Ex-Ante Constraint Elicitation in Incomplete DCOPs

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Abstract

Distributed Constraint Optimization Problems (DCOPs) is a framework for representing and solving distributed combinatorial problems, where agents exchange messages to assign variables they own, such that the sum of constraint costs is minimized. When agents represent people (e.g., in meeting scheduling problems), the constraint information that the agents hold may be incomplete. For such scenarios, researchers proposed Incomplete DCOPs (I-DCOPs), which allow agents to elicit from their human users some of the missing information. Existing I-DCOP approaches evaluate solutions not only by their quality, but also the elicitation costs spent to find them (*ex-post*). Unfortunately, this may result in the agents spending a lot of effort (in terms of elicitation costs) to find high-quality solutions, and then ignoring them because previous lower-quality solutions were found with less effort.

Therefore, we propose a different approach for solving I-DCOPs by evaluating solutions based on their quality and considering the elicitation cost beforehand (*ex-ante*). Agents are limited in the amount of information that they can elicit and, therefore, need to make smart decisions on choosing which missing information to elicit. We propose several heuristics for making these decisions. Our results indicate that some of the heuristics designed produce high-quality solutions, which significantly outperform the previously proposed ex-post heuristics.

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1 Introduction

The Distributed Constraint Optimization Problem (DCOP) formulation is widely used for representing and solving combinatorial optimization problems that are distributed by nature [5, 7, 15]. It includes agents holding variables, which are constrained with variables held by other agents (their neighbors) and attempt to find an optimal assignment to their variables that minimizes constraint costs, while exchanging messages with their neighbors.

When agents represent humans, such as in meeting scheduling problems [4, 1], the information held by agents regarding the preferences of the humans that they represent may be incomplete. Agents can elicit information from the humans by introducing queries to their human users, However, humans might find that answering these queries is a tedious task and may abandon the use of the system if the burden is too heavy. Thus, there is a clear need to limit the amount of queries that the human users need to answer.

In order to represent such situations and allow agents to select high-quality assignments to their variables, while taking into consideration incomplete information, and make a limited use of elicitation queries, the Incomplete DCOP (I-DCOP) model was proposed [10, 11, 12]. I-DCOP enables the representation of partial information by having agents hold constraint tables in which some entries include the costs for the corresponding combination of assignments and some do not. The agents can use elicitation queries to fill some of the empty entries and then use the information available to them in order to select the solution to the problem.

Tabakhi *et al.* [11, 12] proposed limiting the use of elicitation queries by evaluating the outcome of the I-DCOP solving process as a weighted sum of the quality of the selected solution and the effort (e.g., number of queries asked) for producing it. Thus, the agents aimed to find a solution that has not only a high quality, but also a low effort to find it. While this evaluation of outcomes incentivizes the algorithm to make efficient use of the human query resources, from a practical point of view, this method for evaluating possible outcomes does not make sense.

For example, imagine that an agent is searching for a hotel for the next trip of the person it represents. After a small search effort c_1, the agent finds a decent hotel with solution quality q_1. Then, the agent decides to spend more effort, searching for a better hotel and, after a costly effort $c_2 >> c_1$, it manages to find one that is slightly better $q_2 > q_1$. According to the evaluation method proposed [11, 12], the agent will choose the first hotel because $c_1 - q_1 < c_2 - q_2$.[1] In other words, the second hotel is not as good because the marginal increase in quality is not worth the large amount of effort spent for it. However, intuitively, since the search effort was already spent, it does not make sense to not use the better solution found.

The key issue with the prior approach is that the search effort considered is done ex-post – *after* the effort was spent – when it should be done ex-ante – *before* the effort was spent. With this insight in mind, we propose a different approach for solving incomplete DCOPs. Inspired by others [3], we limit the amount of queries that agents can use (i.e., a query "budget") and propose different heuristic strategies for the agents to follow when they decide what information to elicit. We compare the success of the proposed strategies in comparison with the existing ex-post approach, in combination with a complete SyncBB algorithm [2] and two incomplete DSA and MGM [15, 17] algorithms.

Our results indicate that all the ex-ante heuristic strategies we proposed outperformed the existing ex-post heuristic. Moreover, the heuristics that spend effort in identifying parts of the search space that have higher probability to be part of a high-quality solution are more successful.

2 Background

In this section, we present DCOPs and three algorithms for solving them: SyncBB, DSA, and MGM.

2.1 Distributed Constraint Optimization Problems

Without loss of generality, in the rest of this paper, we will assume that all problems are minimization problems, as it is common in the DCOP literature [1]. Thus, we assume that all constraints define costs and not utilities.

[1] We assume that we are minimizing costs in this paper.

A DCOP is defined by a tuple $\langle \mathcal{A}, \mathcal{X}, \mathcal{D}, \mathcal{R} \rangle$. $\mathcal{A}$ is a finite set of agents $\{A_1, A_2, \ldots, A_n\}$. $\mathcal{X}$ is a finite set of variables $\{X_1, X_2, \ldots, X_m\}$. Each variable is held by a single agent, and an agent may hold more than one variable. $\mathcal{D}$ is a set of domains $\{D_1, D_2, \ldots, D_m\}$. Each domain D_i contains the finite set of values that can be assigned to variable X_i. We denote an assignment of value $x \in D_i$ to X_i by an ordered pair $\langle X_i, x \rangle$. $\mathcal{R}$ is a set of relations (constraints). Each constraint $R_j \in \mathcal{R}$ defines a non-negative *cost* for every possible value combination of a set of variables, and is of the form $R_j : D_{j_1} \times D_{j_2} \times \ldots \times D_{j_k} \rightarrow \mathbb{R}^+ \cup \{0\}$. A *binary constraint* refers to exactly two variables and is of the form $R_{ij} : D_i \times D_j \rightarrow \mathbb{R}^+ \cup \{0\}$. We say that a variable is *involved* in a constraint if it is one of the variables the constraint refers to and that an agent is involved in a constraint if one of its variables is involved in the constraint. We assume that agents hold all constraints that they are involved in. For each binary constraint R_{ij}, there is a corresponding cost table T_{ij} with dimensions $|D_i| \times |D_j|$ in which the cost in every entry e_{xy} is the cost incurred when x is assigned to X_i and y is assigned to X_j. A *binary DCOP* is a DCOP in which all constraints are binary. A *partial assignment* is a set of value assignments to variables, in which each variable appears at most once. $vars(PA)$ is the set of all variables that appear in partial assignment PA (i.e., $vars(PA) = \{X_i \mid \exists x \in D_i \wedge \langle X_i, x \rangle \in PA\}$). A constraint $R_j \in \mathcal{R}$ of the form $R_j : D_{j_1} \times D_{j_2} \times \ldots \times D_{j_k} \rightarrow \mathbb{R}^+ \cup \{0\}$ is *applicable* to PA if each of the variables $X_{j_1}, X_{j_2}, \ldots, X_{j_k}$ is included in $vars(PA)$. The set of constraints that are applicable to a partial assignment PA will be denoted by R_{PA}. $R_j(PA)$ is the cost of incurred which corresponds to R_j with respect to PA. When R_j does not apply to (PA), $R_j(PA) = 0$. The *cost of a partial assignment* $C(PA)$ is the sum of costs of all constraints that are applicable to PA, i.e., $C(PA) = \sum_{R_j \in R_{PA}} R_j(PA)$. A *complete assignment* (or a *solution*) is a partial assignment that includes all the DCOP's variables (i.e., $vars(PA) = \mathcal{X}$). An *optimal solution* is a complete assignment with minimal cost.

