

# 15th International Conference on Interactive Theorem Proving

ITP 2024, September 9–14, 2024, Tbilisi, Georgia

Edited by

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## ■ Preface

The International Conference on Interactive Theorem Proving (ITP) is the main venue for the presentation of research into interactive theorem proving frameworks and their applications. It has evolved organically starting with a HOL workshop in 1988, gradually widening to include other higher-order systems and interactive theorem provers generally, as well as their applications. This year’s conference takes place in Tbilisi in Georgia.

Previous ITP conferences took place in Edinburgh 2010, Nijmegen 2011, Princeton 2012, Rennes 2013, Vienna 2014, Nanjing 2015, Nancy 2016, Brasilia 2017, Oxford 2018, Portland 2019, Paris 2020, Rome 2021, Haifa 2022, and Białystok 2023; those in 2010, 2014, 2018 and 2022 were under the umbrella organization of the Federated Logic Conference (FLoC).

This year’s conference attracted a total of 71 submissions. Each paper was systematically reviewed by at least three program committee members or appointed external reviewers and 39 papers were finally selected for presentation at the conference (36 regular papers and 3 short papers). We thank the authors of both accepted and rejected papers for their submissions, as well as the programme committee members and the external reviewers for their invaluable work.

As well as all the selected papers, we are very pleased to have invited keynote talks by Frédéric Blanqui (Centre Inria de l’Université de Lorraine, Nancy) and Tobias Nipkow (Technische Universität München). The present volume collects all the accepted papers contributed to the conference and abstracts for the two invited talks. This is the fifth time that the ITP proceedings are published by the LIPIcs series. We thank all the colleagues at Dagstuhl for their responsive feedback on all matters associated with the production of the finished proceedings.

We are grateful to all of the local organizers and thankful to the ITP Steering Committee for their guidance. This conference received partial support from the European Union COST Action CA20111 “European Research Network on Formal Proofs”, and from Inria, France.

Yves Bertot, Temur Kutsia, and Michael Norrish





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