Assumptionless Bounds for Random Trees

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Abstract

Let T be any Galton-Watson tree. Write $\mathrm{vol}(T)$ for the volume of T (the number of nodes), $\mathrm{ht}(T)$ for the height of T (the greatest distance of any node from the root) and $\mathrm{wid}(T)$ for the width of T (the greatest number of nodes at any level). We study the relation between $\mathrm{vol}(T)$, $\mathrm{ht}(T)$ and $\mathrm{wid}(T)$.

In the case when the offspring distribution $p = (p_i, i \geq 0)$ has mean one and finite variance, both $\mathrm{ht}(T)$ and $\mathrm{wid}(T)$ are typically of order $\mathrm{vol}(T)^{1/2}$, and have sub-Gaussian upper tails on this scale. Heuristically, as the tail of the offspring distribution becomes heavier, the tree T becomes "shorter and bushier". I will describe a collection of work which can be viewed as justifying this heuristic in various ways In particular, I will explain how classical bounds on Lévy's concentration function for random walks may be used to show that for any offspring distribution, the random variable $\mathrm{ht}(T)/\mathrm{wid}(T)$ has sub-exponential tails. I will also describe a more combinatorial approach to coupling random trees with different degree sequences which allows the heights of randomly sampled vertices to be compared.

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