For simplicity, we make the common assumption that each agent holds exactly one variable (i.e., $n = m$) and we concentrate on binary DCOPs. These assumptions are common in the DCOP literature [7, 13]. That being said, we emphasize that all methods and heuristics we propose in this paper apply to k-ary constraints as well, for $2 \leq k \leq n$.

2.2 Synchronous Branch-and-Bound (SyncBB)

Synchronous Branch-and-Bound (SyncBB) [2] is a complete, synchronous, search-based algorithm that can be considered as a distributed version of a standard branch-and-bound algorithm. It uses a complete ordering of the agents to extend a *Current Partial Assignment* (CPA) via a synchronous communication process. The CPA is exchanged by the agents according to the order. Agents add the assignments to their variables before sending the CPA forward and remove their assignments before sending it backwards. The CPA also functions as a mechanism to propagate bound information. The algorithm prunes those parts of the search space whose solution quality is sub-optimal by exploiting the bounds that are updated at each step of the algorithm. In other words, an agent backtracks when the cost of the CPA is not smaller than the cost of the best complete solution found so far.

The algorithm begins by the first agent in the order, which generates the CPA, assigns it a value and forwards it to the next in the order. The CPA includes a lower bound, which is the current cost of the partial assignment carried by the CPA and an upper bound (UB), which is the cost of the best solution found so far by the algorithm (initially infinity), When an agent A_i receives a CPA, it attempts to assign its variable X_i with one of the values $x \in D_i$ and send it forward. When it is received back from the agent following it in the order (A_{i+1}, it attempts to reassign X_i with a different value from D_i. A CPA is sent back when the agent

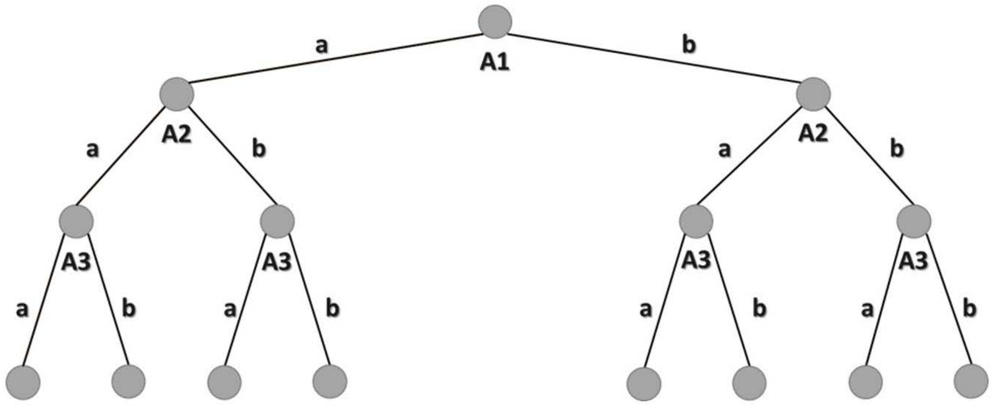


Figure 1 Example of a search tree.

cannot assign a value to the CPA that has not been assigned to the CPA with the specific context (the partial assignment) before, or that does not cause a breach of UB. When the last agent in the order manages to assign its variable, without breaching UB, a new solution is generated and stored, and UB is updated with its cost. The algorithm terminates when the first agent sends the CPA back. The solution reported is the last complete assignment (solution) that caused an update of UB.

In order to analyze the performance of complete search algorithms, such as SyncBB, when solving constraint reasoning problems, such as DCOPs, it is common to use a search tree. The search tree is a tool that allows one to follow the advancement of the search process and analyze its properties. The root of the search tree is the first variable in the order, and each of the edges connecting it to its children represents a possible value assignment. Similarly the second layer represents the possible assignments of the second variable in the order and so forth, until the leaves of the tree, which represent the value assignments of the last variable in the order [14]. Thus, each value assignment is the root of a sub-tree in this search tree.

Figure 1 presents an example of a search tree with three agents A_1, A_2, and A_3, each holding one variable with two values in its domain a and b.

2.3 Distributed Stochastic Algorithm (DSA)

The Distributed Stochastic Algorithm (DSA) [15] is a simple distributed local search algorithm in which, following an initial step where agents (randomly) choose an initial value for their variable, the agents perform a series of steps (looped iteratively) until some termination condition is met. In every step, an agent sends its value assignment to its neighbors in the constraint graph and collects the value assignments of its neighbors. Once the value assignments of all its neighbors have been collected, an agent decides whether to keep its value assignment or to modify it. This decision has a significant effect on the performance of the algorithm. If an agent in DSA cannot upgrade its current state by substituting its present value, it does not do so. On the other hand, if the agent can improve (or maintain, depending on the version used) its current state, it decides whether to replace its value assignment using a stochastic strategy.

2.4 Maximum Gain Message (MGM)

Like DSA, Maximum Gain Message (MGM) is a distributed synchronous local search algorithm, in which agents perform in iterations. In each iteration the agents send messages to all their neighbors, receive messages from all of them and perform computation. The main difference from DSA is that, for each decision whether to replace an assignment, two iterations are performed. In the first, like in DSA, the agents exchange their value assignments. In the second, the agents exchange the maximal improvement they can achieve by replacing assignments. Only agents that suggested a positive improvement that is greater than all their neighbors (ties are broken deterministically according to the agents' identifying indexes), replace their assignments.

