

# Distributive Laws for Lawvere Theories

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## Abstract

Distributive laws give a way of combining two algebraic structures expressed as monads; in this work we propose a theory of distributive laws for combining algebraic structures expressed as Lawvere theories. We propose four approaches, involving profunctors, monoidal profunctors, an extension of the free finite-product category 2-monad from  $\mathbf{Cat}$  to  $\mathbf{Prof}$ , and factorisation systems respectively. We exhibit comparison functors between  $\mathbf{CAT}$  and each of these new frameworks to show that the distributive laws between the Lawvere theories correspond in a suitable way to distributive laws between their associated finitary monads. The different but equivalent formulations then provide, between them, a framework conducive to generalisation, but also an explicit description of the composite theories arising from distributive laws.

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