Brief Annoucement: On Extending Brandt's Speedup Theorem from LOCAL to Round-Based Full-Information Models

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Abstract

Given any task Π, Brandt's speedup theorem (PODC 2019) provides a mechanical way to design another task Π' on the same input-set as Π such that, for any $t \geq 1$, Π is solvable in t rounds in the LOCAL model if and only if Π' is solvable in $t-1$ rounds in the LOCAL model. We dissect the construction in Brandt's speedup theorem for expressing it in the broader framework of all round-based models supporting full information protocols, which includes models as different as asynchronous wait-free shared-memory computing with iterated immediate snapshots, and synchronous failure-free network computing.

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1 Introduction

Given a complexity or computability result established for a distributed computing model $\mathcal{M}_1$, several questions can be raised. Does this result hold for another model $\mathcal{M}_2$? What makes this result true for $\mathcal{M}_1$ but not for $\mathcal{M}_2$, or what are the features common to $\mathcal{M}_1$ and $\mathcal{M}_2$ that make the result true for both models? For instance, if a result holds in the LOCAL model [4,5], is it because the model is synchronous? Is it because processes and communication links are failure-free? Is it because the network satisfies some property (e.g., large girth)? Is it because the problem satisfies some property (e.g., local checkability)? A typical example is Brandt's speedup theorem [3]. This theorem essentially provides a mechanical way to construct a task Π' from any task Π, on the same input set as Π, such that, for every $t \geq 1$, Π is solvable in t rounds in LOCAL if and only if Π' is solvable in $t-1$ rounds in LOCAL. This theorem is an efficient tool for designing lower bounds. Indeed, starting from a task Π, iterating the construction results in a series of tasks $\Pi^{(r)}, r \geq 1$, such that, for every $t \geq 1$, Π is solvable in t rounds if and only if $\Pi^{(r)}$ is solvable in $t-r$ rounds. In particular, $\Pi^{(t)}$ is solvable in zero rounds, and demonstrating that $\Pi^{(t)}$ is actually not solvable in zero rounds establishes the lower bound $t+1$ for the round-complexity of Π.

Brandt's speedup theorem does not directly applies to LOCAL, but to an anonymous variant of LOCAL on graphs with sufficiently large *girth*. This is because the presence of identifiers assigned to the nodes prevents *local-independence* to be satisfied, where the latter is a property that is essential for establishing the theorem. It is not trivial to formally express this property, but, roughly speaking, given the radius-$(t-1)$ views of two adjacent nodes v and v' in some network G, the presence of identifiers results in the fact that one cannot guarantee

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that two independent extensions of these two views into radius-t views are compatible. Local independence also imposes to consider graphs G with girth $g > 2t-1$. Indeed, in graphs with girth $g \leq 2t-1$, two independent radius-t extensions of the radius-$(t-1)$ views of v and v' may include a same node w provided with different identifiers, or with different inputs. This would result into two non-compatible radius-t extensions in the sense that there are no instances yielding the simultaneous presence of these two radius-t views at two adjacent nodes. Also, Brandt's speedup theorem requires the tasks at hand to be *locally checkable*. This property essentially says that, given an assignment of input-output values to the nodes, the correctness of the collection of output values with respect to the collection of input values can be established by merely inspecting the values of each node and of its neighbors in the network. In other words, a task is locally checkable if the correctness of an assignment of values to the nodes is defined as the conjunction of the local correctness of this assignment, where "local" refers to the closed neighborhood of each node. Proper coloring and maximal independent set (MIS) are typical examples of locally checkable tasks in LOCAL.

We can now rephrase our original questioning in the specific case of Brandt's speedup theorem: does this theorem holds in other models? For such a question to make sense, we restrict attention to models in which the notion of *rounds* is defined, which naturally include synchronous models in networks with multiparty interactions, namely *hypergraphs*, and synchronous models in networks that evolve with time, namely, *dynamic networks*. Round-based models however include far more than just synchronous models in networks. For instance, asynchronous shared-memory computing with *iterated immediate snapshots*, referred to as WAIT-FREE in the following, which is computationally equivalent to asynchronous read/write shared-memory computing with crash-prone processes, is round-based. The same holds for t-resilient computing, $0 \leq t \leq n-1$, which is essentially the same as WAIT-FREE, but where at most t processes can crash [1]. The LOCAL model has another feature. It supports *full information* communication protocols. That is, whenever a process receives information from another process, one can assume that the latter has sent *all* the data it acquired before the communication took place. This assumption enables the design of strong lower bounds, which hold even if the processes are not restricted in term of volume of communication. Also, the LOCAL model does not restrict the individual computational power of the processes. This assumption enables the design of unconditional lower bounds, which hold independently from complexity or computability assumptions regarding the computing power of each individual process. All the models mentioned above support full-information protocols, and have unlimited individual computational power.