3 Ex-Ante Incomplete DCOP

An Ex-Ante Incomplete DCOP (EAI-DCOP) is defined by a tuple $\langle \mathcal{A}, \mathcal{X}, \mathcal{D}, \mathcal{R}, \tilde{\mathcal{R}}, \mathcal{E}, \mathcal{B} \rangle$, where $\mathcal{A}$, $\mathcal{X}$, $\mathcal{D}$ and $\mathcal{R}$ are defined the same as in DCOP. For each constraint $R_j \in \mathcal{R}$, there is a corresponding incomplete constraint $\tilde{R}_j \in \tilde{\mathcal{R}}$, where $\tilde{R}_j \in \tilde{\mathcal{R}} : D_{j_1} \times D_{j_2} \times \ldots \times D_{j_k} \to \mathbb{R}^+ \cup \{0, ?\}$, where each of the D_{j_q} in $\tilde{R}_j$ is also a member in R_j and ? is a special element denoting that the cost for a given combination of value assignments is not known to the agent. In I-DCOP, it is assumed that an agent does not hold the set of constraints that it is involved in, but rather the set of incomplete constraints that it is involved in.

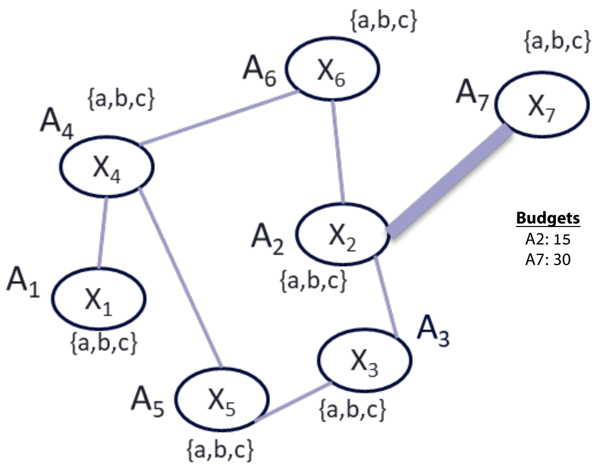
For every incomplete constraint $\tilde{R}_j$, there is an elicitation cost function $E_j \in \mathcal{E}$, such that for each unknown cost of a combination of assignments $r \in R_j$ there is a positive elicitation cost in $e(r) \in E_j$ that the agent will need to "pay" for eliciting this constraint. An explored solution space $\tilde{\mathbf{x}}$ is the union of all solutions explored so far by a particular algorithm. $\tilde{\mathbf{x}}_{PA}$ is the explored solution space at the time that PA was generated. The cumulative elicitation cost $\mathcal{E}(\tilde{\mathbf{x}})$ (and $\mathcal{E}(\tilde{\mathbf{x}}_{PA})$ respectively) is $\sum_{r \in R} e(r)$ such that r is an unknown constraint in $\tilde{\mathcal{R}}$, but it is not an unknown constraint in $\tilde{\mathbf{x}}$. In other words, it is the sum of the elicitation costs of all elicitation queries conducted while exploring $\tilde{\mathbf{x}}$.

In standard (Ex-Post) I-DCOP [12], the cost $C(PA)$ of a partial assignment is calculated as follows: $C(PA) = \sum_{R_j \in R_{PA}} C(R_j) + \mathcal{E}(\tilde{\mathbf{x}}_{PA})$, where R_{PA} is the set of constraints whose variables are in $vars(PA)$. In an Ex-Ante I-DCOP, the solution cost, like in standard DCOP, is $C(PA) = \sum_{R_j \in R_{PA}} C(R_j)$. However, agents are limited in the amount of information they can elicit. We formulate this limitation using a budget $\mathcal{B} = \{B_1, B_2, ..., B_n\}$, where B_i is the amount of elicitation cost agent A_i may spend. These are taken into consideration during the search process and, thus, the agents take the budget limitations into consideration **before** they decide whether to elicit some information.

Figure 2 includes an example of an EAI-DCOP with seven agents. Each agent holds one variable with three values in its domain, and has an elicitation budget (we only present the budgets of A_2 and A_7, which are relevant to the example). On the right hand side the cost table and the elicitation cost table of constraint $R_{2,7}$ are presented. There are three unknown costs in the cost table. Agent A_2's budget allows it to elicit the cost for $\langle X_2 = a, X_7 = a \rangle$ or $\langle X_2 = b, X_7 = a \rangle$. Agent X_7 can afford to elicit any of the three missing costs, and even to elicit the costs for both $\langle X_2 = a, X_7 = a \rangle$ and $\langle X_2 = b, X_7 = a \rangle$.

4 Solving EAI-DCOPs

We propose ex-ante elicitation heuristics for three algorithms solving EAI-DCOPs – SyncBB [2], DSA [15], and MGM [4]. These well-known algorithms were selected for their simplicity, in order to emphasize the effect of the selected elicitation heuristic on the search process. SyncBB and MGM were also used in the previous I-DCOP studies [11, 12].



Constraint

X_2/ X_7	a	b	c
a		3	2
b		5	0
c	6	0	

Elicitation

X_2/ X_7	a	b	c
a	10	0	0
b	15	0	0
c	0	0	30

Figure 2 Example of an EAI-DCOP.

4.1 Solving EAI-DCOPs with SyncBB

The main difference between agents performing SyncBB to solve EAI-DCOPs from the agents performing SyncBB to solve standard DCOPs is that, in EAI-DCOPs, agents do not attempt to assign all values to their variables. Instead, when an agent that has partial information regarding the constraint costs of its variable receives a CPA, it needs to decide whether to elicit missing information and which missing information to elicit. We will assume that regardless of the heuristic being used, an agent will first attempt to assign values to its variable, for which it knows all costs of constraints with the value assignments included in the CPA. For the values in its domain for which it does not know all the constraint costs, the agent can decide either to elicit this information, and pay the corresponding cost (which is deducted from its budget), or to avoid eliciting this information. Obviously, if its budget is smaller than the elicitation cost, the first option is ruled out. After eliciting the constraint costs corresponding to a value $x \in D_i$, agent A_i treats x as any other value in its domain for which it has complete knowledge regarding its constraints, that is, it tries to assign x to X_i and send the CPA forward. On the other hand, if A_i decides not to elicit the costs that correspond to x, the subtree rooted by x (in the search tree) will not be explored. We propose the following heuristics for deciding whether to elicit the cost information by agents solving EAI-DCOPs with SyncBB.

