When Far Is Better: The Chamberlin-Courant Approach to Obnoxious Committee Selection

Sushmita Gupta
The Institute of Mathematical Sciences, HBNI, Chennai, India

Tanmay Inamdar
Indian Institute of Technology Jodhpur, Jodhpur, India

Pallavi Jain
Indian Institute of Technology Jodhpur, Jodhpur, India

Daniel Lokshtanov
University of California Santa Barbara, CA, USA

Fahad Panolan
School of Computer Science, University of Leeds, UK

Saket Saurabh
The Institute of Mathematical Sciences, HBNI, Chennai, India
University of Bergen, Norway

Abstract

Classical work on metric space based committee selection problem interprets distance as "near is better". In this work, motivated by real-life situations, we interpret distance as "far is better". Formally stated, we initiate the study of "obnoxious" committee scoring rules when the voters' preferences are expressed via a metric space. To accomplish this, we propose a model where *large distances imply high satisfaction* (in contrast to the classical setting where shorter distances imply high satisfaction) and study the egalitarian avatar of the well-known Chamberlin-Courant voting rule and some of its generalizations. For a given integer value λ between 1 and k, the committee size, a voter derives satisfaction from only the λth favorite committee member; the goal is to maximize the satisfaction of the least satisfied voter. For the special case of $\lambda = 1$, this yields the egalitarian Chamberlin-Courant rule. In this paper, we consider general metric space and the special case of a d-dimensional Euclidean space.

We show that when λ is 1 and k, the problem is polynomial-time solvable in $\mathbb{R}^2$ and general metric space, respectively. However, for $\lambda = k - 1$, it is NP-hard even in $\mathbb{R}^2$. Thus, we have "double-dichotomy" in $\mathbb{R}^2$ with respect to the value of λ, where the extreme cases are solvable in polynomial time but an intermediate case is NP-hard. Furthermore, this phenomenon appears to be "tight" for $\mathbb{R}^2$ because the problem is NP-hard for general metric space, even for $\lambda = 1$. Consequently, we are motivated to explore the problem in the realm of (parameterized) approximation algorithms and obtain positive results. Interestingly, we note that this generalization of Chamberlin-Courant rules encodes practical constraints that are relevant to solutions for certain facility locations.

2012 ACM Subject Classification Theory of computation → Facility location and clustering; Theory of computation → Fixed parameter tractability

Keywords and phrases Metric Space, Parameterized Complexity, Approximation, Obnoxious Facility Location

Digital Object Identifier 10.4230/LIPIcs.FSTTCS.2024.24

Related Version *Full Version*: https://arxiv.org/abs/2405.15372 [36]

Funding Sushmita Gupta and Pallavi Jain acknowledge support from SERB-SUPRA (grant number SPR/2021/000860).
Sushmita Gupta: The author acknowledges the support from SERB-MATRICS (grant number MTR/2021/000869).

Tanmay Inamdar: The author acknowledges support from IITJ Research Initiation Grant (grant number I/RIG/TNI/20240072).
Pallavi Jain: The author acknowledges support from IITJ Seed Grant (grant number I/SEED/PJ/20210119).
Daniel Lokshtanov: The author acknowledges support from NSF award CCF-2008838.
Saket Saurabh: The author acknowledges support from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 819416); and he also acknowledges the support of Swarnajayanti Fellowship (grant number DST/SJF/MSA-01/2017-18).

1 Introduction

Initiated in the 18th century, the multiwinner election problem, also known as the committee selection problem, has been central to social choice theory for over a century [16, 68, 53] and in the last decade and a half it has been among the most well-studied problems in computational social choice [3, 29, 9, 8]. In this problem, given a set of candidates $\mathcal{C}$, a profile $\mathcal{P}$ of voters' preferences, and an integer k; the goal is to find a k-sized subset of candidates (called a *committee*) using a multiwinner voting rule. The committee selection problem has many applications beyond parliamentary elections, such as selecting movies to be shown on a plane, making various business decisions, choosing PC members for a conference, choosing locations for fire stations in a city, and so on. For more details on the committee selection problem, we refer the reader to [29, 46].

