Visual Methods for Representing Flow Space with Vector Fields

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Abstract

The issue of human mobility has been a focal point of research among numerous scholars in the field of geography for decades. Among them, the visualization of origin-destination (OD) data is an important branch of geographic flow studies. In this paper, we vectorize and represent immigration flows using OD flow data of U.S. immigrants in the year 2000, constructing an immigration space. Through data validation, it is confirmed that the vector field satisfies the Gauss's theorem and is irrotational, demonstrating that the field can be derived from a potential and that the field is uniquely determined by the potential. Scalar potential fields are inferred from the vector field, providing a more intuitive and convenient description of the underlying flow patterns in geographical interaction matrices. Additionally, this paper employs potential fields and applies a density-equalizing areal cartogram to reconstruct the global representation of functional space, constructing cartogram maps based on potential magnitudes.

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1 Introduction

The issue of human mobility has long been a focus of research in the field of geography. These studies encompass various areas such as urban spatial structure, urban and regional development, transportation and infrastructure planning, environmental pollution [1], elections and political polarization [3], among others. The visualization of OD data is an important branch of geographic flow research. Plane used a reverse doubly constrained gravity model to calibrate and estimate the cognitive or functional distances between states based on observed interstate migration flows, choosing distances to represent observed flows and visualizing "migration space" as distorted maps [6]. In 1976, Tobler introduced the concept of vector fields and proposed a vector representation method based on OD data, considering the scalar potential of vector fields as a way to describe hidden forces [7]. In recent years, scholars have

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applied vector fields to the study of urban spatial structure. For example, Mazzoli et al. used data from multiple cities to demonstrate that vector fields constructed from flow data satisfy the Gauss (divergence)'s theorem and possess irrotationality. They also explored the utility performance of gravity and radiation models in vector fields based on flow data [4]. Yang, H et al. based on Mazzoli's vector representation method, defined anomalous fields, source fields, and dispersion fields to identify abnormal human flows [9]. Furthermore, scholars have used spatio-temporal potential fields to predict traffic flows [8] and analyzed trade flows in regional science and spatial economics using vector gradient methods and gravity models [5].

Tobler proposed that M_{ij} refers to the flow from location i to location j. Flows occur between various scales, such as communities, cities, and regions. The OD matrix M_{ij} only contains information about the origin and destination of trips and does not include information about intermediate points along the trajectories or visits. The directional attribute of the vector representing the flow is represented by a unit vector from location i to location j. Tobler demonstrated through a series of algebraic transformations that the frequency of the differences between interactions in both directions divided by their sum (i.e. $(M_{ij}$- $M_{ji})/(M_{ij} + M_{ji}))$ is considerably high and robust. Therefore, this quantity is introduced as a component for constructing vectors. Finally, the vectors pointing from the origin location i to the destination location j are summed, defining a vector field in space. Another approach to construct vectors is directly using net migration flow as the numerator in the vector representation [4]. Additionally, besides the aforementioned methods of constructing vectors, error terms can also be utilized [2]. The vector representation method based on error terms incorporates detailed errors related to observed flows between regions (such as model errors and missing variables like individual preferences), measurement errors (pertaining to the variables themselves), and pure random effects. On the other hand, the vector $\vec{c_i}$ represents half the difference between opposing direction error terms.

Every vector field can be written as the gradient of a scalar field plus an additional vector field. These two components are respectively referred to as the scalar potential and the vector potential. If the second field is everywhere zero and only then, the original vector field can be identified as the gradient of a scalar field. In order to recover this scalar potential, the gradient operation can be reversed through integration to compute the scalar potential. Therefore, if we want to determine the scalar potential of a vector field, it is necessary to ensure that the rotational vector at every point of this vector field is zero, indicating a curl-free condition. The curl-free property of a field implies that the field can be derived from a potential, where the field is uniquely determined by the potential alone.

