Eating Ice-Cream with a Colander

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Abstract

k-order α-hull is a generalization of both k-hull and α-shape (which are generalizations of convex hull); since its introduction in a 2014 IPL paper (which also established its combinatorial properties and gave efficient algorithms to compute it), it was used in a variety of applications (as witnessed by 38 citations in Google Scholar) ranging from computer graphics to hydrology to seismology. The subject must have been so rich and complex that it took more than a year to review the submission at IPL (which was chosen as the venue "Devoted to the Rapid Publication"), as may be witnessed by the timeline in the paper header. Nonetheless it was not rich enough to warrant publication at SODA 2009 and WADS 2009 (the reviews saying it is not yet ready for the prime time – cited from memory) nor in FUN 2010 to which the paper was submitted under the title "Eating Ice-Cream with Colander"

2012 ACM Subject Classification Theory of computation → Computational geometry

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1 The surveyed paper

This note is surveys the paper [6]

D. Krasnoshchekov and V. Polishchuk. Order-k α-hulls and α-shapes.
Information Processing Letters, 114(1-2):76–83, 2014.
`https://doi.org/10.1016/j.ipl.2013.07.023`
Publicly available version: `https://www.itn.liu.se/~valpo40/pages/ka.pdf`

Convex hull of a point set $S \subset \mathbb{R}^2$, which may be defined as the complement of the union of all halfplanes that contain no points of S, gives a rough description of the "shape" of S (Fig. 1, left). α-hull (the complement of the union of all disks of radius α that contain no points of S; in particular, convex hull is α-hull for $\alpha = \infty$) and α-shape (a "straight-line" version of α-hull) [2] of S outline the shape more precisely (Fig. 1, right).

As rightly stated in [6], "Often the set of points representing a geometric object is obtained by sampling the object in the presence of noise, which can introduce outliers in the data"; the outliers may distort the shape (Fig. 2a,b). The surveyed paper defined generalizations of α-hull and α-shape: *k-order α-hull* (the complement of the union of all disks of radius α that contain less than k points of S) and *k-order α-shape* (a "straight-line" version of k-order α-hull– the exact definition is somewhat technical; see [6, Definition 2.6]). The generalizations are "capable of ignoring a certain amount of outliers, which results in a more robust shape reconstruction" (Fig. 2c,d).

Another generalization of convex hull of S is k-hull (the locus of points such that any halfplane through a point in the k-hull contains at least k points from S) [1], widely used in statistics where it is known under the name of k-depth contour because it is the level set

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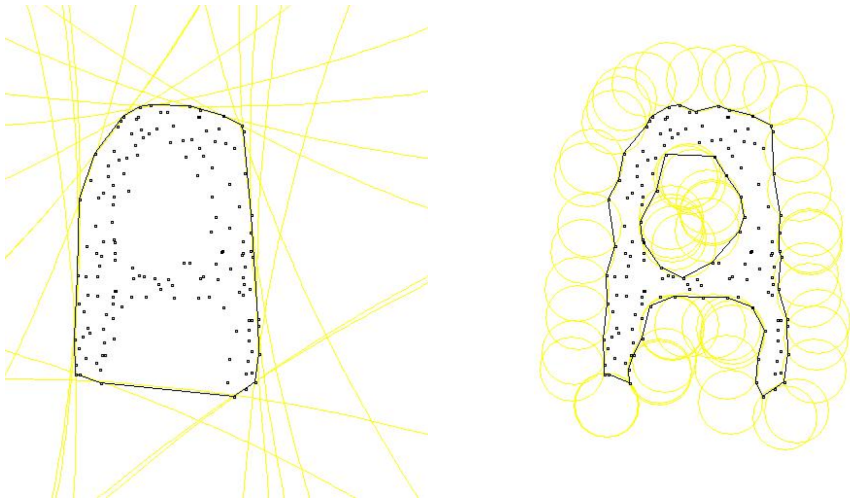

Figure 1 Convex hull (left) misses important features of the shape (e.g., the hole). α-shape (right) does a better job. Yellow are the empty circles; for the convex hull, the circles are infinite-radius (halfplanes). The images are generated with the applet [7] (downloadable from [11]) accompanying the surveyed paper.

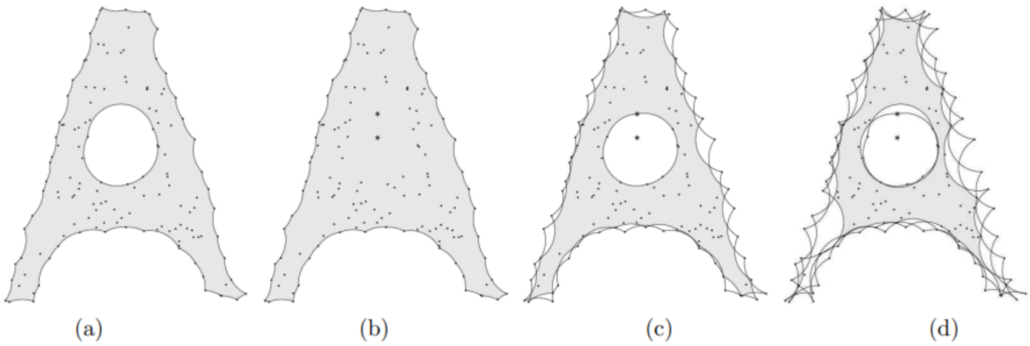

Figure 3: Order-k α-hull ignores the outliers. (a): An α-hull. (b): Two outliers change the picture. (c) and (d): Order-k α-hull with $k = 2, 3$ brings it back.