So, making our questioning even more specific: Is there an analog of Brandt's speedup theorem for all round-based models supporting full-information protocols with unlimited individual computational power? If not, what make the LOCAL model so special? If yes, for which models? Under which conditions?

2 Our Results

We refer to [2] for a complete description of our results. Using the framework provided by combinatorial topology applied to distributed computing, we give a general definition of speedup tasks for round-based models supporting full-information protocols (see Fig. 1). Given a task Π in the LOCAL model, Brandt's speedup theorem constructs such a speedup task $\Pi' = \Phi(\Pi)$. We then revisit Brandt's construction, that is, we dissect the nature of the operator Φ transforming any task Π into a task $\Pi' = \Phi(\Pi)$, for identifying the central assumptions allowing this construction to work in LOCAL.

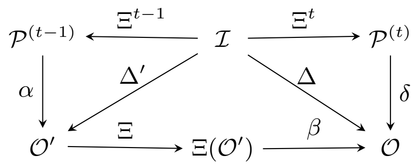

Figure 1 The task $\Pi' = (\mathcal{I}, \mathcal{O}', \Delta')$ is a speedup task for $\Pi = (\mathcal{I}, \mathcal{O}, \Delta)$, where $\mathcal{I}$ and $\mathcal{O}$ denote the input and output complexes, respectively, and Δ denotes the input-output specification. Ξ denotes the map corresponding to the communication model $\mathcal{M}$ at hand, and $\mathcal{P}^{(t)}$ denotes the protocol complex at round t. All maps α, β, and δ are simplicial.

They are two central assumptions in Brandt's construction: *local checkability* and *local independence*. We extend these two notions from the LOCAL model to round-based models supporting full-information protocols. We also extend Brandt's operator Φ to all such models. We denote by $\Phi^\star$ this extension. As a result, we are able to express a general speedup theorem, which roughly reads as follows.

▶ **Theorem 1.** *Let $\mathcal{M}$ be a round-based model supporting full-information protocols, let Π be a task, and let $t \geq 1$. The task $\Phi^\star(\Pi)$ satisfies the following:*

1. *Assume that Π satisfies $(t-1)$-independence with respect to $\mathcal{M}$. If Π is solvable in at most t rounds, then $\Phi^\star(\Pi)$ is solvable in at most $t-1$ rounds.*
2. *Assume that Π is locally checkable in $\mathcal{M}$. If $\Phi^\star(\Pi)$ is solvable in at most $t-1$ rounds, then Π is solvable in at most t rounds.*

Statement 1 guarantees that the task $\Phi^\star(\Pi)$ is at least "1-round faster" than the original task Π. Note that that local independence is actually sufficient for deriving lower bounds. Statement 2 guarantees that $\Phi^\star(\Pi)$ is no more than "1-round faster", and, in particular, that $\Phi^\star(\Pi)$ is not a "trivial" task. Observe that the sets of hypotheses required for each of the two statements are different. Concretely, our general construction $\Phi^\star$ allows us to directly extend Brandt's speedup theorem to various kinds of synchronous models in networks, including directed graphs, hypergraphs, dynamic networks, and even to graphs including short cyclic dependencies between processes (i.e., small girth). Interestingly, our general construction also enables to extend Brandt's speedup theorem to asynchronous failure-prone computing models such as WAIT-FREE. In particular, we provide a new impossibility proof for consensus and for perfect renaming in 2-process systems. The case of consensus is an example of a non locally checkable task for which our approach still provide a non-trivial lower bound.

3 Conclusion

Our construction $\Phi^\star$ is based on identifying specific subcomplexes of the output complex, for generalizing Brandt's construction. This approach is well suited to LOCAL, and to its extensions to hypergraphs and dynamic networks. However, it does not satisfactorily match the characteristics of WAIT-FREE for large systems, essentially because WAIT-FREE does not satisfy the local independence property whenever $n > 2$. Nevertheless, we conjecture that there might be another way to decompose the output complex into subcomplexes that would provide a speedup theorem for models not satisfying local independence (e.g., WAIT-FREE), but this decomposition still remains to be found.

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