_{\mathrm{E}}(\lambda \cdot v) \in E(v)$. Strategies of Abélard are defined likewise, and usually denoted φ_{A}.

Fix a strategy φ_{E} for Éloïse and a strategy φ_{A} for Abélard. This induces a random walk on G. Indeed, define $\mathrm{Plays}^{\varphi_{\mathrm{E}},\varphi_{\mathrm{A}}}$ to be the set of all possible *plays* when the game starts on v_0 and when Éloïse and Abélard chooses their moves accordingly to φ_{E} and φ_{A}. Formally, an infinite play $v_0 v_1 v_2 \cdots \in V^\omega$ belongs to $\mathrm{Plays}^{\varphi_{\mathrm{E}},\varphi_{\mathrm{A}}}$ if for every $i \geq 0$ one has $v_i \in V_{\mathrm{E}} \Rightarrow v_{i+1} = \varphi_{\mathrm{E}}(v_0 \cdots v_i)$, $v_i \in V_{\mathrm{A}} \Rightarrow v_{i+1} = \varphi_{\mathrm{A}}(v_0 \cdots v_i)$ and $v_i \in V_R \Rightarrow v_{i+1} \in E(v_i)$.

Define a partial play as a prefix of a play in $\mathrm{Plays}^{\varphi_{\mathrm{E}},\varphi_{\mathrm{A}}}$: with any partial play λ, the *cone* for λ is the set $cone(\lambda) = \lambda \cdot V^\omega \cap \mathrm{Plays}^{\varphi_{\mathrm{E}},\varphi_{\mathrm{A}}}$ of all infinite plays with prefix λ. Denote by Cones the set of all possible cones and let $\mathcal{F}$ be the Borel σ-field generated by Cones considered as a set of basic open sets (*i.e.* $\mathcal{F}$ is the smallest set containing Cones and closed under complementation, countable union): then $(\mathrm{Plays}^{\varphi_{\mathrm{E}},\varphi_{\mathrm{A}}}, \mathcal{F})$ is a σ-algebra.

A pair of strategies $(\varphi_{\mathrm{E}}, \varphi_{\mathrm{A}})$ induces a probability space over $(\mathrm{Plays}^{\varphi_{\mathrm{E}},\varphi_{\mathrm{A}}}, \mathcal{F})$. Indeed one can define a measure $\mu^{\varphi_{\mathrm{E}},\varphi_{\mathrm{A}}} : \mathrm{Cones} \to [0,1]$ on cones (this task is easy as a cone is uniquely defined by a finite partial play) and then uniquely extends it to a probability measure on $\mathcal{F}$ using the Carathéodory Unique Extension Theorem. For this, one defines $\mu^{\varphi_{\mathrm{E}},\varphi_{\mathrm{A}}}$ inductively on cones:

[1] We do not consider randomised strategies as pure strategies are the right model here.

- $\mu^{\varphi_E,\varphi_A}(cone(v)) = 1$ if $v = v_0$, and $\mu^{\varphi_E,\varphi_A}(v) = 0$ otherwise.
- For every partial play λ ending in $V_E \cup V_A$, let $\mu^{\varphi_E,\varphi_A}(cone(\lambda \cdot v)) = \mu^{\varphi_E,\varphi_A}(cone(\lambda))$ if $v = \varphi_E(\lambda)$ or $v = \varphi_A(\lambda)$, and $\mu^{\varphi_E,\varphi_A}(cone(\lambda \cdot v)) = 0$ otherwise;
- For every partial play λ ending in $v \in V_R$, let $\mu^{\varphi_E,\varphi_A}(cone(\lambda \cdot v')) = \mu^{\varphi_E,\varphi_A}(cone(\lambda)) \cdot \delta(v)(v')$.

Denote by $\Pr^{\varphi_E,\varphi_A}$ the unique extension of $\mu^{\varphi_E,\varphi_A}$ to a probability measure on $\mathcal{F}$. Then $(\text{Plays}^{\varphi_E,\varphi_A}, \mathcal{F}, \Pr^{\varphi_E,\varphi_A})$ is a probability space.

A *winning condition* is a subset[2] $\Omega \subseteq V^\omega$ and a (two-player perfect information) *stochastic game* is a pair $\mathbb{G} = (\mathcal{G}, \Omega)$. A game is *deterministic* whenever $V_R = \emptyset$ (and in this case we omit both V_R and δ in the notations).

A strategy φ_E for Éloïse is *surely winning* if $\text{Plays}^{\varphi_E,\varphi_A} \subseteq \Omega$ for every strategy φ_A of Abélard; it is *almost-surely winning* if $\Pr^{\varphi_E,\varphi_A}(\Omega) = 1$ for every strategy φ_A of Abélard. Similar notions for Abélard are defined dually.

A *reachability game* is one with a winning condition of the form V^*FV^ω, *i.e.* winning plays are those that eventually visit a vertex in F (we refer to vertices in F as final ones). We consider the *parity* winning condition: a *colouring* function ρ is a mapping $\rho : V \to \text{Col} \subset \mathbb{N}$ where Col is a *finite* set of colours; the *parity condition* associated with ρ is the set $\Omega_\rho = \{v_0v_1 \cdots \in V^\omega \mid \liminf(\rho(v_i))_{i\geq 0} \text{ is even}\}$. A *parity game* is a game equipped with a parity winning condition and we shall denote it $\mathbb{G} = (\mathcal{G}, \rho)$ (*i.e.* writing ρ instead of Ω_ρ). *Büchi games* are those where $\text{Col} = \{0, 1\}$ and we refer to vertices v such that $\rho(v) = 0$ as *Büchi vertices*. The dual is *co-Büchi*, for $\text{Col} = \{1, 2\}$.