Depth Dependent (DD). The decision whether to elicit the cost information for a value is decided stochastically. The probability for an agent A_i to elicit the costs of a value in its domain is calculated using the Sigmoid function: $p(i) = \frac{e^i}{e^{n/2}+e^i}$, where i is the depth of the agent's variable in the search tree and n is the number of agents.[2] Note that this function does not distinguish between the values within a domain of a variable held by some agent and, thus, if the decision is to elicit, the agent will elicit the constraint cost information for all its values, until the budget is exhausted.

[2] Sigmoid functions are used as activation functions in neural networks [9], but are not related to their use here.

The intuition that led to the design of this heuristic was that the deeper an agent is in the search tree, the larger is the chance that a solution improving on former solutions will be found. This is because every layer in the search tree can require additional elicitation.

Distance from Bound (DB). The decision whether to elicit the cost information for a value $x \in D_i$ is based on its distance Δ_x of the cost of the CPA from the upper bound (UB) maintained by SyncBB and a threshold $t(i)$ of the agent a_i. Specifically, the costs for x are elicited if $\Delta_x > t(i)$.

The distance from the upper bound Δ_x for each value $x \in D_i$ is calculated as follows: $\Delta_x = UB - (C(PA) + \delta_x)$, where $C(PA)$ is the cost of the current partial assignment and δ_x is the lowest cost that was generated from constraints with assignments of variables held by agents that come after A_i in the order, in previous attempts to assign x to X_i. If there were no previous attempts, $\delta_x = 0$.

The threshold $t(i)$ is calculated as follows: $t(i) = g \cdot \left(1 - \frac{e^i}{e^{n/2}+e^i}\right)$, where g is a constant that is dependent on the distribution of constraint costs in the problem.

The intuition is that when the distance from the upper bound is larger, there is more chance that this part of the search tree will include relevant solutions, since for a small distance an additional cost is expected to breach the bound and cause a backtrack.

Elicitation Required in Subtree (ERS). The decision whether to elicit the costs corresponding to a value assignment is done according to the number of unknown constraints in the subtree (of the search tree) rooted by this value. The intuition is that there is a larger chance that in a subtree with a small number of unknown constraints, complete solutions will be found with low elicitation cost.

In EAI-DCOPs, it is possible to count the number of unknown constraints in each such subtree. The process is performed bottom up, having the last agent in the order count the unknown constraints for each of the values in its domain, and sending this information up to the agent preceding it in the order. This agent adds the amount of constraints for each of its value and sends it up to the agent preceding it, and so forth. The process ends when the first agent in the order, holding the root variable of the subtree, updates the number of unknown constraints in each of the subtrees rooted by the values in its domain.

In more details, after an agent A_i receives a message that includes a number C_{i+1} of unknown constraints from the agent following it in the order, it can calculate, for each value $x \in D_i$, the number of constraints it is involved in among them. Thus, in order to calculate the number of unknown constraints in the subtree rooted by x, it reduces from C_{i+1} the number of constraints that all values $x' \in D_i, x' \neq x$ are involved in. To this number, it adds the number of unknown constraints that x is involved in with variables held by agents that are before A_i in the order. The total number of unknown constraints that A_i sends to A_{i-1} is then calculated as follows: $C_i = \sum_{x \in D_i} C_x$.

Each agent performs the initial calculations of C_x for every value x in the domain of its variables before the algorithm begins as a prepossessing procedure. After the algorithm starts, these parameters are updated following each elicitation as follows: When agent A_i elicits the unknown constraints for a value $x \in D_i$, it updates the number of unknown constraints for this value, and updates each of the agents involved in the constraints that were elicited. These agents reduce the corresponding number of unknown constraints for the subtrees of the corresponding values.

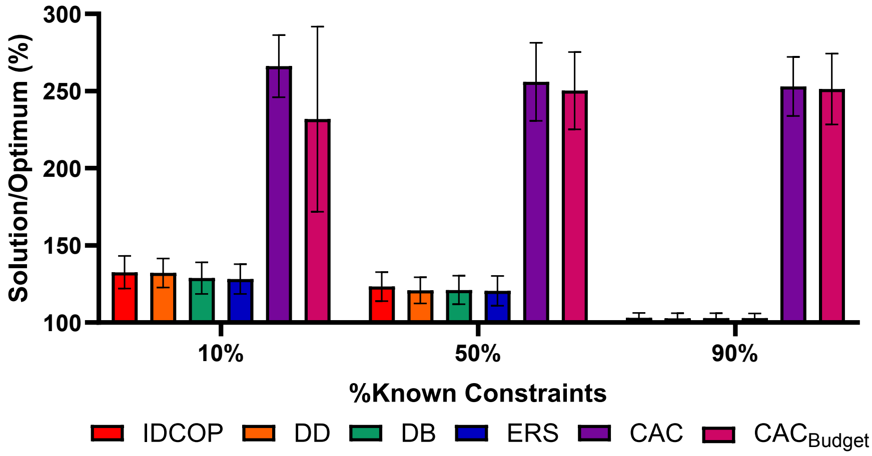


Figure 3 Solution costs when agents have a total budget of 105.

4.2 Solving EAI-DCOPs with DSA and MGM

Like in the case of SyncBB, the main challenge when solving EAI-DCOPs using distributed local search algorithms, such as the DSA and MGM, is in deciding which constraints to elicit. However, in contrast to SyncBB, DSA and MGM are incomplete and, thus, not all values with known constraints need to be assigned. Therefore, the decision whether to elicit needs to take into consideration the amount of uncertainty regarding the cost of a possible value assignment and the potential improvement it offers. In more details, we propose a heuristic that we incorporated in both DSA and MGM, according to which the elicitation decision for value $x \in D_i$ is performed if the following two conditions hold:

1. The number of unknown constraints that x is involved in is smaller than $q \cdot CN_x$, where CN_x is the total number of constraints that x is involved in and $0 \leq q \leq 1$.
2. The difference between the sum of the known constraints that x is involved in and the cost of the current partial assignment is larger than $g \cdot \frac{e^i}{e^{n/2}+e^i}$. Here, g is a constant as defined above for the formula in SyncBB and i is the iteration number.