The Chamberlin-Courant (CC) committee is a central solution concept in the world of committee selection. Named after Chamberlin and Courant [16], it is derived from a multiwinner voting rule where the voter's preference for a given k-sized committee is evaluated by adding the preference of each voter for its *representative*, the most preferred candidate in the committee.The CC committee is one with the highest value. There has been a significant amount of work in computational social choice centered around this concept and has to date engendered several *CC-type rules* that can be viewed as a generalization of the above. Specifically, *ordered weighted average*(OWA) operator-based rules such as the *median scoring rule*, defined formally later in [62, 4] can be seen as a direct generalization of CC. Moreover, there are other notions of generalization based on the preference aggregation principle: the original CC rule is *utilitarian*, that is, it takes the summation of each voter's preference value toward its representative, [62, 10, 37]. The *egalitarian* variant studied by Aziz et al. [4] and Gupta et al. [37] is one where only the least satisfied voter's preference value towards its representative is taken. Clearly, there could be many other variants where some other aggregation principle is considered. We refer to all these variants collectively as the CC-*type* rules and the egalitarian variants as the *egalitarian* CC-*type* rules. The egalitarian rules, also known as Rawlsian rules, are based on the highly influential Rawls's theory of "justice as fairness" [58, 59] that favors equality in some sense by maximising the minimum satisfaction. Egalitarian rules are very well-studied in voting theory [44, 4, 37, 24, 66].

In this paper, we consider the situation where the voters' preferences are expressed via a metric space, a natural setting in the facility location problem. Facility location can be viewed as an application of the committee selection (also known multi-winner voting) problem [17] and spatial voting [48]. Furthermore, we consider egalitarian scoring rules, which aim to maximize the "satisfaction" of the least satisfied voter. Moreover, Gupta et al. [37] study a wide range of egalitarian rules, called the *egalitarian median rule*, which is a generalization of the egalitarian CC rule and is defined as follows: for a voter v and a committee S, let pos_v^S (called a *position vector*) be the vector of positions of candidates in S in the ranking of v in increasing order. For example, for the voter $v\colon a \succ b \succ d \succ c$ and set $S = \{c, d\}$, the vector $pos_v^S = [3, 4]$. In the egalitarian median rule, given a value $1 \leq \lambda \leq k$, the satisfaction of a voter v for a committee S is given by $m - pos_v^S[\lambda]$, where m is the total number of candidates and $pos_v^S[\lambda]$ is the position value of the λ^{th} candidate in S according to v's preference list. Note that when $\lambda = 1$, we get the egalitarian CC scoring rule. Gupta et al. [37] proved that the egalitarian median rule is NP-hard for every $\lambda < k$.

In contrast to the aforementioned intractibility results for egalitarian CC-type rules, Betzler et al.[7] show that for $\lambda = 1$, the egalitarian median rule is polynomial-time solvable for 1-dimensional Euclidean preferences because such preferences are single-peaked. This motivates us to study egalitarian CC-type rules in the metric space setting that goes beyond dimension 1.

Preferences via a metric space. For a given metric space, preferences are encoded in the following manner: each candidate and each voter is represented by a point in the metric space. In earlier works, distance is viewed as being inversely proportional to preference, that is, a voter is said to have a higher preference for candidates who are closer to her than those that are farther in the metric space, [65, 13, 41, 34, 29, 64, 20]. In this paper, we consider the opposite scenario where *distance is directly proportional to preference*, that is, a candidate who is *farther away is more preferred* than the one who is closer. Inspired by obnoxious facility location [47, 67, 21], we call our problem *obnoxious committee selection*. Before we delve into the formal definition of our problem, we discuss the use of metric space to encode preference in earlier works.

- The facility location problem is actually equivalent to the committee selection problem, where we assume that the closer facility is more preferred. The well-known k-CENTER problem (also known as the MINIMAX FACILITY LOCATION problem in metric space) is equivalent to the egalitarian CC committee selection problem when the voters' preferences are encoded in a metric space, where higher preference is given to the candidate that is closer. For some applications, it is natural to demand more than one facility in the vicinity, e.g., convenience stores, pharmacies, healthcare facilities, playgrounds, etc. This is known as the *fault tolerant* k-CENTER problem and is captured by egalitarian median rules in a metric space [19]. Facility location is among the most widely studied topics in algorithms, and we point the reader to some recent surveys [1, 22, 35] on the topic and to [17] for a survey on facility location in mechanism design.
- In the spatial theory of voting, voters and candidates are embedded in the d-dimensional Euclidean space, and each voter ranks the candidates according to their distance from them [38, 48].

