



# Rule-Based Temporal Reasoning: Exploring DatalogMTL

Przemysław Andrzej Wałęga  

University of Oxford, UK

Queen Mary University of London, UK

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## Abstract

I will introduce DatalogMTL – an extension of Datalog, augmenting it with operators known from metric temporal logic (MTL). DatalogMTL is an expressive language which allows us for complex temporal reasoning over a dense timeline and, at the same time, remains decidable. I will provide an overview of research on DatalogMTL by discussing its computational complexity, syntactic and semantic modifications, practical reasoning approaches, applications, and future research directions.

**2012 ACM Subject Classification** Computing methodologies → Temporal reasoning

**Keywords and phrases** Temporal Datalog, Temporal Logic Programming, Temporal Reasoning

**Digital Object Identifier** 10.4230/LIPIcs.TIME.2024.3

**Category** Invited Talk

**Funding** This research was partially supported by the EPSRC projects OASIS (EP/S032347/1), ConCuR (EP/V050869/1) and UK FIRES (EP/S019111/1), as well as SIRIUS Centre for Scalable Data Access and Samsung Research UK.

**Acknowledgements** The research surveyed in this talk is a result of a joint work with Bernardo Cuenca Grau, Mark Kaminski, Egor V. Kostylev, Michał Zawidzki, Dingmin Wang, David Tena Cucala, Pan Hu, Matthias Lanzinger, Markus Nissl, and Emanuel Sallinger.

## Extended Abstract

DatalogMTL [2] is a temporal extension of Datalog which allows one to perform complex reasoning over a dense timeline, using metric temporal logic (MTL) operators. For example it makes it possible to write temporal rules involving recursion over time, as in the program:

$$\begin{aligned} \text{TransactionChain}(x, y) &\leftarrow \text{Transaction}(x, y) \wedge \text{RedList}(x), \\ \text{TransactionChain}(x, z) &\leftarrow \diamond_{[0,24]} \text{TransactionChain}(x, y) \wedge \text{Transaction}(y, z), \\ \boxplus_{[0,\infty)} \text{Suspect}(y) &\leftarrow \text{TransactionChain}(x, y) \wedge \text{HighRisk}(y). \end{aligned}$$

The first two rules recursively define relation *TransactionChain*. The first rule initialises the definition by stating that *TransactionChain* holds between  $x$  and  $y$  at some time point, if at that time point a financial *Transaction* was recorded between  $x$  and  $y$ , and moreover,  $x$  was on a bank's *RedList*. The second rule, in turn, states that *TransactionChain* holds between  $x$  and  $z$  at a time point  $t$ , if *TransactionChain*( $x, y$ ) did hold sometime in the interval  $[t - 24, t]$  (expressed with the MTL operator  $\diamond_{[0,24]}$ ) and *Transaction*( $y, z$ ) holds at  $t$ . Hence, *TransactionChain*( $x, y$ ) means that there is a chain of transactions from  $x$  to  $y$ , such that the period between consecutive transactions in this chain is at most 24 hours and  $x$  was on *RedList* at the time of the first transaction. The third rule states that if  $y$  is in a *TransactionChain* (started by some  $x$ ) and they are a *HighRisk* customer, then  $y$  will be treated as a *Suspect* in indefinite future (expressed with  $\boxplus_{[0,\infty)}$ ).



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31st International Symposium on Temporal Representation and Reasoning (TIME 2024).

Editors: Pietro Sala, Michael Sioutis, and Fusheng Wang; Article No. 3; pp. 3:1–3:3

Leibniz International Proceedings in Informatics



LIPICs Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

The above program shows some non-trivial temporal reasoning aspects allowed by DatalogMTL, which involve recursion over time as well as reasoning about time intervals and time distances. Due to succinct representation and high modeling capabilities, DatalogMTL has been applied to a number of tasks, including temporal ontology-based query answering, stream reasoning, modelling smart contracts, verification of banking agreements, fact-checking economic claims, and even for describing dance movements. On the other hand, the high expressive power of DatalogMTL leads to challenging computational behaviour, namely consistency checking and fact entailment are EXPSPACE-complete in combined complexity [2] and PSPACE-complete in data complexity [4]. As a result, research on DatalogMTL has been concentrated on establishing low complexity fragments and developing practical reasoning algorithms.

The main approaches to obtaining low complexity fragments consist in: restricting the form of rules, for example by assuming their linearity or by limiting the form of allowed temporal operators, assuming a discrete time line, and considering an alternative event-based semantics. There are also several extensions of DatalogMTL, allowing for additional features such as temporal aggregation, existential rules, and (stratified and non-stratified) negation-as-failure under the stable model semantics [6].

Regarding the practical reasoning approaches, non-recursive programs can be rewritten into SQL, which was implemented within a temporal extension of the Ontop platform [3]. For DatalogMTL programs which are finitely materialisable [8] – that is the forward-chaining procedure for them takes a finite number of steps – reasoning can be performed using a standard chase, which was implemented within Vadalog system [1]. The first system allowing for reasoning in full DatalogMTL, called MeTeoR [5], combines materialisation-based and automata-based techniques, and has also been applied for solving stream reasoning tasks [7].

During the talk, I will describe both the theoretical results and practical reasoning approaches established for DatalogMTL. I will also present the ongoing and future research directions.

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