Currently, research related to representing flows using vectors is relatively scarce. Besides, the majority of existing flow visualization techniques primarily involve flow mapping, which provides a descriptive representation of the flow. In contrast, our method employs the concept of potentials to interpret the flow, offering a distinct visualization approach. As a result, these two methods are not directly comparable. In this study, based on interstate migration data from the United States, we will investigate the problem starting from the core concept of vector space. We will begin with the vector field constructed from OD flow data and generate a potential field through integration.

2 Data and Methods

2.1 Data

The primary data for this study consists of population migration data between states in the United States from the 1965–1970, 1975–1980, 1985–1990, and 1995–2000 censuses. Additionally, data on the physical distances between states during each time period and the population centroid coordinates for each state are included.

2.2 Theoretical models

In terms of vector definition, two different approaches have been identified based on methods applied in various literature. The first approach, proposed by Tobler, involves constructing vectors based on relative net flow. The second approach, employed by Mazzoli, utilizes absolute flow.

(1) Vector Representation Based on Relative Migration Flow

As mentioned earlier, d_{ij} represents the physical distance between the population centroids of locations i and j; $\vec{C_i}$ denotes the vector aggregation centered at the population centroid of location i, originating from the origin point O. The vector space should contain 48 vectors located at the population centroids of each state, with distinct directions and magnitudes. The formulas are shown in (1) and (2).

$$d_{ij}^2 = (X_j - X_i)^2 + (Y_j - Y_i)^2 \tag{1}$$

$$\vec{C_i} = \frac{1}{n-1} \sum_{j=1}^{n} \frac{M_{ij} - M_{ji}}{M_{ij} + M_{ji}} \frac{1}{d_{ij}} [(X_j - X_i), (Y_j - Y_i)] \tag{2}$$

(2) Vector Representation Based on Absolute Migration Flow

T_{ij} represents the migration flow from location i to j, $\vec{u_{ij}}$ represents the unit vector (directional attribute) from i to j. Then, the vectors pointing to all destination locations j are summed to obtain the resulting vector at each location i. m_i represents the total outflow from location i. Finally, the vector $\vec{W_i}$ can be constructed as shown in equation (3) These vectors define a field in space, determining the average outward direction of movement at each point.

$$\vec{W_i} = \frac{\vec{T_i}}{m_i} = \sum_{j}^{n} \frac{T_{ij}}{m_i} \vec{u_{ij}} \tag{3}$$

$$m_i = \sum_{j}^{n} T_{ij} \tag{4}$$

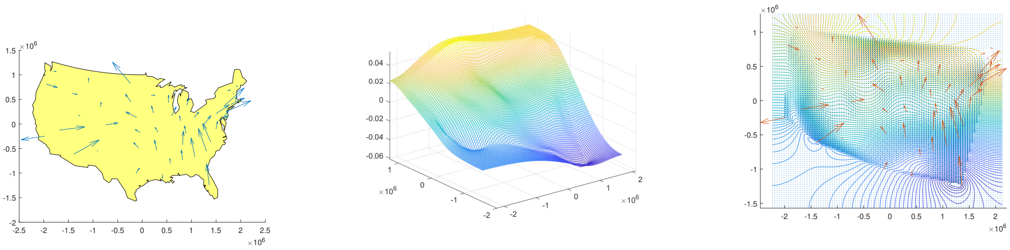
3 Results

Using US migration data from the period of 1995–2000, we constructed vector fields, three-dimensional grid visualizations of migration potential, and contour maps of migration potential based on the different vector representation methods (Figure 1 and 2).