A positional strategy φ is one that does not require memory, *i.e.* such that for any two partial plays of the form $\lambda \cdot v$ and $\lambda' \cdot v$, one has $\varphi(\lambda \cdot v) = \varphi(\lambda' \cdot v)$, equivalently φ only depends on the current vertex. It is well-known that positional strategies suffice to surely win in deterministic parity games (see *e.g.* [27]).

▶ **Theorem 1** (Positional determinacy). *Let $\mathbb{G}$ be a deterministic parity game. Then either Éloïse or Abélard has a positional surely winning strategy.*

Working with deterministic parity games we only consider positional strategies and see them as maps from V_E (or V_A depending on the player) to V.

For stochastic games the following result is well-known (see *e.g.* [15] for a slightly more general result).

▶ **Theorem 2.** *Let $\mathbb{G}$ be a stochastic parity game played on a* finite *arena. If Éloïse almost-surely wins then she can do so using a positional strategy.*

To the best of our knowledge, no extension of this result is known when dropping the assumption that the arena is finite. We give such an extension (for Büchi games on so-called chronological arenas of finite out-degree) in Theorem 7.

2.2 Alternating Parity Tree Automata

An *alternating parity tree automaton* is a tuple $\mathcal{A} = (Q_\exists, Q_\forall, \Sigma, \Delta, q_{\text{in}}, \rho)$, where $Q_\exists$ is a set of existential states and $Q_\forall$ is a set of universal states such that $Q_\exists$ and $Q_\forall$ are disjoint (we let $Q = Q_\exists \uplus Q_\forall$), $q_{\text{in}} \in Q$ is an initial state, Σ is a labelling finite alphabet, $\Delta \subseteq Q \times \Sigma \times Q \times Q$ is a (finite) transition relation and $\rho : Q \to \mathbb{N}$ is a colouring function. We additionally assume

[2] Formally one needs to require that Ω is measurable for all $\Pr^{\varphi_E,\varphi_A}$, which will be trivially true for all cases considered here as Ω will always be Borel.

without loss of generality that for all $(q,\sigma) \in Q \times \Sigma$ there is at least one $(q,\sigma,q_0,q_1) \in \Delta$. A *non-deterministic* parity tree automaton is an alternating automaton in which all states are existential (hence, we omit $Q_\forall$ in this case).

In the following, we use tree automata as acceptors for tree languages, and acceptance is defined by means of a perfect information parity game. We will define two semantics for acceptance: *classical* and *qualitative*.

Fix an alternating parity tree automaton $\mathcal{A} = (Q_\exists, Q_\forall, \Sigma, \Delta, q_{\text{in}}, \rho)$ and a Σ-labelled tree t. From $\mathcal{A}$ and t, we define two games: $\mathbb{G}_{\mathcal{A},t}$ and $\mathbb{G}^{=1}_{\mathcal{A},t}$.

Intuitively, a play in those two games consists in moving a pebble along a branch of t in a top-down manner: the pebble is attached to a state and in a node n with state q Éloïse (if $q \in Q_\exists$) or Abélard (if $q \in Q_\forall$) picks a transition $(q, t(n), q_0, q_1) \in \Delta$, and then *Abélard* (in $\mathbb{G}_{\mathcal{A},t}$) or *Random* (in $\mathbb{G}^{=1}_{\mathcal{A},t}$) chooses to move down the pebble either to $n \cdot 0$ (and update the state to q_0) or to $n \cdot 1$ (and update the state to q_1).

Formally, one let $G = (V_\exists \uplus V_\forall \uplus V_\Delta, E)$ with $V_\exists = \{0,1\}^* \times Q_\exists$, $V_\forall = \{0,1\}^* \times Q_\forall$ and $V_\Delta = \{(n,q,q_0,q_1) \mid n \in \{0,1\}^* \text{ and } (q,t(n),q_0,q_1) \in \Delta\}$ and

$$\begin{aligned} E \quad = \quad & \{((n,q),(n,q,q_0,q_1)) \mid (n,q,q_0,q_1) \in V_\Delta\} \quad \cup \\ & \{((n,q,q_0,q_1),(n \cdot x, q_x)) \mid x \in \{0,1\} \text{ and } (n,q,q_0,q_1) \in V_\Delta\} \end{aligned}$$

Then let $\mathcal{G}_{\mathcal{A},t} = (G, V_{\text{E}}, V_{\text{A}}, (\varepsilon, q_{\text{in}}))$ be the deterministic arena defined by letting $V_{\text{E}} = V_\exists$ and $V_{\text{A}} = V_\forall \cup V_\Delta$ and let $\mathcal{G}^{=1}_{\mathcal{A},t} = (G, V_{\text{E}}, V_{\text{A}}, V_R, \delta, (\varepsilon, q_{\text{in}}))$ be the stochastic arena defined by letting $V_{\text{E}} = V_\exists$, $V_{\text{A}} = V_\forall$, $V_R = V_\Delta$ and $\delta((n,q,q_0,q_1))$ be the distribution $(n \cdot 0, q_0) \mapsto \frac{1}{2}$ and $(n \cdot 1, q_1) \mapsto \frac{1}{2}$.

Extend ρ on V by letting $\rho((n,q)) = \rho((n,q,q_0,q_1)) = \rho(q)$. Finally one let $\mathbb{G}_{\mathcal{A},t} = (\mathcal{G}_{\mathcal{A},t}, \rho)$ and $\mathbb{G}^{=1}_{\mathcal{A},t} = (\mathcal{G}^{=1}_{\mathcal{A},t}, \rho)$.