5 Experimental Evaluation

In order to evaluate the success of our approach for generating high-quality I-DCOP solutions, under different elicitation "budget" restrictions, we performed experiments in which we compared the different heuristics that we proposed for solving EAI-DCOPs incorporated in DCOP algorithms, with the previous proposed approach for solving I-DCOP [12]. All our experiments were performed on a simulator, implemented in Python, on a Lenovo Carbon X1 Gen 9 computer with an 11th Generation Intel(R) Core (TM) i7-1165G7 @ 2.80GHz 2.80 GHz processor.

The complete algorithms solved I-DCOPs including seven agents, each holding one variable with four values in its domain. The average number of neighbors that agents had was 3. Constraint costs for combinations of assignments of neighboring agents were selected uniformly between 2 and 5. The percentage of unknown constraints varied. We generated problems in which the fraction of known constraints was 10%, 50%, and 90%. For each unknown cost, an elicitation cost was selected uniformly from the range $[0, 20]$.

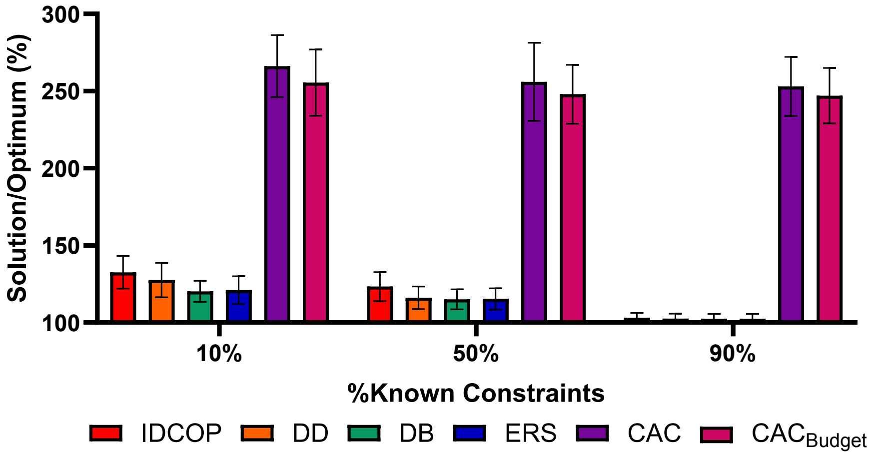


Figure 4 Solution costs when agents have a total budget of 525.

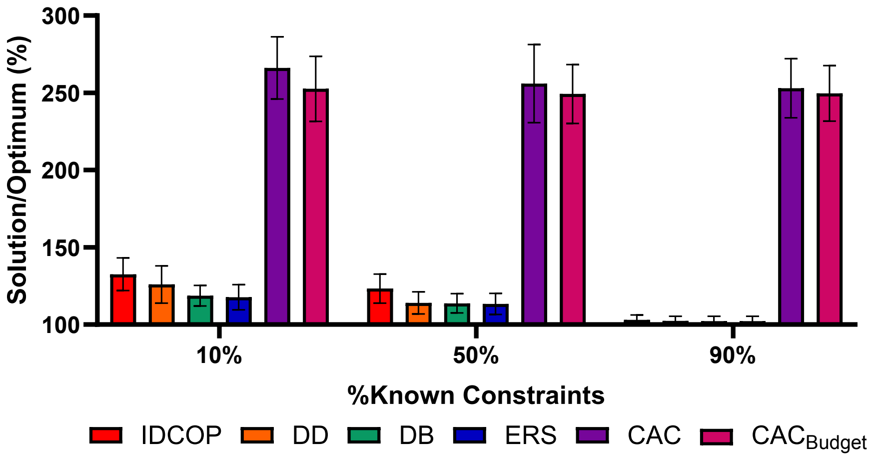


Figure 5 Solution costs when agents have a total budget of 1050.

5.1 SyncBB with Global Budgets

Previous I-DCOP approaches [12] with SyncBB assumed agents considered global elicitation costs, which are summations of elicitation costs over all agents. Therefore, to fairly compare against them, we also consider a variant, where agents have access to a global budget that can be accessed by all agents. Specifically, we set the global budget to either 105, 525, or 1050, so that the highest budget was an order of magnitude more than the lowest and they are easy to split among the agents in the personal budget version, which will be presented next. (We consider the variant where agents have personal budgets in the next section.)

Figure 3 depicts the average overhead in the solution quality with respect to the optimal solution (when all information is known), of solutions that the different algorithms produced when solving the problems with SyncBB, when agents were allocated the smallest global elicitation budget (105). The figure includes three batches of bars, one for each percentage of knowledge known to the agents in the beginning of the run of the algorithm (from left

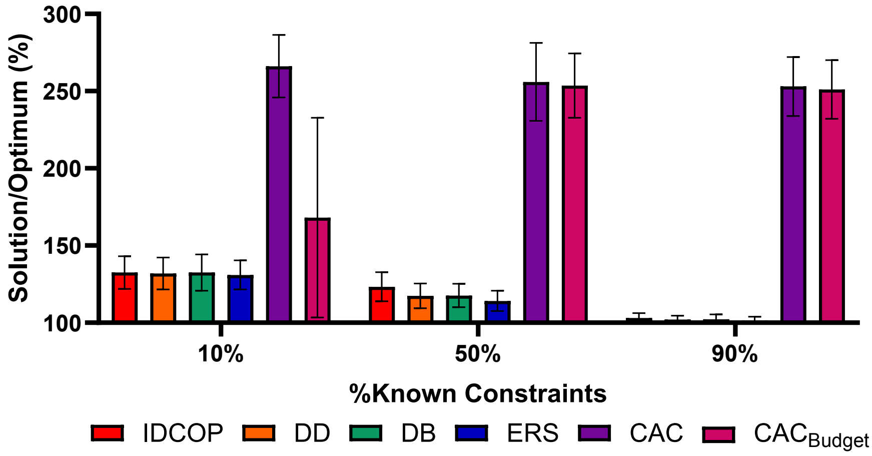


Figure 6 Solution costs when agents have a personal budget of 15.

