# Reachability Problems for Continuous Linear Dynamical Systems*

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## Abstract

This talk is about reachability problems for continuous-time linear dynamical systems. A central decision problem in this area is the Continuous Skolem Problem [1], which asks to determine the existence of a zero of a real-valued function $f$ satisfying an ordinary linear differential equation

$$f^{(n)} + a_{n-1}f^{(n-1)} + \ldots + a_0 f = 0$$

with coefficients $a_0, \ldots, a_{n-1} \in \mathbb{Q}$ and initial conditions $f(0), \ldots, f^{(n-1)}(0) \in \mathbb{Q}$. An alternative formulation of the problem asks whether the solution $x(t) \in \mathbb{R}^n$ of a given differential equation $x' = Ax + b$, with $A$ a rational $n \times n$ matrix and $b$ a rational $n$-dimensional vector, reaches a given halfspace.

The nomenclature *Continuous Skolem Problem* arises by analogy with the Skolem Problem for linear recurrence sequences [4]. The latter problem asks whether a sequence of integers satisfying a given linear recurrence has a zero term. Decidability is open for both the discrete and continuous versions of the Skolem Problem.

We show that the Continuous Skolem Problem lies at the heart of many natural computational problems on linear dynamical systems, such as reachability in continuous-time Markov chains and linear hybrid automata. We describe some recent work, done in collaboration with Chonev and Ouaknine [2, 3], that uses results in transcendence theory and real algebraic geometry to obtain decidability for certain variants of the problem. In particular, we consider a bounded version of the Continuous Skolem Problem, corresponding to time-bounded reachability. We prove decidability of the bounded problem assuming Schanuel's conjecture, a central conjecture in transcendence theory. We also describe some partial decidability results in the unbounded case in the case of functions $f$ satisfying differential equations of fixed low order.

Finally, we give evidence of significant mathematical obstacles to proving decidability of the Continuous Skolem Problem in full generality by exhibiting some number-theoretic consequences of the existence of a decision procedure for this problem.

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