A tree t is *accepted* (*resp.* *qualitatively accepted*) by $\mathcal{A}$ if Éloïse has a surely (*resp.* almost-surely) winning strategy in the game $\mathbb{G}_{\mathcal{A},t}$ (*resp.* $\mathbb{G}^{=1}_{\mathcal{A},t}$). Finally, we define the set $L(\mathcal{A})$ as the set of trees accepted by $\mathcal{A}$ and the set $L^{=1}(\mathcal{A})$ as the set of trees qualitatively accepted by $\mathcal{A}$.

The languages of the form $L(\mathcal{A})$, *regular tree languages*, have many remarkable properties and characterisations (see *e.g.* [16]). The languages of the form $L^{=1}(\mathcal{A})$ when $\mathcal{A}$ is non-deterministic, *qualitative tree languages*, have been introduced in [6] and also enjoy many good properties: for instance they are closed under union and intersection, and their emptiness can be tested in *polynomial* time.

▶ **Example 3.** A typical language $L^{=1}(\mathcal{A})$ with $\mathcal{A}$ being non-deterministic is the set of trees that have almost all their branches containing infinitely many a's. An example of a language $L^{=1}(\mathcal{A}_\mathcal{B})$ with $\mathcal{A}_\mathcal{B}$ being alternating is the set of trees such that all subtrees belongs to some $L^{=1}(\mathcal{B})$ where $\mathcal{B}$ is non-deterministic.

▶ Remark. There are several definitions of alternating tree automata, and another popular one is by not distinguishing between existential and universal states but replacing the transition relation by a map $\delta : Q \times \Sigma \to \mathcal{B}^+(Q \times \{0,1\})$ where $\mathcal{B}^+(X)$ denotes the positive Boolean formulas over X (see *e.g.* [19]). Our model is easily seen to be equi-expressive with that one.

▶ Remark. Any *positional* strategy for Éloïse in $\mathbb{G}_{\mathcal{A},t}$ or $\mathbb{G}^{=1}_{\mathcal{A},t}$ can be described as a function $\varphi : \{0,1\}^* \times Q_\exists \to Q \times Q$ that satisfies the following property: $\forall n \in \{0,1\}^*$, if $\varphi(n,q) = (q_0,q_1)$ then $(q,t(n),q_0,q_1) \in \Delta$. Equivalently, in a curryfied form, φ is a map $\{0,1\}^* \to (Q_\exists \to Q \times Q)$. Hence, if one let $\mathcal{T}$ be the set of functions from $Q_\exists$ into $Q \times Q$, Éloïse's positional strategies are in bijection with $\mathcal{T}$-labelled binary trees.

2.3 Imperfect Information Stochastic Parity Games

In the following we introduce a quite restrictive class of games with imperfect information which is essentially a stochastic version of the model in [10].

An *arena of imperfect information* is a tuple $\mathcal{G} = (S, s_0, A, T, \sim)$ where S is a finite set of states, $s_0 \in S$ is an initial state, A is the finite alphabet of Éloïse's actions, $T \subseteq S \times A \times \mathcal{D}(S)$ is a (finite) stochastic transition relation and $\sim$ is an equivalence relation over S. We additionally require that for all $(s, a) \in S \times A$ there is at least one $\delta \in \mathcal{D}(S)$ such that $(s, a, \delta) \in T$. If for all probability distributions in T the support is a singleton, we say that $\mathcal{G}$ is an imperfect information *deterministic* arena and we see T as a subset of $S \times A \times S$. An *imperfect information stochastic parity game* is a pair $\mathbb{G} = (\mathcal{G}, \rho)$ where $\mathcal{G}$ is an arena of imperfect information with set of states S and $\rho : S \to \mathbb{N}$ is a colouring function defining a parity condition $\Omega_\rho \subseteq S^\omega$. The game is *deterministic* if its arena is deterministic. A *play* starts from the initial state s_0 and proceeds as follows: Éloïse plays an action $a_0 \in A$, then Abélard resolves the non-determinism by choosing a distribution δ_0 such that $(s_0, a_0, \delta_0) \in T$ and finally a new state is randomly chosen according to δ_0. Then Éloïse plays a new action, Abélard resolves the non-determinism and a new state is randomly chosen and so on forever. Hence a play is an infinite word $s_0a_0\delta_0s_1a_1\delta_1s_2 \cdots \in (S \times A \times \mathcal{D}(S))^\omega$ and is won by Éloïse if $s_0s_1s_2 \cdots \in \Omega_\rho$. A *partial play* is a prefix of a play.

Imperfect information is modeled thanks to the equivalence relation $\sim$ with the meaning that Éloïse cannot distinguish two states that are $\sim$-equivalent which is important when defining strategies for Éloïse. Intuitively, she should not play differently in two indistinguishable plays, where the indistinguishability of Éloïse is based on *perfect recall* [13], that is: Éloïse cannot distinguish two plays $s_0a_0\delta_0s_1a_1\delta_1 \cdots s_\ell$ and $s'_0a'_0\delta'_0s'_1a'_1\delta'_1 \cdots s'_\ell$ with $s_i \sim s'_i$ for all $i \leq \ell$ and $a_i = a'_i$ for all $i < \ell$. Hence, a strategy for Éloïse is a function $\varphi : (S_{/\sim} \times A)^* \cdot (S_{/\sim}) \to A$ assigning an action to every set of indistinguishable plays (here $S_{/\sim}$ denotes the set of equivalence classes of $\sim$ in S, and for every $s \in S$, we shall write $[s]_\sim$ for its $\sim$-equivalence class). Éloïse *respects a strategy* φ during a play $\lambda = s_0a_0\delta_0s_1a_1\delta_1 \cdots$ if $a_{i+1} = \varphi([s_0]_\sim a_0[s_1]_\sim \cdots [s_i]_\sim)$, for all $i \geq 0$. A strategy φ for Éloïse is *surely winning* if Éloïse wins all plays consistent with φ and it is *almost-surely winning* if Éloïse wins almost all plays[3] consistent with φ.