Among them, the visualizations of the vector representation method based on net flow (Tobler's method) for the time periods of 1965–1970, 1975–1980, 1985–1990 are shown in the following figures (Figure 3, 4, and 5):

4 Discussion

Based on the migration flows in and out of each state, California consistently ranks at the top in terms of both incoming and outgoing population numbers among all states. Following closely are states like Texas, Florida, and New York, which also exhibit significant flows of population in and out. It is noteworthy that Texas and Florida are located in the moderate climate region known as the "Sun Belt". This aligns with the notable trend of population

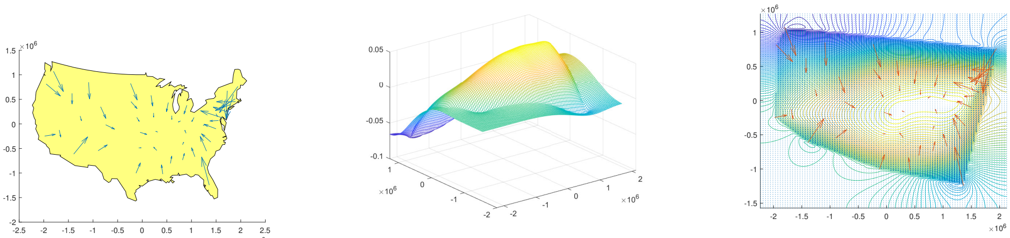


(a) Migration vectors.

(b) Mesh grid visualization of migration potential shown in 3-D.

(c) contour map of migration potential with observed (red) and interpolated (blue) vector fields.

Figure 1 Vector representation based on relative net flow(1995–2000).

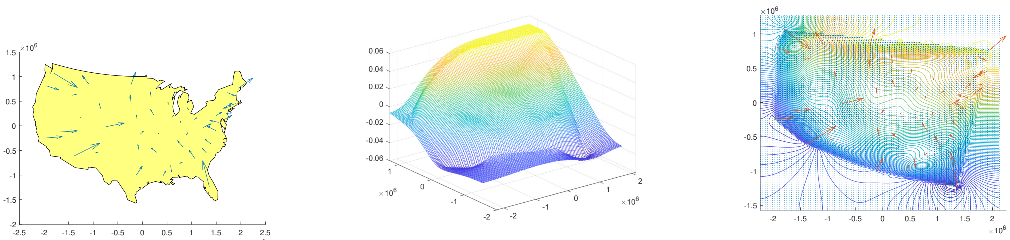


(a) Migration vectors.

(b) Mesh grid visualization of migration potential shown in 3-D.

(c) contour map of migration potential with observed (red) and interpolated (blue) vector fields.

Figure 2 Vector representation based on absolute traffic(1995–2000).

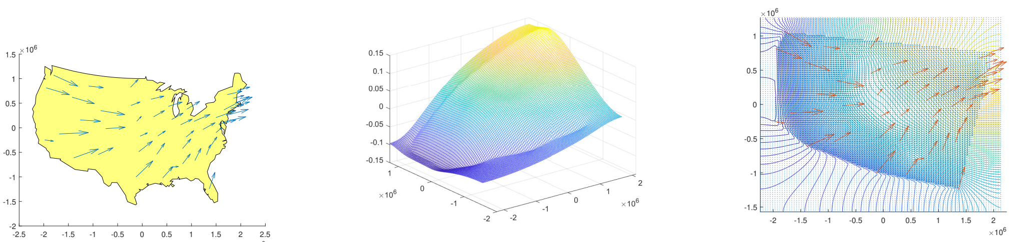


(a) Migration vectors.

(b) Mesh grid visualization of migration potential shown in 3-D

(c) contour map of migration potential with observed (red) and interpolated (blue) vector fields.

Figure 3 Vector representation based on relative net flow(1965–1970).

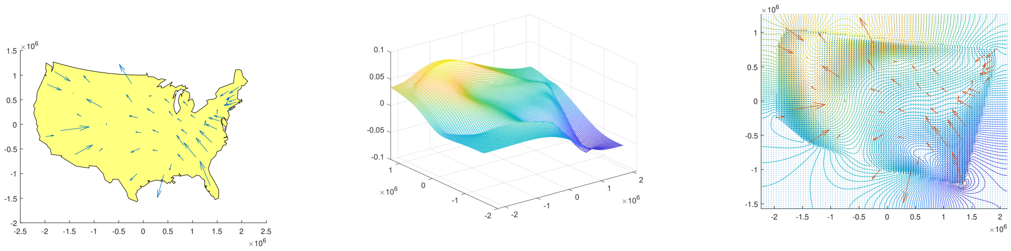


(a) Migration vectors.

