

# Introducing Fairness in Graph Visualization

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## Abstract

Information visualization tools are an essential component of many data-driven decision-making systems that rely on human feedback. The aim of this paper is to propose a novel research direction focused on *fair visualizations of graphs*.

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## Supplementary Material

*Software (Source Code)*: <https://github.com/tommaso-piselli/fairness-MLVis/tree/main>

archived at [swh:1:dir:378c9ba331d5a22b515264c6de24c45f6a40c065](https://swh.io/1/dir/378c9ba331d5a22b515264c6de24c45f6a40c065)

## 1 Motivation and Contribution

In a recent survey focused on bias in machine learning, Mehrabi et al. [7] define fairness as the absence of any prejudice or favoritism toward an individual or a group based on their inherent or acquired characteristics.

Information visualization tools are an essential component of many data-driven decision-making systems that rely on human feedback. Although there is a substantial body of literature on fairness in artificial intelligence and related fields, fairness issues in information visualization have been surprisingly overlooked. The aim of this paper is to propose a novel research direction on this topic, focused on *fair visualizations of graphs*.

Borrowing an example from [3], imagine two competing parties, the reds and the blues. Also, suppose we are given a visualization of the graph modeling the relationships among the parties' members. Using recent layout algorithms, we can optimize a desired set of quality criteria (see, e.g., [1]), hence producing a readable and effective layout of our graph. However, the global optimization process underlying our layout algorithm will not provide any guarantee that the readability of the visualization “around” red vertices will be of the same quality as for blue vertices. In fact, while nearly every graph drawing algorithm optimizes global metrics for the computed layout and can readily incorporate local constraints, only a few algorithms are capable of handling more general constraints at the subgroup level [2, 5]. In contrast, a fair visualization should ensure that no party is favored in terms of readability.



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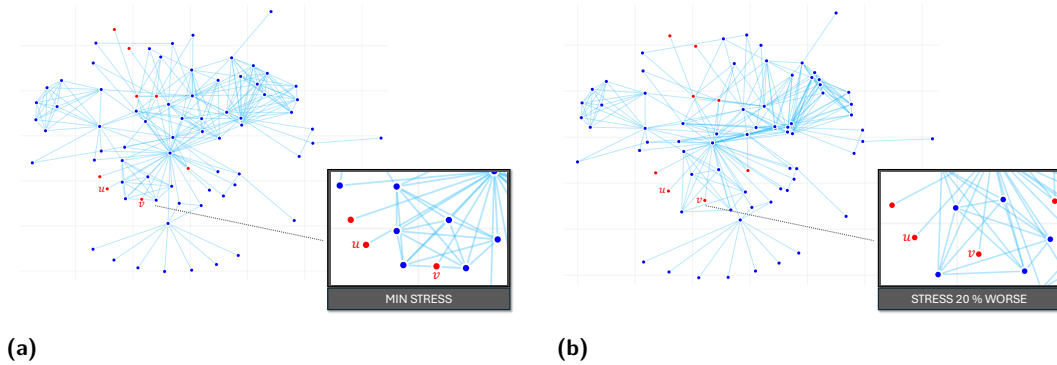
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■ **Figure 1** Two straight-line drawings of the same graph. (a) is obtained by optimizing the stress function, while (b) is obtained starting from (a) and by subsequently optimizing the fairness function without worsening the stress by more than 20%. One can observe (zoomed windows) the increase on the readability around the two red vertices  $u$  and  $v$  when optimizing fairness (the red vertices are fewer than the blue ones). In particular, in (a), the edge incident to  $u$  overlaps with a blue vertex, while  $v$  overlaps with an edge between two blue vertices. Both ambiguities are resolved in (b).

This means that the potential visual complexity of the representation is equally distributed between the two sets, which becomes especially challenging when the cardinalities of the two sets are unbalanced. Although a fair drawing might be suboptimal in terms of overall readability, it offers greater insight to end users by balancing readability between the two vertex groups. Figure 1 illustrates an example of the impact of fairness on the readability of a straight-line drawing with several blue vertices and few red vertices.

Our results are as follows.

- We provide a conceptual contribution by formalizing the notion of fair straight-line graph drawings, based on the concept of stress, a well-known and widely adopted quality function (see, e.g., [4]). Clearly, the concept of fair straight-line drawings can be transferred to other quality criteria, as well as to other graph drawing paradigms.
- We present empirical results concerning the price of fairness to be paid in terms of additional stress with respect to stress-minimal (but potentially unfair) solutions. To this aim, we implement a gradient-descent based algorithm that can optimize multiple drawing criteria. Our investigation reveals that multi-objective functions that optimize fairness and stress together can output straight-line drawings with good fairness at the expenses of a relatively small increment of global stress.

Due to space limitations, we present below our fairness model for straight-line drawings, and we point the reader to [6] for an extended abstract of the paper.

## 2 Fairness of Straight-line Drawings

Let  $G = (V, E)$  be a graph and let  $\Gamma$  be a straight-line drawing of  $G$ . For a pair of vertices  $u, v \in V$ , let  $\delta(u, v)$  be the length of any shortest path in  $G$  between  $u$  and  $v$ . Also, let  $\|\Gamma(u) - \Gamma(v)\|_2$  be the Euclidean distance of  $u$  and  $v$  in  $\Gamma$ . Moreover, let  $\omega : V \times V \rightarrow \mathbb{Q}$  be a weighting function. The *stress* of  $\Gamma$  is defined as follows:

$$\text{stress}(\Gamma) = \sum_{u, v \in V} \omega(u, v) (\|\Gamma(u) - \Gamma(v)\|_2 - \delta(u, v))^2.$$

Assume now that the vertex set  $V$  of  $G$  is the union of two non-empty disjoint subgroups of vertices, that is,  $V = V_R \cup V_B$  (with  $V_R \neq \emptyset$  and  $V_B \neq \emptyset$ ); vertices in  $V_R$  ( $V_B$ ) are called *red* (*blue*). Thus, let  $G = (V_R \cup V_B, E)$  be a graph and let  $\Gamma$  be a straight-line drawing of  $G$ . To convey the notion of fairness in  $\Gamma$ , we can refine the concept of stress by either focusing exclusively on the red vertices or on the blue vertices.

$$\text{stress}_R(\Gamma) = \sum_{u \in V_R, v \in V} \omega(u, v) (\|\Gamma(u) - \Gamma(v)\|_2 - \delta(u, v))^2$$

$$\text{stress}_B(\Gamma) = \sum_{u \in V_B, v \in V} \omega(u, v) (\|\Gamma(u) - \Gamma(v)\|_2 - \delta(u, v))^2$$

Ideally,  $\Gamma$  should not be unfair to any of the two sets of vertices, that is, the difference between  $\text{stress}_R(\Gamma)$  and  $\text{stress}_B(\Gamma)$  normalized by their cardinalities should be as close to zero as possible. More formally, we conveniently define the *unfairness*  $\lambda(\Gamma)$  of  $\Gamma$ , whose minimization leads to a fair drawing:  $\lambda(\Gamma) = \left( \frac{\text{stress}_R(\Gamma)}{|V_R|} - \frac{\text{stress}_B(\Gamma)}{|V_B|} \right)^2$ .

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