▶ Remark. Our model of imperfect information games belongs is quite restrictive compared to general models developed in [17, 3, 9, 7], as here Abélard is perfectly informed. However, our model turns out to be expressive enough for our purpose.

▶ Remark. It is important to note that Éloïse may not observe the colour of the current state in general, as we do not require that $s \sim s' \Rightarrow \rho(s) = \rho(s')$. In particular, this has to be taken into account when eventually solving the game.

3 Revisiting the Emptiness using Imperfect Information Game

In this section, we introduce our technique by revisiting emptiness checking of regular tree languages when described by an alternating parity tree automaton. The standard technique [21] first removes alternation and then reduces emptiness to decide the winner in a finite perfect information game; our technique goes directly to decide the winner in a game but

[3] To formally define what it means to win almost all plays one needs to define Abélard's strategies and explain how a pair of strategies for Éloïse and Abélard induces a probabilistic space over plays consistent with those strategies. This is essentially identical to what we did in Section 2.1 for perfect information games.

as a drawback imperfect information is needed. Our construction is reminiscent of the notion of *blindfold games* introduced by Reif in [23], and later thoroughly extended [12, 25, 26].

We first give our construction and then compare it to the classical one, arguing that rather than being novel our technique is indeed a change of perspective. Later in Section 4 we use this new perspective to design an emptiness test for alternating qualitative tree automata for which the classical perspective was unsuccessful.

Fix an alternating parity tree automaton $\mathcal{A} = (Q_\exists, Q_\forall, \Sigma, \Delta, q_{\text{in}}, \rho)$. Our goal is to check whether $L(\mathcal{A}) = \emptyset$ and for this we design an imperfect information *deterministic* parity game. We first present the game and show how it permits to decide emptiness; then we compare our construction with the standard approach.

3.1 An Imperfect Information Emptiness Game

We define an imperfect information deterministic parity game $\mathbb{G}_\mathcal{A}$ that intuitively works as follows. Éloïse describes both a tree t and a positional strategy φ_t for her in the game $\mathbb{G}_{\mathcal{A},t}$; the strategy φ_t is described as a $\mathcal{T}$-labeled tree (where $\mathcal{T}$ is the set of functions from $Q_\exists$ into $Q \times Q$, see Remark 2.2). As the plays are of ω-length, she actually does not fully describe t and φ_t but only a branch: this branch is chosen by Abélard, who also takes care of computing the sequence of states along it (either by updating an existential state accordingly to φ_t or, when the state is universal, by choosing an arbitrary valid transition of the automaton). In this game, Éloïse observes the directions, but not the actual control state of the automaton (indeed, otherwise she could easily "cheat").

Formally, we let $\mathcal{G}_\mathcal{A} = (S, s_{\text{in}}, A, T, \sim)$ where $S = (Q \times \{0,1\}) \cup \{(q_{\text{in}}, \varepsilon)\}$ and $s_{\text{in}} = (q_{\text{in}}, \varepsilon)$; $A \subseteq \Sigma \times \mathcal{T}$ is the set of pairs (a, τ) such that for all $q \in Q_\exists$ we have that $(q, a, q_0, q_1) \in \Delta$ where $\tau(q) = (q_0, q_1)$, $(q, i) \sim (q', i)$ for all $q, q' \in Q$ and $i \in \{0, 1\}$, and

$$T = \{((q,i),(a,\tau),(q_0,0)),((q,i),(a,\tau),(q_1,1)) \mid q \in Q_\exists \text{ and } \tau(q) = (q_0,q_1)\}$$
$$\cup\ \{((q,i),(a,\tau),(q_0,0)),((q,i),(a,\tau),(q_1,1)) \mid q \in Q_\forall \text{ and } (q,a,q_0,q_1) \in \Delta\}$$

Finally we let $\mathbb{G}_\mathcal{A} = (\mathcal{G}_\mathcal{A}, \rho_\mathcal{A})$ be the imperfect information *deterministic* parity game obtained by letting $\rho_\mathcal{A}(q, i) = \rho(q)$ for any $(q, i) \in S$.

▶ **Lemma 4.** *Éloïse has a surely winning strategy in $\mathbb{G}_\mathcal{A}$ iff $L(\mathcal{A}) \neq \emptyset$.*

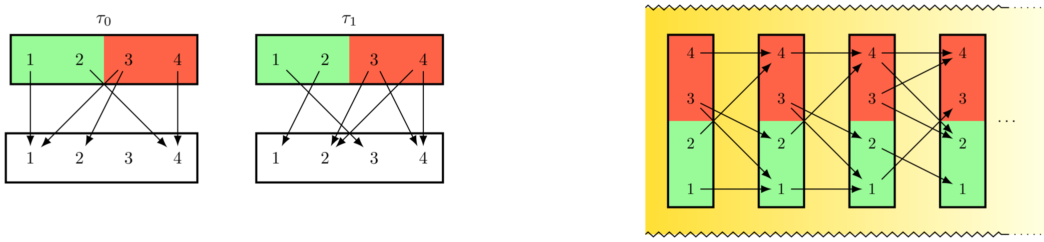
▶ Remark. From the proof of Lemma 4, one can also conclude that if $L(\mathcal{A}) \neq \emptyset$ then $L(\mathcal{A})$ contains a regular tree (*i.e.* the unfolding of a finite graph). Indeed, this is a direct consequence of the fact that if Éloïse has a surely winning strategy in $\mathbb{G}_\mathcal{A}$, then she has one that uses finite memory [10].