In the last few years, a fair amount of research centered on the theme of voting, committee selection, especially the CC rule, in metric spaces has appeared in theory and economics and computation venues [64, 55, 18, 71, 38, 54, 51, 2]. Motivated by the applications stated above, we consider the general metric spaces as well as the Euclidean space for our study. Next, we discuss our motivation for studying *obnoxious committee* selection before presenting the formal definition.

Why Obnoxious Committee? The committee selection problem has been studied for the metric space in literature [27, 34, 43, 63, 64]. All these papers use the "closer is better" perspective and thus candidates that are closer are preferred over those that are farther. Motivated by real-life scenarios where *every kind of facility is not desirable in the vicinity* such as is the case with factories, garbage dumps and so on, we want to study a problem which allows us to *restrict the number of facilities in the vicinity.* This is particularly relevant for facilities that bring some utility but too many lead to loss in value or even to negative utility. In order to design an appropriate solution concept for scenarios such as these we associate higher preference to facilities (i.e candidates) that are far and set the value of λ in a situation-specific way. For example, consider a situation where a local government is searching locations to build k factories, with the constraint that each of the k factories is located far from every neighborhood. This can be modeled by our problem by setting each of the neighborhoods as voters, each potential factory location as a candidate, and $\lambda = k$. Moreover, for facilities such as garbage recycling, we can set $\lambda = k - \mathcal{O}(1)$ so that all but few facilities are located far from any neighborhood. Since the value of λ can depend on k (which is part of the input), we take λ to be part of the input. Overall, we observe that as far as satisfaction is concerned, different facilities bring different levels of satisfaction depending on how many of them are in the vicinity. Consequently, it is desirable to have a model which is robust enough to capture this nuance. This translates to λ being user defined, and is thus specified as part of the input to the problem.

Formal definition. We introduce some notation before giving a formal definition of the problem studied in this paper. For a given metric space $\mathscr{M} = (X, d)$, a point $x \in X$, a subset $S \subseteq X$, and $\lambda \in [k]$, we define $d^\lambda(x, S)$ to be the distance of x to the λ^{th} farthest point in S. To define this notion formally, we may sort the distances of a point x to each $s \in S$ in non-increasing order (breaking ties arbitrarily, if needed), and let these distances be $d(x, s_1) \geq d(x, s_2) \geq \ldots \geq d(x, s_k)$. Then, $s_\lambda \in S$ is said to be the λ^{th} farthest point from x in S, and $d^\lambda(x, S) = d(x, s_\lambda)$. Note that $d^1(x, S)$ is the distance of x from a farthest point in S. For a point $p \in X$ and a non-negative real r, $B(p, r) := \{q \in X : d(p, q) \leq r\}$ denotes the ball of radius r centered at p.

Obnoxious Egalitarian Median Committee Selection
(Obnox-Egal-Median-CS, in short)
Input: A metric space $\mathscr{M}$ consisting of a set of voters, $\mathscr{V}$, a set of candidates, $\mathcal{C}$; positive integers k and $\lambda \in [k]$; and a positive real t.
Question: Does there exist a subset $S \subseteq \mathcal{C}$ such that $|S| = k$ and for each $v \in \mathscr{V}$, $d^\lambda(v, S) \geq t$?

When $\lambda = 1$, we give the problem a special name, Obnoxious-Egal-CC, due to its similarity with the egalitarian CC rule. Note that the egalitarian (resp. utilitarian) CC rule itself is the special case of the egalitarian (resp. utilitarian) median rule when $\lambda = 1$.

Our Contributions. In the following, we discuss the highlights of our work in this paper and the underlying ideas used to obtain the result.

- We begin with studying Obnoxious-Egal-CC, that is, Obnox-Egal-Median-CS with $\lambda = 1$, and show that it is polynomial-time solvable when voters and candidates are embedded in $\mathbb{R}^2$ with Euclidean distances, Theorem 1. To design this algorithm, we first observe that the above setting can be equivalently reformulated as the following geometric problem. Given $\mathscr{V}$, and a set of equal-sized disks $\mathcal{D}$, find a k-size subset $\mathcal{D}' \subseteq \mathcal{D}$ such that no point of $\mathscr{V}$ belongs to the common intersection region of $\mathcal{D}'$. Following that we use geometric properties of equal-sized disks to design an algorithm that uses dynamic programming to inductively build such a region. This algorithmic result contrasts with the intractability of the non-obnoxious version (the k-Center problem) which is known to be NP-hard in $\mathbb{R}^2$.