(b) Mesh grid visualization of migration potential shown in 3-D.

(c) contour map of migration potential with observed (red) and interpolated (blue) vector fields.

Figure 4 Vector representation based on relative net traffic(1975–1980).



(a) Migration vectors.

(b) Mesh grid visualization of migration potential shown in 3-D.

(c) contour map of migration potential with observed (red) and interpolated (blue) vector fields.

Figure 5 Vector representation based on relative net flow(1985–1990).

migration from northern regions of the United States to warmer southern areas, often referred to as the "Sun Belt." During the time period of 1985–1990, there was a significant decline in both incoming and outgoing population flows across states. This could be attributed to various socio-economic and political factors, including the stabilization of the U.S. economy during that period. Such factors likely influenced people's decisions regarding migration, leading to a decrease in population mobility between states.

The results obtained from the vector representation method based on absolute flow exhibit overall similarities to those obtained from Tobler's method, depicting similar patterns at a macro level. This discrepancy arises from the different definitions of vector flows, leading to distinct underlying interpretations of the vector fields. The three-dimensional grid visualization of the scalar potential in Figure 1b) exhibits greater complexity compared to Figures 2b), featuring two peaks. This complexity is particularly meaningful in exploring the implicit forces underlying interactions. Similar to how Tobler conceptualizes the flow field as "wind," this wind implies a potential function that facilitates interactions in specific directions, aiding in uncovering the causes of asymmetric interactions.

In the vector representation method based on relative net flow, the visualizations in Figures 3, 4, and 5 reveal certain migration patterns during the 1970s and around that time, indicating that the central region of the United States was a primary destination for population movements (Figure 3a)). From the mid to late 1980s, there were noticeable fluctuations in both the origins and destinations of migrants, gradually shifting towards the northeastern part of the country (Figure 4a)). The trend of migration towards the central region persisted in the 1990s (Figure 5a)). However, in the period from 1995 to 2000, significant changes in migration patterns occurred, particularly in California, where a major shift in immigration patterns was observed, along with slight outward migration from some northeastern states (Figure 3a)). Figure 3b), 4b), and 5b) represents the migration potential, revealing implicit forces that can be further explored.

5 Conclusion

Functional spaces are closely connected to human perception and utilization of physical spaces. Conceptualizing the spatial patterns of functional relationships embedded within physical spaces is crucial for understanding the spatial interaction processes that shape geographical phenomena. The use of vector fields to introduce and describe implicit forces in interactions proves valuable. Vector fields approximate the gradient of scalar potentials, which can be used to explain flows. While vector field methods do not directly transform relative distances into functional spaces, they offer an alternative perspective for integrating spatial interaction patterns and aid in developing a global view of functional spaces.

From a visualization perspective of geographic flows, this paper proposes a methodological framework for constructing migration flow vector spaces. Two vector construction methods are introduced: 1) constructing interaction force fields based on the difference between interactions in two directions divided by their sum; 2) representing vectors based on absolute flow quantities to establish the vector space. After demonstrating the irrotational of the field and satisfying the Gauss (divergence)'s theorem, a scalar potential field is inferred through integration, facilitating the description of implicit flow patterns within geographic interaction matrices.

This research exploration can be further extended to different aspects of vector space, such as different vector expressions, vector aggregation methods (at the origin or destination), vector weighting approaches, and vector field superposition methods. By constructing different vector representations of OD flows and comparing their visual effects and inherent properties, we can explore their suitability for various research topics and applications. Furthermore, the migration space constructed in this study mainly focuses on inter-city or inter-state scales. There is potential to investigate vector fields and scalar potentials of intra-city commuting flows. This would provide valuable insights into urban spatial organization, city centers, urban boundaries, infrastructure planning, and public services, among other aspects.

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