Figure 2 Figure 3 from [6], together with its caption.

of the "location depth" (aka halfspace depth). The rich field of statistical data depth has produced a number of depth functions (many of which generalize convex hull in the sense that the points on the convex hull have depth 1) which are unimodal functions of distance to the "center" – the deepest point defined by S (the center may not belong to S). k-order α-hull, which is a generalization of k-hull, allows the depth to have local maxima, allowing each cluster of data to have its own depth function (Fig. 3).

Fischer [4], inspired by Edelsbrunner and Mücke [3], had a fun view of the space as an infinite piece of icecream, with points S being chocolate chips in it; then α-hull is the icecream that remains after a person, allergic to chocolate, eats as much icecream as possible with a scoop of radius α. The surveyed paper extended this fun view of α-hulls, suggesting how convex hull, k-hull and k-order α-hull may be obtained by eating as much icecream as possible with various kitchenware (Fig. 4).

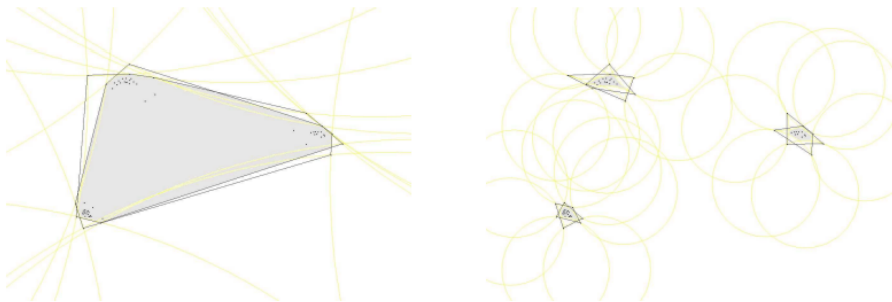

Figure 6: Points come in 3 clusters. Left: The 2-hull (2-depth contour) oversmoothes the data. Right: Order-2 α-hull (for a suitable $\alpha < \infty$) better fits the data by identifying 2-depth contours independently within each cluster.

Figure 3 Figure 6 from [6], together with its caption.

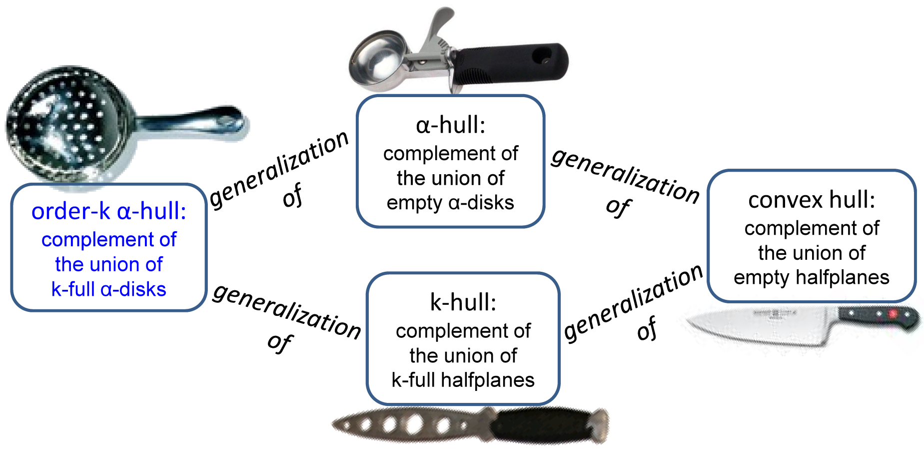

Figure 4 Figure 2 from [6]: The family picture.

2 The paper history

The paper was born in discussions between geophysicists (who needed to do robust shape analysis of seismological data) and algorithmists (computational gasometers who were interested in developing extensions of α-shapes and k-hulls); the results of the joint work were presented in a poster already in 2008 [5]. The algorithmic/combinatorial results on k-order α-hulls and α-shapes were submitted to SODA 2009 and WADS 2009 none of which saw the potential of the introduced notions; FUN 2010 (to which the paper was submitted as “Eating Ice-Cream with Colander”) did not find the kitchenware amusing enough.

Due to lack of luck with algorithmic conferences, the obtained results (both seismic data analysis and algorithmic/combinatorial results on k-order α-hulls and α-shapes) were submitted to Geophysical Journal International which liked the seismology results, but fairly suggested that the algorithmic part should be reviewed by experts in an algorithmic journal (to get a “stamp of correctness”). By this time, the seismology colleagues were rightly anxious to get the results out, so IPL was chosen as the algorithms venue “Devoted to the Rapid Publication” [10]. After a 1.5-year review (see the paper header: “Received 20 January 2012, Revised 24 July 2013, Accepted 27 July 2013”) the paper was accepted, and it was no problem to publish the results from application of the techniques immediately [8, 9]. By now, the surveyed paper gathered 38 citations on Google Scholar, in subjects ranging from computer graphics to hydrology to seismology to urban airspace optimization. Overall, the subject is quite pictorial, see, e.g., the video [7] that accompanied the presentation of interactive applet [11] at SoCG 2010 (the web version of the applet requires Java in the browser, but the standalone version, downloadable from [11], may be run on any machine).

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