Approximating Fixpoints of Approximated Functions

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Abstract

There is a large body of work on fixpoint theorems, guaranteeing the existence of fixpoints for certain functions and providing methods for computing them. This includes for instance Banachs's fixpoint theorem, the well-known result by Knaster-Tarski that is frequently employed in computer science and Kleene iteration.

It is less clear how to compute fixpoints if the function whose (least) fixpoint we are interested in is not known exactly, but can only be obtained by a sequence of subsequently better approximations. This scenario occurs for instance in the context of reinforcement learning, where the probabilities of a Markov decision process (MDP) – for which one wants to learn a strategy - are unknown and can only be sampled. There are several solutions to this problem where the fixpoint computation (for determining the value vector and the optimal strategy) and the exploration of the model are interleaved. However, these methods work only well for discounted MDPs, that is in the contractive setting, but not for general MDPs, that is for non-expansive functions.

After describing and motivating the problem, we will in particular concentrate on the non-expansive case. There are many interesting systems who value vectors can be obtained by determining the fixpoints of non-expansive functions. Other than contractive functions, they do not guarantee uniqueness of the fixpoint, making it more difficult to approximate the least fixpoint by methods other than Kleene iteration. And also Kleene iteration fails if the function under consideration is only approximated.

We hence describe a dampened Mann iteration scheme for (higher-dimensional) functions on the reals that converges to the least fixpoint from everywhere. This scheme can also be adapted to functions that are approximated, under certain conditions.

We will in particular study the case of MDPs and consider a related problem that arises when performing model-checking for quantitative μ-calculi, which involves the computation of nested fixpoints.

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