- In Theorem 7, we consider Obnoxious-Egal-CC in general metric spaces. We show that it is NP-hard, and in fact, the optimization variant is also W[2]-hard to approximate beyond a factor of $1/3$, parameterized by k, the committee size. Informally speaking, this implies that no algorithm with running time $f(k)n^{\mathcal{O}(1)}$ is likely to exist, assuming widely believed complexity-theoretic assumptions. For more background on parameterized complexity, the reader may refer to the full version [36], or more generally, a textbook on the topic [23].
- Notwithstanding these negative results, we show that Obnoxious-Egal-CC admits a factor $1/4$ approximation algorithm that runs in polynomial time, Theorem 9. In this algorithm, we first compute a "$t/2$-net" $S \subseteq \mathcal{C}$, i.e., S satisfies the following two properties: (1) $d(c, c') > t/2$ for any distinct $c, c' \in S$, and (2) for any $c \notin S$, there exists some $c' \in \mathscr{M}$ such that $d(c, c') \leq t/2$. Now, consider a point $p \in \mathscr{V}$ and a $c^* \in \mathcal{C}$, such that $d(p, c^*) \geq t$. Then, by using the two properties of S, we argue that there exists a point in $c' \in S$ that is "near" c, and hence, "far from" p. More specifically, we can show that $d(p, c') \geq t/4$, leading to a $1/4$-approximate solution.
- Our work on Obnox-Egal-Median-CS for $\lambda > 1$ reveals that for $\lambda = k$, the problem can be solved in polynomial time due to the fact that every committee member needs to be at least t distance away from every voter. So, if possible, we can choose any k candidates that are t-distance away from every voter; otherwise, a solution does not exist. The algorithm is same as the one in Proposition 3 in [4], but here we can have ties. We show that for $\lambda = k - 1$, Obnox-Egal-Median-CS is NP-hard (Theorem 11) even when the voters and candidates are points in $\mathbb{R}^2$. Furthermore, we show that the intractability results we have for Obnoxious-Egal-CC in Theorem 7 carry forward to $\lambda > 1$, as shown in Theorem 13.
- For an arbitrary value of λ in $\mathbb{R}^d$ space, we exhibit a *fixed-parameter tractable approximation scheme*, that is, an algorithm that returns a solution of size k, in time FPT in (ϵ, λ, d), such that for every point $v \in \mathscr{V}$ there are at least λ points in the solution that are at distance at least $(1-\epsilon)t$ from v, Theorem 22. Note that $\lambda \leq k$, thus, this algorithm is also FPT in (ϵ, k, d). To obtain this result, we first observe that it is possible to further refine the idea of $t/2$-net, and define a set of "representatives", if the points belong to a Euclidean space. In this setting, for any $0 < \epsilon < 1$, we can compute a candidate set $\mathcal{R}$ of representatives, such that for every relevant $c \in \mathcal{C}$, there exists a $c' \in \mathcal{R}$ such that $d(c, c') \geq \epsilon/2$. Moreover, $\mathcal{R}$ is bounded by a function of λ, d, and ϵ. Thus, we can find an $(1 - \epsilon)$-approximation by enumerating all size-k subsets of $\mathcal{R}$.

Related works. Much of the research on multiwinner voting is concentrated on the computational complexity of computing winners under various rules, because for many applications it is crucial to be able to efficiently compute exact winners. As might be expected, computing winners under some committee scoring rules can be done in polynomial time (e.g., k-Borda [29]), while for many of the others the decision problem is NP-hard.

Effort towards applying the framework of parameterized complexity to these problems has primarily focused on parameters such as the committee size k and the number of voters, n. Indeed, this line of research has proven to be rather successful (see, e.g., [11, 10, 28, 31, 30, 4, 7, 6, 32, 70, 72, 49, 5, 52, 37, 69]). The problem has also been studied through the perspectives of approximation algorithms [55, 12] and parameterized approximation algorithms [60, 61, 10].