Lemma 4 provides a reduction of the emptiness problem to deciding the existence of a surely winning strategy in an imperfect information deterministic game. We prove a converse result.

▶ **Lemma 5.** *For any imperfect information deterministic parity game $\mathbb{G}$ one can construct an alternating parity tree automaton $\mathcal{A}_\mathbb{G}$ such that Éloïse surely wins in $\mathbb{G}$ iff $L(\mathcal{A}_\mathbb{G}) \neq \emptyset$. Moreover in $\mathcal{A}_\mathbb{G}$ all states are universal.*

3.2 Comparison with the Standard Approach

The usual roadmap to check emptiness of an alternating tree automaton is as follows. First one builds an equivalent non-deterministic automaton thanks to Simulation Theorem (see below) and then one checks emptiness of this latter automaton by solving an associated



(a) (b)

Figure 1 **(a)** Semi-tiles τ_0 and τ_1 such that (τ_0, τ_1) represents the tile $\{(1,1,3),(2,4,1),(3,1,2),(3,2,4),(4,4,2),(4,4,4)\}$ with $Q_\exists = \{1,2\}$ and $Q_\forall = \{3,4\}$.
(b) Representation of the sequence of semi-tiles $\tau_0\tau_0\tau_1\cdots$.

perfect information game. It is a well-known result that alternating and non-deterministic automata are equi-expressive [21].

▶ **Theorem 6** (Simulation Theorem). *Let $\mathcal{A}$ be an alternating parity tree automaton with n states and using k colours. Then one can effectively construct a non-deterministic parity tree automaton $\mathcal{B}$ such that $L(\mathcal{A}) = L(\mathcal{B})$. The automaton $\mathcal{B}$ has $2^{\mathcal{O}(nk\log(nk))}$ states and it uses $\mathcal{O}(nk)$ colours.*

Proof. We do not give a complete proof of this classical result [21] but we rather exhibit the crucial arguments here to later revisit the emptiness problem for alternating parity tree automata.

Fix an alternating parity tree automaton $\mathcal{A} = (Q_\exists, Q_\forall, \Sigma, \Delta, q_{\text{in}}, \rho)$. For any letter $\sigma \in \Sigma$, call a *σ-tile* any subset $\tau \subseteq Q \times Q \times Q$ such that

- for all $q, q_0, q_1 \in Q$, if $(q, q_0, q_1) \in \tau$ then $(q, \sigma, q_0, q_1) \in \Delta$;
- for all $q \in Q_\exists$ there exists a *unique* $(q_0, q_1) \in Q^2$ such that $(q, q_0, q_1) \in \tau$;
- for all $q \in Q_\forall$ and for all $(q, \sigma, q_0, q_1) \in \Delta$, $(q, q_0, q_1) \in \tau$.

Hence, one should think of a σ-tile as a description of the local value of a *positional* strategy for Éloïse in a node labeled by σ from a tree t in the game $\mathbb{G}_{\mathcal{A},t}$ (the case of $q \in Q_\forall$ is here to leave all options of Abélard). In the following it is convenient to think of a tile τ as a pair (τ_0, τ_1) of *semi-tiles* where $\tau_0, \tau_1 \subseteq Q \times Q$ and $(q, q_0) \in \tau_0$ (*resp* $(q, q_1) \in \tau_1$) if and only if there exists $p \in Q$ such that $(q, q_0, p) \in \tau$ (*resp.* $(q, p, q_1) \in \tau$). See Figure 1a for an example.

Now an equivalent non-deterministic automaton $\mathcal{A}'$ is obtained by choosing as control states the set S of all possible semi-tiles augmented with an extra dummy initial state s_{in}. The transition relation Δ' consists of all those elements of the form $(s, \sigma, \tau_0, \tau_1)$ where s is any state and (τ_0, τ_1) is a σ-tile. Acceptance for $\mathcal{A}'$ is then defined by means of a game $\mathbb{G}_{\mathcal{A}',t}$ as previously except that the winning condition is more involved than a parity condition. A play is an element $\lambda = v_0 v_1 \cdots \in ((\{0,1\}^* \times S) \cdot (\{0,1\}^* \times S \times S \times S))^\omega$ to which we can associate a sequence of semi-tiles $\pi(\lambda) = s_1 s_2 \cdots$ where $v_{2i} = (n_i, s_i)$ for all integer $i \geq 1$ (we ignore the dummy initial state). The sequence $\pi(\lambda)$ can be seen as a set of infinite paths in an infinite ribbon obtained by gluing together the semi-tiles $s_1, s_2, \ldots$ (see Figure 1b). An infinite sequence $q_1 q_2 \cdots$ is an infinite path in $\pi(\lambda)$ if and only if for all $i \geq 1$ one has $(q_i, q_{i+1}) \in s_i$; it is *good* if $\liminf(\rho(q_i))_{i\geq 1}$ is even; and $\pi(\lambda)$ is *good* if all plays in it are good. Finally we define those winning plays for Éloïse as those plays λ such that $\pi(\lambda)$ is good.