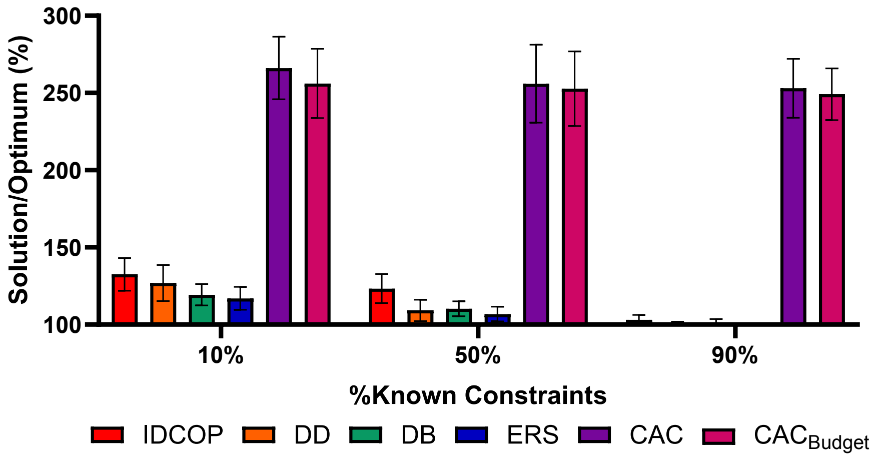


Figure 7 Solution costs when agents have a personal budget of 75.

to right: 10%, 50%, and 90%). The two bars on the right in each batch are the results of the I-DCOP algorithm using the *CAC* heuristic proposed previously [12]. In one version (labeled $\text{CAC}_{\text{budget}}$), the algorithm was limited by a budget, similar to the amount used by the EAI-DCOP heuristics (i.e., 105). In the second (labeled CAC), the algorithm was not bounded by a budget. Surprisingly, the version that was limited by a budget produced better results on average. We assume that a limited budget limits also the elicitation cost that is taken into consideration in the lower bound of the algorithm and, thus, the agents explore more solutions when they have a limited budget. It is apparent that both of these versions produce solutions with much greater costs than the solutions produced by the other versions. Among them, the bar on the left (labeled IDCOP) is a version of the algorithm that does not perform elicitation at all, while the three others present the average results of the algorithm using the limited budget according to the three heuristics described in Section 4.1.

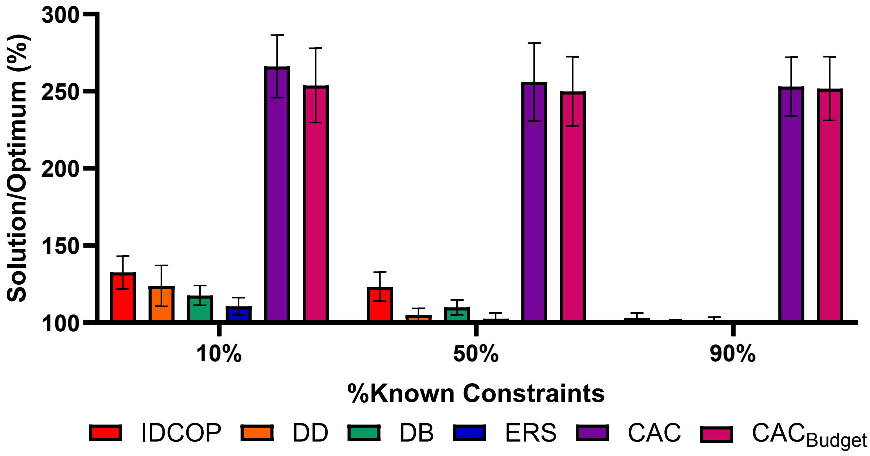


Figure 8 Solution costs when agents have a personal budget of 150.

The results clearly indicate that it is enough to solve the I-DCOP using SyncBB with no elicitation, in order to get a much better solution in comparison with the solutions produced by the algorithm implementing the ex-post approach and heuristic suggested by the literature [12]. Yet, performing elicitation using the allocated budget can reduce costs further. The best result is achieved by the ERS heuristic. However, its advantage over DD and DB is not significant. Figures 4 and 5 present similar results produced by scenarios in which agents had larger budgets (525 and 1050 respectively). While the trends seem similar, it is clear that the advantage of the proposed heuristics over the I-DCOP version without elicitation, is more apparent (as expected).

5.2 SyncBB with Personal Budgets

Figures 6, 7, and 8 present results for the same algorithms solving the same problems, only in this case the EAI-DCOP heuristics, that is, DD, DB, and ERS use personal budgets instead of a global budget as used in the experiments presented above. We divided the global costs such that there will be no difference in the total budget used by the agents. However, these scenarios present the more realistic case where an agent represents a user, and the budget limits the effort a user must spend in replying to queries during search. In these personal budget scenarios, the versions using the EAI-DCOP heuristics produced solution with a more significant advantage in general over the vanilla I-DCOP version. The difference was most apparent when the agents had medium or high budgets, and the initial knowledge available was 50%. The EAI-DCOP heuristics were able to produce solutions with a significant advantage over the vanilla I-DCOP version, and for the 150 budget per agent, the DD and ERS heuristics produce solutions that their quality was close to optimal. It seems that, besides being a more realistic scenario in a multi-agent environment, the budget per agent settings allows the algorithm to use elicitation in different parts of the search space and explore high quality solutions.

Figures 9, 10, and 11 present the runtimes in a logarithmic scale, of the algorithms in terms of NCLOs [16, 6]. In each figure, the budget allocated to the agents were different. The differences between the complete DCOP version and the I-DCOP version is identical in all figures because it is not affected by the budget. It is however affected by the amount

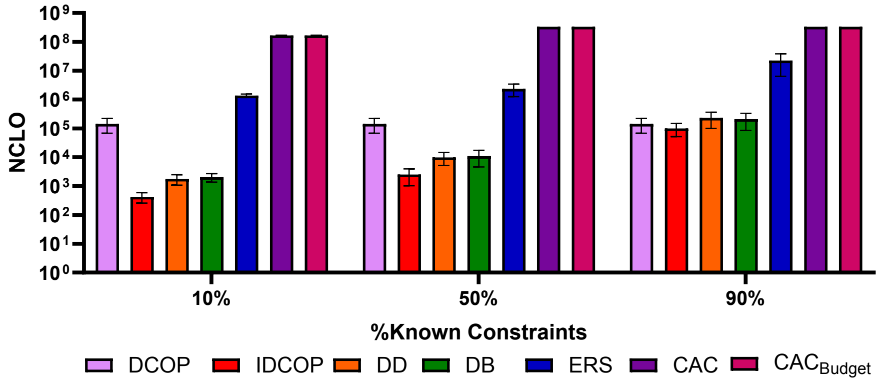


Figure 9 Runtime in terms of NCLOs with a budget of 105.