It is worth noting the similarities between our model and that of the *fault tolerant* versions of clustering problems, such as k-Center or k-Median [45, 39, 14], also [15]. In the latter setting, the clustering objective incorporates the distance of a point to its λ^{th} closest chosen center. Here, $\lambda \geq 1$ is typically assumed to be a small constant. Thus, even if $\lambda - 1$ centers chosen in the solution undergo failure, and if they all happen to be nearby a certain point p, we still have some (upper) bound on the distance of p to its now-closest center. Note that this motivation of fault tolerance translates naturally into our setting, where we want some (lower) bound on the distance of a voter to its λ^{th} *farthest* candidate, which may be useful of the $\lambda - 1$ farthest *candidates* are unable to perform their duties.

Preliminaries. In the optimization variant of OBNOX-EGAL-MEDIAN-CS, the input consists of $(\mathscr{M}, \mathscr{V}, \mathcal{C}, k, \lambda)$ as defined above, and the goal is to find the largest t^* for which the resulting instance is a yes-instance of OBNOX-EGAL-MEDIAN-CS, and we call such a t^* the optimal value of the instance. We say that an algorithm has an approximation guarantee of $\alpha \leq 1$, if for any input $(\mathscr{M}, \mathscr{V}, \mathcal{C}, k, \lambda)$, the algorithm finds a subset $S \subseteq \mathcal{C}$ of size k such that for each $v \in \mathscr{V}$, $d^\lambda(v, S) \geq \alpha \cdot t^*$.

For more details on parameterized complexity, we refer the reader to the textbooks [23, 33, 25, 56].

2 Obnoxious Egalitarian Chamberlin-Courant (CC)

We begin our study with OBNOXIOUS-EGAL-CC. Recall that OBNOX-EGAL-MEDIAN-CS with $\lambda = 1$ is OBNOXIOUS-EGAL-CC. We begin with the Euclidean space, followed by the general metric space.

2.1 Polynomial Time algorithm in $\mathbb{R}^2$

In this section, we design a polynomial time algorithm when the voters and candidates are embedded in $\mathbb{R}^2$. In particular, we prove the following result.

▶ **Theorem 1.** *There exists a polynomial-time algorithm to solve an instance of* OBNOXIOUS-EGAL-CC *when $\mathscr{V} \cup \mathcal{C} \subset \mathbb{R}^2$, and the distances are given by Euclidean distances.*

Overview. Before delving into a formal description of the polynomial-time algorithm, we start with a high-level overview of the result. For simplicity of the exposition, we assume that $t = 1$ (this can be easily achieved by scaling $\mathbb{R}^2$, and thus all points in the input, by a factor of t). For each $c \in \mathcal{C}$, let $D(c)$ denote a *unit disk* (i.e., an open disk of *diameter* 1) with c as its center. In the new formulation, we want to find a subset $S \subseteq \mathcal{C}$ of size k, such that for each $v \in \mathscr{V}$, the solution S contains at least one candidate c, such that v is outside $D(c)$ (which is equivalent to saying that the euclidean distance between v and c is larger than 1, which was exactly the original goal). This is an equivalent reformulation with a more geometric flavor, thus enabling us to use techniques from computational geometry.

First, we perform some basic preprocessing steps, that will be help us in the main algorithm. First, if there is a disk $D(c)$ that does not contain a voter, then any set containing c is a solution. Similarly, if we have two disjoint unit disks $D(c)$ and $D(c')$ centered at distinct $c, c' \in \mathcal{C}$, then any superset of $\{c, c'\}$ of size k is a valid solution, which can be found and returned easily. We check this condition for all subsets of size 2. In the final step of preprocessing, we iterate over all subsets of candidates of size 3, and check whether the common intersection of the corresponding three disks is empty – if we find such a set, then it is easy to see that any of its k-sized superset forms a solution. Now, assuming that the preprocessing step does not already give the solution, we know that each subset of unit disks of size at most 3 have a common intersection. By a classical result in discrete geometry called Helly's theorem [50], this also implies that each non-empty subset must have a common intersection. Our goal is to find a smallest such subset S, for which, the common intersection region is devoid of all voters $v \in \mathscr{V}$. We design a dynamic programming algorithm to find such a subset. Note that each subset is in one-to-one correspondence with a convex region defining the boundary of the common intersection, and the boundary of the common intersection consists of portions of boundaries of the corresponding unit disks (also known as "arcs"). The dynamic programming algorithm considers partial solutions defined by a consecutive sequence of arcs that can be attached end-to-end, while at the same time, ensuring that the common intersection does not contain any voter $v \in \mathscr{V}$. When we are trying to add another arc to the boundary, we have to make sure that (i) one of the endpoints of the arc is the same as one of the endpoints of the last arc defining the partial boundary, and (ii) the new area added to the "partial common intersection" does not contain a voter. We need to introduce several defintions and handle several special cases in order to formally prove the correctness of this strategy, which we do next.

