Algebraic Algorithms for Finding Patterns in Graphs

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— Abstract

I will give a gentle introduction to algebraic graph algorithms by showing how to determine if a given graph contains a simple path of length k. This is a famous problem admitting a beautiful and widely-known algorithm, namely the colour-coding method of Alon, Yuster and Zwick (1995). Starting from this entirely combinatorial approach, I will carefully develop an algebraic perspective on the same problem. First, I will explain how the colour-coding algorithm can be understood as the evaluation of a well-known expression (sometimes called the "walk-sum" of the graph) in a commutative algebra called the zeon algebra. From there, I will introduce the exterior algebra and present the algebraic framework recently developed with Brand and Dell (2018).

The presentation is aimed at a combinatorially-minded audience largely innocent of abstract algebra.

2012 ACM Subject Classification Theory of computation \rightarrow Fixed parameter tractability; Mathematics of computing \rightarrow Paths and connectivity problems; Mathematics of computing \rightarrow Graph algorithms

 $\mathsf{Keywords}$ and $\mathsf{phrases}$ paths, exterior algebra, wedge product, color-coding, parameterized complexity

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Category Invited Talk

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— References

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