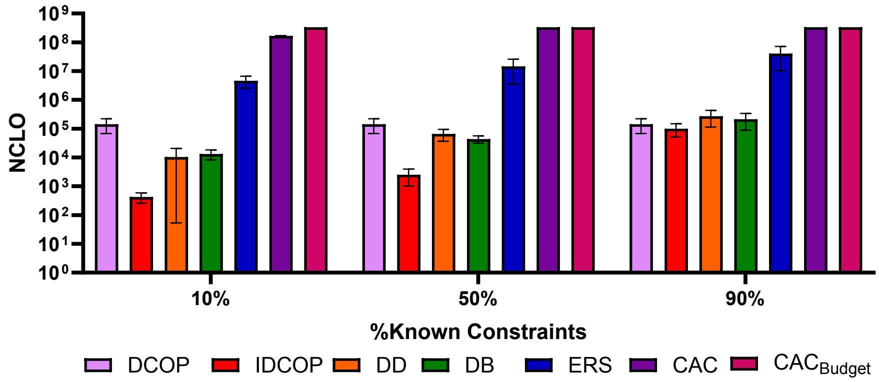


Figure 10 Runtime in terms of NCLOs with a budget of 525.

of knowledge known to agents: In the 10% scenario, the algorithm solving I-DCOP is much faster than the algorithm solving the complete DCOP while, in the 90% scenario, the runtimes are much closer. It is also apparent that the CAC and ERS versions of the algorithms are much slower than all other versions. The reason is that these heuristics require exponential computation before the algorithm begins its search (preparing the heuristic data in preprocessing). Moreover, unlike the SyncBB algorithms that performs some level of pruning, the preprocessing heuristics aggregate information from the entire search space and thus, their runtime is orders of magnitude larger. On the other hand, The DD and DB heuristics are much faster. Additionally, their advantage over the ERS heuristic in runtime is much more significant than the advantage that ERS provides in solution quality. Similar results were obtained when agents used personal budget as presented in Figures 12, 13 and 14.

5.3 MGM and DSA with Personal Budgets

In the second set of experiments we performed, we compared incomplete algorithms, that is, MGM and DSA, which were implemented in the ALS framework [17], in order to produce the anytime solutions as was done previously in the literature [12]. In this set of experiments, the problems included 50 agents, each holding a single variable with 10 values in its domain. Agents had 20 neighbors in average. The costs of constraints were randomly selected between 2 and 5. We present the solution costs of the algorithms in the first 50 iterations, because

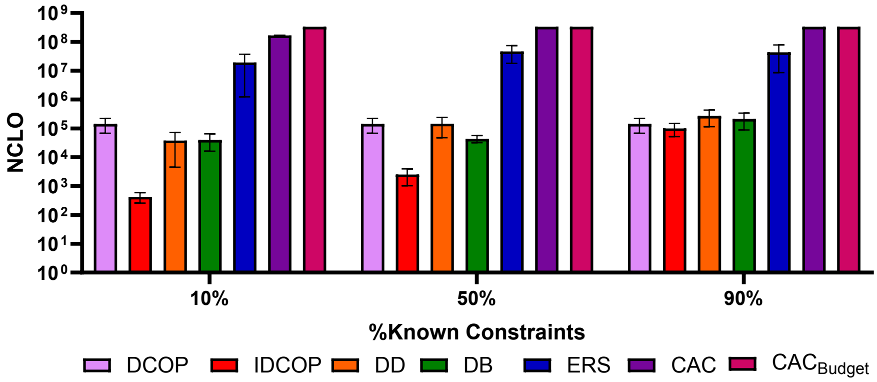


Figure 11 Runtime in terms of NCLOs with a budget of 1050.

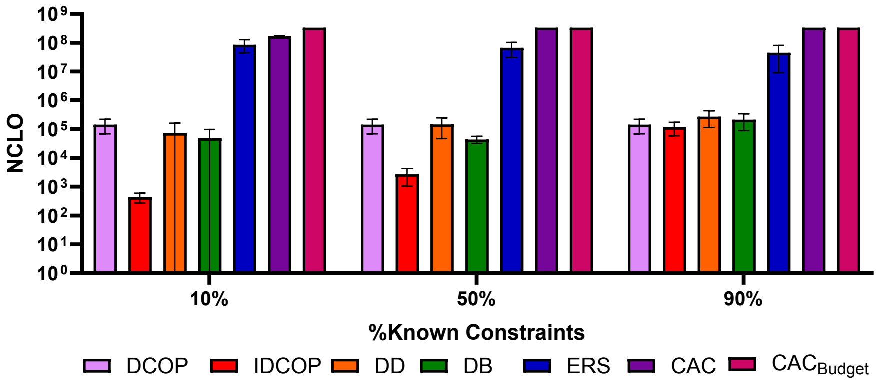


Figure 12 Runtime in terms of NCLOs when each agent had a personal budget of 15.

that was the number of iterations that were required for the algorithms to converge.

We first discuss the MGM results, since this was the algorithm that was implemented in the literature [12]. Figure 15 presents the solution cost as a function of the number of iterations performed by the algorithms. The DCOP version is the omniscient algorithm that knows all constraints. The dashed lines are the versions implementing the NHC heuristic [12]. The different lines represent the amount of initial knowledge. The other solid lines represent the results of the EAI-DCOP version in which the elicitation was performed according to the heuristic presented in Section 4.2. Each agent was allocated 180 elicitation queries. The significant advantage of the EAI-DCOP version is apparent regardless of the amount of initial knowledge the agents had.

Figure 16 presents the results of MGM and DSA performing the EAI-DCOP heuristic for deciding on elicitation, with an allocation 180 queries per agent (with 50% initial knowledge available). Surprisingly, the EAI-DCOP version of MGM is more successful than the DSA version. This is in contrast to the well-known advantage that DSA has over MGM when solving standard DCOPs.