Formal description. We work with the rescaled and reformulated version of the problem, as described above. Further, we assume, by infinitesimally perturbing the points if required (see, e.g., [26]), that the points $\mathcal{C} \cup \mathscr{V}$ satisfy the following general position assumption: no three unit disks centered at distinct candidates intersect at a common point. Note that this assumption is only required in order to simplify the algorithmic description.

For each candidate $c \in \mathcal{C}$, let $D(c)$ denote the unit disk (i.e., an open disk of radius 1) with c as center. In the following, we will often omit the qualifier *unit*, since all disks are assumed to be open unit disks unless explicitly mentioned otherwise. Note that our original problem is equivalent to determining whether there exists a subset $S \subseteq \mathcal{C}$ of size k such that for every $v \in \mathscr{V}$, there exists a candidate $c \in S$ such that $v \notin D(c)$. Equivalently, we want to find a set $S \subseteq \mathcal{C}$ such that $\left(\bigcap_{c \in S} D(c)\right) \cap \mathscr{V} = \emptyset$. For a subset $S' \subseteq \mathcal{C}$, we let $I(S') \coloneqq \bigcap_{c \in S'} D(c)$, and let $D(S') = \{D(c) : c \in S'\}$. We design a polynomial-time algorithm to find a smallest-sized subset $S' \subseteq \mathcal{C}$ such that $I(S') \cap \mathscr{V} = \emptyset$. For any two points $x, y \in \mathbb{R}^2$, let $\overline{xy}$ be the straight-line segment joining x and y.

We first perform the following preprocessing steps to handle easy cases. For $k = 1$, we try each $c \in \mathcal{C}$ and check whether $d(v, c) \geq 1$ for all voters $v \in \mathscr{V}$. Now suppose $k \geq 2$. First, we check whether there exists a pair of disks centered at distinct $c_1, c_2 \in \mathcal{C}$ such that the distance between c_1, c_2 is at least 2. Then, for any voter $v \in \mathscr{V}$, if $d(v, c_1) < 1$, then $d(v, c_2) > 1$ by triangle inequality. Therefore, $\{c_1, c_2\}$ can be augmented by adding arbitrary set of $k-2$ candidates in $\mathcal{C} \setminus \{c_1, c_2\}$ to obtain a solution. Now suppose that neither of the previous two steps succeeds. Then, we try all possible subsets $S' \subseteq \mathcal{C}$ of size at most 3, and check whether $I(S') = \emptyset$, that is, no point in $\mathbb{R}^2$ belongs to $I(S')$ (note that this specifically implies that $I(S') \cap \mathscr{V} = \emptyset$). If we find such a set S', then we can add an arbitrary subset of $\mathcal{C} \setminus S'$ of size $k - |S'|$ to obtain a set S of size k. Thus, we can make the following assumptions, given that the preprocessing step does not solve the problem.

1. $k \geq 4$,
2. For every $c, c' \in \mathcal{C}$, $D(c) \cap D(c') \neq \emptyset$, and the two disks intersect at two distinct points (this is handled in the second step of preprocessing), and
3. For any subset $\emptyset \neq S \subseteq \mathcal{C}$, $I(S) \neq \emptyset$. In particular, this also holds for sets S with $|S| > 3$ – otherwise by Helly's theorem [50], there would exist a subset $S' \subseteq S$ of size 3 such that $I(S') = \emptyset$, a case handled in the preprocessing step.