Then one can easily remark that the set of all λ such that $\pi(\lambda)$ is good is an ω-regular language over S, hence is accepted by a deterministic parity ω-word automaton $\mathcal{C}$. Considering a "synchronised" product of $\mathcal{C}$ together with $\mathcal{A}'$ leads $\mathcal{B}$. The desired complexity is achieved by carefully constructing $\mathcal{C}$. ◀

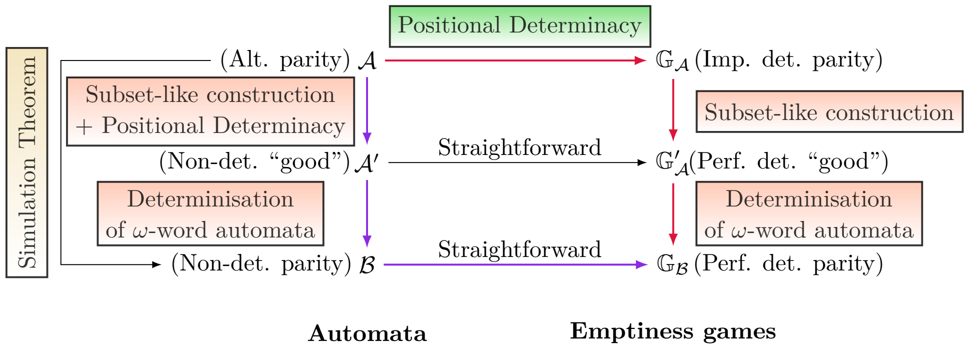


Figure 2 Roadmaps to emptiness checking: the classical one (purple) *vs* ours (in red).

Consider now a *non-deterministic* parity tree automaton $\mathcal{K} = (Q, \Sigma, \Delta, q_{\text{in}}, \rho)$. A perfect information emptiness game for $\mathcal{K}$ is built as follows. We let $G_{\mathcal{K}} = (V_{\text{E}} \uplus V_{\text{A}}, E)$ where $V_{\text{E}} = Q$, $V_{\text{A}} = \Delta$ and $E = \{(q, (q, \sigma, q_0, q_1)),\ ((q, \sigma, q_0, q_1), q_0), ((q, \sigma, q_0, q_1), q_1) \mid (q, \sigma, q_0, q_1) \in \Delta\}$. We define $\mathbb{G}_{\mathcal{K}} = (\mathcal{G}_{\mathcal{K}}, \rho)$ with $\mathcal{G}_{\mathcal{K}} = (G_{\mathcal{K}}, V_{\text{E}}, V_{\text{A}}, q_{\text{in}})$ and where we extend ρ by letting $\rho((q, \sigma, q_0, q_1)) = \rho(q)$. Then one easily has that $L(\mathcal{K}) \neq \emptyset$ if and only if Éloïse surely wins in $\mathbb{G}_{\mathcal{K}}$. Indeed, strategies for Éloïse in $\mathbb{G}_{\mathcal{K}}$ are in bijection with pairs made of a tree t and a strategy for Éloïse in $\mathbb{G}_{\mathcal{K},t}$, and this bijection preserves the fact that a strategy is surely winning.

Now think of adapting this construction to the automaton $\mathcal{A}'$ from the proof of the Simulation Theorem and recall that acceptance for $\mathcal{A}'$ was defined thanks to a game as the classical one except that the winning condition was more involved. Then, the same construction provides a game $\mathbb{G}_{\mathcal{A}'}$ where Éloïse's vertices are semi-tiles and Abélard's vertices are tuple made of a semi-tile, a letter in Σ and a tile, and whose winning condition for a play $\lambda = v_0 v_1 \cdots \in ((\Sigma \times S) \cdot (\Sigma \times S \times S \times S))^\omega$ is that $\pi(\lambda) = s_1 s_2 \cdots$ is good (in the previous sense) where $v_{2i} = (\sigma_i, s_i)$ for all integer $i \geq 1$.

Now think back to our reduction from $\mathcal{A}$ to $\mathbb{G}_{\mathcal{A}}$: it makes crucial use of positional determinacy while determinisation of automata on infinite word is implicitly needed when deciding whether Éloïse surely wins in $\mathbb{G}_{\mathcal{A}}$. Indeed, one first applies the subset construction of [10], getting an intermediate perfect information game isomorphic to $\mathbb{G}_{\mathcal{A}'}$ and then, since Éloïse does not observe the colour, one has to embed in the previous subset/tile construction a deterministic parity automaton over infinite words that checks that all plays consistent with the observations fit the parity condition. As this latter automaton is essentially the automaton $\mathcal{C}$ one gets a perfect information parity game isomorphic to $\mathbb{G}_{\mathcal{B}}$. Figure 2 summarises the previous discussion.

4 Checking Emptiness Using an Imperfect Information Game: The Case of $L^{=1}(\mathcal{A})$ for Büchi Condition

We are now coming to the central contributions of this paper. We design an emptiness test for alternating *qualitative* Büchi tree automata adapting the approach developed in Section 3. Recall that it relies on two key arguments: a positionality result and a decidability result for games. Hence, we start by proving a positionality result (Theorem 7), and then explain how to obtain a (decidable) emptiness game (Theorem 10).

4.1 A Positionality Result for Chronological Games

In this subsection, we prove a positionality result for a subclass of infinite arenas, satisfying the following two conditions:

- (finite out-degree) the underlying graph has finite degree, *i.e* for every $v \in V$ there are finitely many outgoing edges from v, and
- (chronological) there exists a function $\text{rank} : V \to \mathbb{N}$ such that $\text{rank}^{-1}(0) = \{v_0\}$ and for $(v, v') \in E$, $\text{rank}(v') = \text{rank}(v) + 1$.

Note that both assumptions hold for $\mathbb{G}_{\mathcal{A}}^{=1}$. Also, observe that for any k, the set $\text{rank}^{-1}(k)$ is finite, and the set V of vertices is countable.

▶ **Theorem 7.** *Let $\mathbb{G}$ be a Büchi game whose arena has finite out-degree and is chronological. Then if Éloïse wins almost-surely, then she has a positional winning strategy.*

We will use the following result for finite arenas. We state it here in a rather weak form (with the chronological assumption), as it can be easily proved by a backward induction, whereas the proof for general finite arenas is more involved. We refer to [11] for the original proof, and to [18] for a nice survey.