Figure 17 presents the EAI-DCOP versions of MGM and DSA, when allocated fewer (120) queries per agent. Again, it is apparent that the MGM versions outperform the DSA versions, except for the versions solving problems with 10% initial knowledge, where both

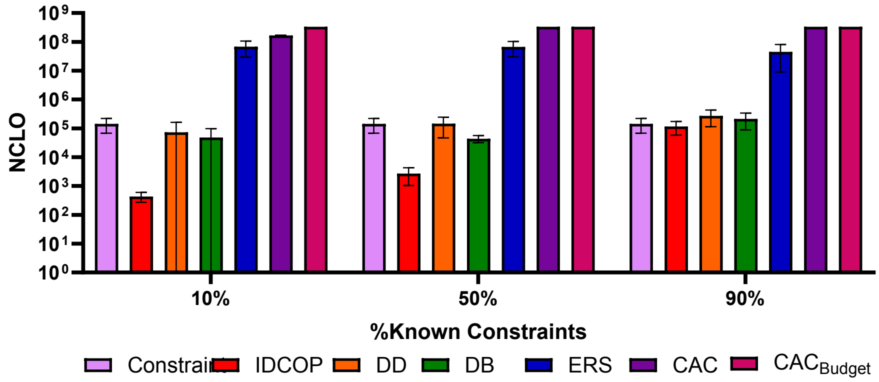


Figure 13 Runtime in terms of NCLOs when each agent had a personal budget of 70.

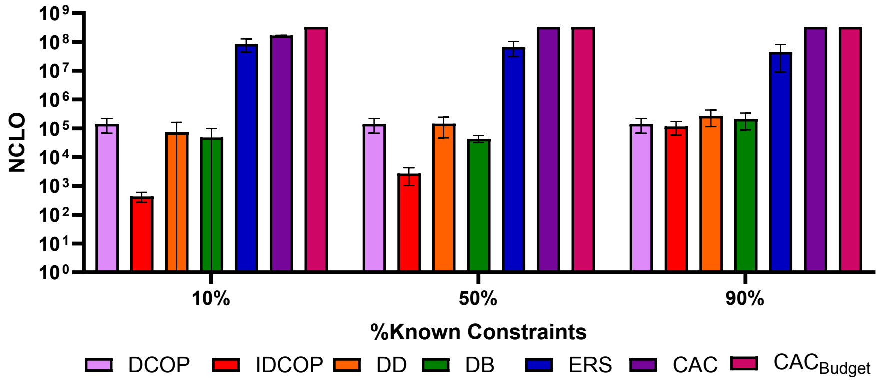


Figure 14 Runtime in terms of NCLOs when each agent had a personal budget of 150.

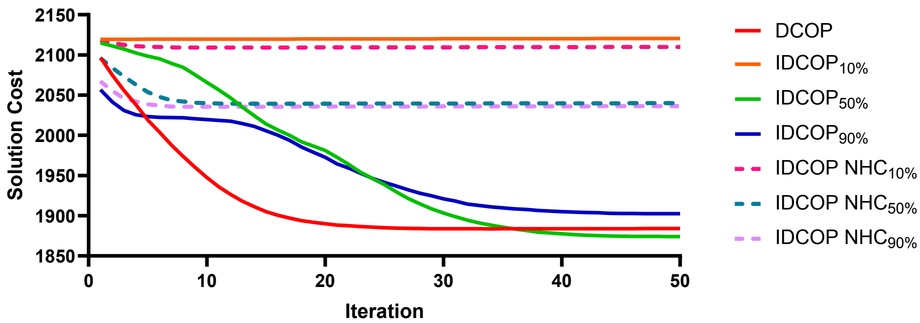


Figure 15 Solution cost as a function of the number of iterations.

algorithms struggle. Moreover, the versions that solved problems with 50% initial knowledge outperform the versions that solved problems with 90% knowledge. We assume that less knowledge resulted in a positive exploration effect, as was reported for environments with imperfect communication [8].

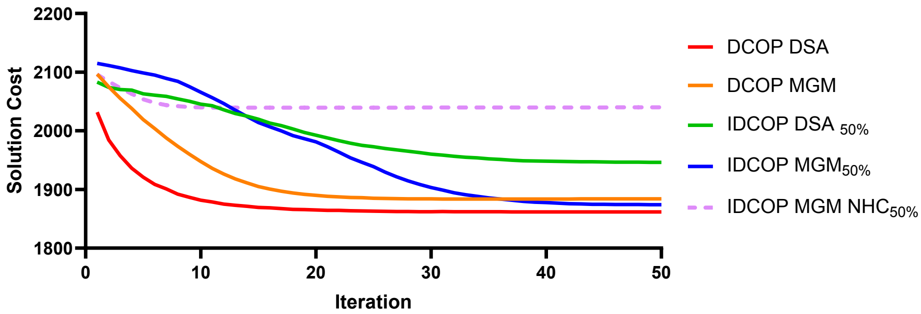


Figure 16 Solution cost as a function of the number of iterations.

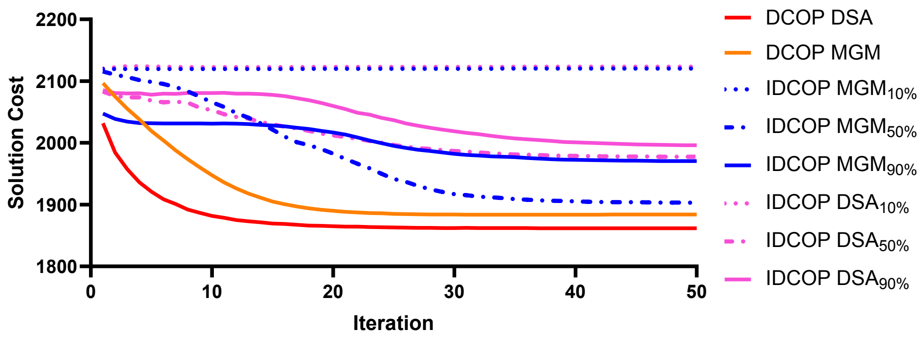


Figure 17 Solution cost as a function of the number of iterations.

6 Conclusions

We introduced a novel approach for solving I-DCOPs in this paper. In contrast to previous studies on I-DCOPs in which elicitation costs were considered *after* elicitations were made, we consider the costs *before* the elicitations in EAI-DCOPs.

The EAI-DCOP approach is not only more realistic, as it is not reasonable that agents will not use a high-quality solution after spending much effort to find it, it is also better in finding higher quality solutions *and* finding them with less runtime. These empirical results were shown on both complete and incomplete algorithms that are commonly used in the literature. Therefore, this seems to be one of the rare occasions where the new approach outperforms prior work on all three key relevant dimensions – practicality, solution quality, and runtime.

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