Let $\mathcal{P}$ be a set of intersection points of the boundaries of the disks $\{D(c) : c \in \mathcal{C}\}$. Note that since the boundaries of every pair of disks intersect exactly twice (this follows from the item (2) above), $|\mathcal{P}| = 2\binom{|\mathcal{C}|}{2}$. Furthermore, for $c \in \mathcal{C}$, let $\mathcal{P}(c) \subset \mathcal{P}$ be the set of intersection points that lie on the boundary of $D(c)$. For $c \in \mathcal{C}$ and distinct $p, q \in \mathcal{P}(c)$, we define $\mathsf{arc}(p, q, c)$ as the *minor arc* (i.e., the portion of the boundary of $D(c)$ that is smaller than a semicircle) of disk $D(c)$ with p and q as its endpoints. Note that p and q are interchangeable in the definition, and $\mathsf{arc}(p, q, c) = \mathsf{arc}(q, p, c)$. For a subset $S' \subseteq \mathcal{C}$, let $\mathcal{A}(S')$ be the set of arcs defining the boundary of the region $I(S')$ – note that since $I(S') \neq \emptyset$ for any $S' \neq \emptyset$, $\mathcal{A}(S')$ is well-defined and is a non-empty set of arcs. We first have the following proposition, the proof of which follows from arguments in planar geometry.

▶ **Proposition 2.** *Fix a set $S \subseteq \mathcal{C}$ with $|S| \geq 2$. Furthermore, assume that S is a* minimal *set with intersection equal to $I(S)$, i.e., there is no subset $S' \subset S$ such that $I(S') = I(S)$. Then, for every $c \in S$, $\mathcal{A}(S)$ contains exactly one arc of the form $\mathsf{arc}(p, q, c)$ for some $p, q \in \mathcal{P}(c)$.*

Proof. First we prove that every arc in $\mathcal{A}(S)$ is a minor arc. Suppose for contradiction that $\mathcal{A}(S)$ contains a non-minor arc A on the boundary of some $D(c)$, $c \in S$. Consider any $c' \in S$ with $c' \neq c$, and let $S' = \{c, c'\}$. Note that $I(S) \subseteq I(S')$ as $S' \subseteq S$ and intersection of disk can only decrease by adding more points to the set. Thus, $\mathcal{A}(S')$ contains an arc A' that is a superset of A. Let p and q denote the endpoints of A', and note that A' is also a non-minor arc. Note that $p \in I(S') = D(c) \cap D(c')$. Let p' denote the point on $D(c)$ that is diametrically opposite to p, and since A' is a major arc, it follows that $p' \in A' \subseteq I(S') = D(c) \cap D(c')$. To summarize, both p and p' belong to both $D(c)$ and $D(c')$. However, since both $D(c)$ and $D(c')$ are unit disks, $\overline{pp'}$ is a common diameter of $D(c)$ and $D(c')$, which contradicts that c and c' are distinct.

Now we prove the second part of the claim, that is, for each $c \in S$, $\mathcal{A}(S)$ contains exactly one minor arc of the form $\mathsf{arc}(\cdot, \cdot, c)$. Suppose there exists some c such that there exist two arcs $A_1 = \mathsf{arc}(p_1, q_1, c)$ and $A_2 = \mathsf{arc}(p_2, q_2, c)$ in $\mathcal{A}(S)$. Note that A_1 and A_2 must be disjoint, otherwise we can concatenate them to obtain a single arc. Suppose, without loss of generality, traversing clockwise along the boundary of $D(c)$, the ordering of the points is p_1, q_1, q_2, p_2. Let $c_1 \in S$ (resp. $c_2 \in S$) be the candidate such that q_1 (resp. q_2) belongs on the boundaries of $D(c)$ and $D(c_1)$ (resp. $D(c)$ and $D(c_2)$). It is clear that $c \neq c_1$ and $c \neq c_2$. We further claim that $c_1 \neq c_2$ – suppose this is not the case. Then, q_1 and q_2 belong to the boundaries of $D(c)$ and $D(c_1)$. In this case, p_1 (or p_2) cannot belong to $D(c) \cap D(c_1) \subseteq I(S)$, which contradicts the assumption that p_1 (or p_2) lie on the boundary of $I(S)$. Thus, we have that c, c_1, c_2 are all distinct. However, again we reach a contradiction since p_1 is outside $D(c) \cap D(c_2) \subseteq I(S)$. It follows that each arc appears at most once in $\mathcal{A}(S)$.

Finally, we consider the case when there is some $c \in S$ such that no arc of the form $\mathsf{arc}(\cdot, \cdot, c)$ belongs to $\mathcal{A}(S)$. In this case, the region bounded by $\mathcal{A}(S)$, i.e., $I(S)$, is completely contained inside $D(c)$. However, this implies that $I(S) = I(S \setminus \{c\})$, which contradicts the minimality of S. ◀

Next, we proceed towards designing our dynamic programming algorithm.