▶ **Lemma 8.** *Let $\mathbb{G}$ be a stochastic reachability game played on a* chronological *and* finite *arena. Let W be the set of vertices from which Éloïse has a strategy ensuring to win with probability at least $\frac{1}{2}$. Then there exists a positional strategy which ensures to win with probability at least $\frac{1}{2}$ from every vertex in W.*

Note that the important point here is that the constructed strategy is *uniform*, *i.e.* the same strategy is winning from every vertex.

We first sketch the proof. The main idea is to note that if Éloïse can ensure to reach F with probability 1 from some initial vertex, then there exists a bound k such that she can ensure to reach F with probability at least $\frac{1}{2}$ within k steps against *any* strategy of Abélard. This allows to "slice" the arena into infinitely many disjoint finite arenas: in each slice Éloïse plays to reach F with probability at least half. Since each slice forms a finite subarena, optimal positional strategies exist by Lemma 8. The resulting strategy consists in playing in turns the above positional strategies; since each slice gives a probability to reach F of at least half before proceeding to the next, the probability to reach F infinitely often is 1.

Proof. We assume that Éloïse almost-surely wins $\mathbb{G}$. Without loss of generality, we can assume that she wins almost-surely from everywhere, by restricting the arena to vertices reachable by a fixed almost-surely winning strategy.

In the next statement and later on, by a strategy in $\mathbb{G}$ from a vertex v we mean a strategy in the game obtained from $\mathbb{G}$ by changing the initial vertex of the arena $\mathcal{G}$ to be v.

▶ **Lemma 9.** *Let φ_{E} be an almost-surely winning strategy for Éloïse in $\mathbb{G}$ from v. There exists an integer k such that for all strategies φ_{A} of Abélard, we have $\mathrm{Pr}^{\varphi_{\mathrm{E}},\varphi_{\mathrm{A}}}(V^{\leq k}FV^{\omega}) \geq \frac{1}{2}$.*

Proof. Toward a contradiction, assume that such a k does not exist. Hence, for each k there exists a strategy $\varphi_{\mathrm{A},k}$ such that $\mathrm{Pr}^{\varphi_{\mathrm{E}},\varphi_{\mathrm{A},k}}(V^{\leq k}FV^{\omega}) < \frac{1}{2}$. Moreover we can assume that the $\varphi_{\mathrm{A},k}$ are positional strategies: indeed, one can pick for $\varphi_{\mathrm{A},k}$ an optimal strategy for Abélard in the finite reachability game obtained by restricting $\mathbb{G}$ to the vertices of rank at most k. As this game is finite, by Lemma 8, an optimal strategy can always be chosen to be positional.

From the sequence $(\varphi_{\mathrm{A},k})_{k\geq 0}$ we can extract a strategy $\varphi_{\mathrm{A},\infty}$ that is consistent, for any $k \geq 0$, with infinitely many $\varphi_{\mathrm{A},h}$ on its k first moves. For this, fix an enumeration $v_1, v_2, \cdots$ of the vertices in V. We will define by induction on i the following objects:

- I_i an infinite set of vertices such that $I_0 \supseteq I_1 \supseteq I_2 \supseteq I_3 \supseteq \cdots$, and
- $\varphi_{A,\infty}$ such that at step i, $\varphi_{A,\infty}$ is defined on $v_1, \ldots, v_i$ and is consistent on these vertices with all those strategies $\varphi_{A,h}$ with $h \in I_i$.

Let $I_0 = \mathbb{N}$ be the set of all positive integers. At step i let us consider the values of $\varphi_h(v_i)$ for all $h \in I_{i-1}$: as $\mathcal{G}$ has finite out-degree, there is one v'_i such that for infinitely many $h \in I_{i-1}$, $\varphi_{A,h}(v_i) = v'_i$. We define $\varphi_{A,\infty}(v_i) = v'_i$ and we let $I_i = \{h \in I_{i-1} \mid \varphi_{A,h}(v_i) = v'_i\}$.

Now for $k \geq 0$ let i be such that $\{v_j \mid j \leq i\}$ contains all vertices of rank at most k: then as required $\varphi_{A,\infty}$ is consistent with infinitely many $\varphi_{A,h}$ — all the $\varphi_{A,h}$ with $h \in I_i$ — on its k first moves. In particular, for all k, there is some $h \geq k$ such that

$$\Pr^{\varphi_E,\varphi_{A,\infty}}(V^{\leq k}FV^\omega) = \Pr^{\varphi_E,\varphi_{A,h}}(V^{\leq k}FV^\omega) \leq \Pr^{\varphi_E,\varphi_{A,h}}(V^{\leq h}FV^\omega) < \frac{1}{2}$$

As $V^*FV^\omega = \bigcup_{k\geq 0} V^{\leq k}FV^\omega$ and as the sequence $(V^{\leq k}FV^\omega)_{k\geq 0}$ is increasing for set inclusion, one concludes that $\Pr^{\varphi_E,\varphi_{A,\infty}}(V^*FV^\omega) = \lim_{k\to\infty} \Pr^{\varphi_E,\varphi_{A,\infty}}(V^{\leq k}FV^\omega) \leq \frac{1}{2} < 1$ which leads a contradiction with φ_E being almost-surely winning. ◀

Let $k < k'$, we define $\mathbb{G}_{[k,k']}$ the reachability game induced by $\mathbb{G}$ restricted to vertices of rank in $[k, k']$. Since $\mathcal{G}$ has finite out-degree, there are finitely many vertices of rank in $[k, k']$, hence $\mathbb{G}_{[k,k']}$ is finite.

We define inductively a sequence of ranks $(k_i)_{i\geq 1}$ together with a sequence of strategies $(\varphi_{E,[k_i,k_{i+1}[})_{i\geq 0}$ such that for all $i \geq 0$, $\varphi_{E,[k_i,k_{i+1}[}$ is a positional strategy, defined on all vertices of rank $[k_i, k_{i+1}[$, such that from all vertices of rank k_i, for all strategies φ_A, we have $\Pr^{\varphi_{E,[k_i,k_{i+1}[},\varphi_A}(V^{\leq \ell}FV^\omega) \geq \frac{1}{2}$, where $\ell = k_{i+1} - k_i$.

Set $k_0 = 0$. Assume the first i ranks and strategies are defined. For each vertex of rank k_i, Lemma 9 shows the existence of a bound; since there are finitely many such vertices, we can consider the maximum of those bounds, and denote it by k_{i+1}. By construction, from all vertices of rank k_i, for all strategies φ_A, we have $\Pr^{\varphi_E,\varphi_A}(V^{\leq \ell}FV^\omega) \geq \frac{1}{2}$, where $\ell = k_{i+1} - k_i$. In other words, Éloïse wins the chronological and finite reachability game $\mathbb{G}_{[k_i,k_{i+1}]}$ with probability at least $\frac{1}{2}$, so, thanks to Theorem 2, there exists a uniform positional strategy ensuring to reach F with probability at least $\frac{1}{2}$, denote it $\varphi_{E,[k_i,k_{i+1}[}$. This concludes the inductive construction.

Now define $\varphi_{E,\infty}$ as the disjoint union of the strategies $\varphi_{E,[k_i,k_{i+1}[}$. This is a positional strategy; we argue that it is almost-surely winning. Indeed, since $\varphi_{E,\infty}$ ensures that going through any slice, a vertex in F will be visited with probability half, the Borel-Cantelli Lemma implies that infinitely many vertices in F will be visited with probability one. ◀

4.2 The Reduction

Fix an alternating Büchi tree automaton $\mathcal{A} = (Q_\exists, Q_\forall, \Sigma, \Delta, q_{\text{in}}, \rho)$. In order to check whether $L^{=1}(\mathcal{A}) = \emptyset$, we design an imperfect information *stochastic* Büchi game $\mathbb{G}^{=1}_{\mathcal{A}}$, in a way similar to the one to decide whether $L(\mathcal{A}) = \emptyset$ taking advantage the positionality result established in Theorem 7. In the game, Éloïse describes both a tree t and a positional strategy φ_t for her in the game $\mathbb{G}^{=1}_{\mathcal{A},t}$; the strategy φ_t is described as a $\mathcal{T}$-labeled tree (where $\mathcal{T}$ is the set of functions from $Q_\exists$ into $Q \times Q$, see Remark 2.2. As the plays are of ω-length, she actually does not fully describe t and φ_t but only a branch: this branch is chosen by Random, and Abélard takes care of computing the sequence of states along it (either by updating an existential state accordingly to φ_t or, when the state is universal, by choosing an arbitrary valid transition of the automaton). Éloïse observes the directions. Formally, we let $\mathcal{G}^{=1}_{\mathcal{A}} = (S, s_{\text{in}}, A, T, \sim)$ where $S = (Q \times \{0, 1\}) \cup \{(q_{\text{in}}, \varepsilon)\}$ and $s_{\text{in}} = (q_{\text{in}}, \varepsilon)$; $A \subseteq \Sigma \times \mathcal{T}$ is the set of pairs (a, τ) such that for all $q \in Q_\exists$, $(q, a, q_0, q_1) \in \Delta$ where $\tau(q) = (q_0, q_1)$ and $T = \{((q, i), (a, \tau), d_{q_0,q_1}) \mid q \in Q_\exists \text{ and } \tau(q) = (q_0, q_1)\} \cup \{((q, i), (a, \tau), d_{q_0,q_1}) \mid q \in Q_\forall \text{ and } (q, a, q_0, q_1) \in \Delta\}$ where d_{q_0,q_1} is the probability distribution $(q_0, 0) \mapsto 1/2$ and $(q_1, 1) \mapsto 1/2$, and $(q, i) \sim (q', i)$ for all $q, q' \in Q$ and $i \in \{0, 1\}$. Define $\mathbb{G}^{=1}_{\mathcal{A}} = (\mathcal{G}_{\mathcal{A}}, \rho_{\mathcal{A}})$ with $\rho_{\mathcal{A}}(q, i) = \rho(q)$ for any $(q, i) \in S$.

With a proof similar to the one of Lemma 4, we have the following result.

▶ **Theorem 10.** *Éloïse almost-surely wins in $\mathbb{G}^{=1}_{\mathcal{A}}$ iff $L^{=1}(\mathcal{A}) \neq \emptyset$.*

From [9, 7], one can decide almost-sure winning in imperfect information Büchi games in EXPTIME, hence the same holds for checking emptiness of languages of the form $L^{=1}(\mathcal{A})$. One can also reduce the emptiness problem for probabilistic ω-words automaton with the almost-sure semantics (in the sense of [2]) to check emptiness of languages of the form $L^{=1}(\mathcal{A})$ hence, it implies lower bounds as well as undecidability results.

▶ **Theorem 11.** *(1) Deciding whether $L^{=1}(\mathcal{A}) = \emptyset$ for a given alternating Büchi tree automaton $\mathcal{A}$ is an EXPTIME-complete problem. (2) Deciding whether $L^{=1}(\mathcal{A}) = \emptyset$ for a given alternating co-Büchi tree automaton $\mathcal{A}$ is an undecidable problem.*

▶ Remark. As one can decide whether $L^{=1}(\mathcal{A}) = \emptyset$ for non-deterministic tree automata [6], Theorem 11 implies that there is no effective simulation theorem for co-Büchi alternating qualitative tree automata.

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