Algorithm. For any $x, y, p \in \mathcal{P}$, $c_1, c \in \mathcal{C}$, and an integer $i \geq 2$, we define a table entry $A[x, y, p, c_1, c, i]$ that denotes whether there exists a region $R(x, y, p, c_1, c) \subset \mathbb{R}^2$ with the following properties:

- $R = R(x, y, p, c_1, c)$ is a convex region bounded by a set $\mathcal{A}(R)$ of $i - 1$ circular arcs, and straight-line segment $\overline{xy}$, such that $\mathcal{A}(R)$ contains:
  - At most one arc of the form $\mathsf{arc}(\cdot, \cdot, c')$ for every $c' \in \mathcal{C}$.
  - Exactly one arc of the form $\mathsf{arc}(x, \cdot, c_1)$, which is the *first* arc traversed along the boundary of R in clockwise direction, starting from x. Note that c_1 is the center of this arc.
  - $\mathsf{arc}(y, p, c)$
- $R \cap \mathscr{V} = \emptyset$.

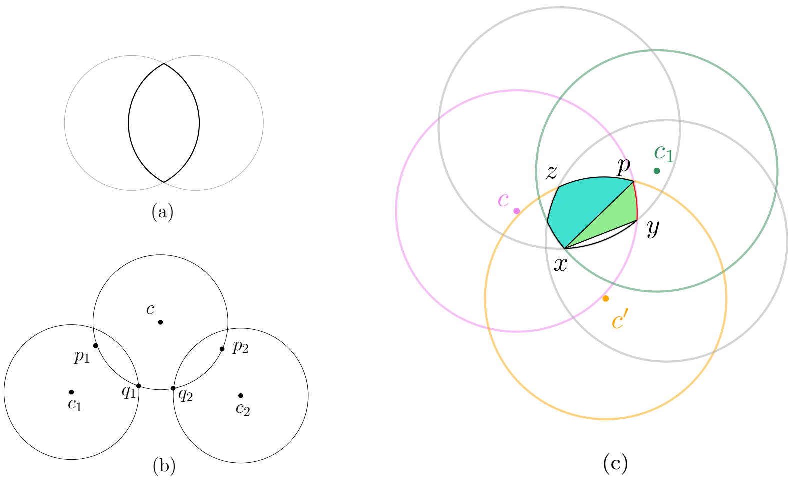

Figure 1 Illustration for the proof of Proposition 2 and the algorithm. Fig (a): intersection of boundaries of two unit disks is defined by two minor arcs. Fig (b): Two disjoint arcs $\mathsf{arc}(p_1, q_1, c)$ and $\mathsf{arc}(p_2, q_2, c)$ cannot appear on the boundary of a common intersection, since they correspond to disjoint regions. Fig (c): Illustration for the dynamic program. A region formed by intersection of 5 disks is shown. $\mathsf{arc}(p, y, c)$ is shown in red. Blue region corresponds to the entry $A[x, p, z, c_1, c', 3]$, and green region corresponds to the newly added region to the blue region, corresponding to the entry $A[x, y, p, c_1, c, 4]$.

Note that if $\mathsf{arc}(y, p, c)$ is not defined, or any of the other conditions do not hold, then a region with the required properties does not exist.

First, we compute all entries $A[x, y, p, c, c_1, i]$ with $i \leq 3$. Note that the number of arcs of the form $\mathsf{arc}(\cdot, \cdot, \cdot)$ is bounded by $O(|\mathcal{P}|^2 \cdot |\mathcal{C}|) = O(|\mathcal{C}|^3)$, and since $i \leq 3$, we can explicitly construct all such candidate regions in polynomial time. Thus, we can correctly populate all such table entries with `true` or `false`.

Now, we discuss how to fill a table entry $A[x, y, p, c_1, c, i]$ with $i \geq 4$. We fix one such entry and its arguments. If the region $R(x, p, y, c_1, c)$ bounded by $\overline{xp}, \overline{xy}$ and $\mathsf{arc}(y, p, c)$ contains a point from $\mathscr{V}$, then the entry $A[x, p, y, c_1, c, i]$ is defined to be `false`. Note that this can easily be checked in polynomial time. Otherwise, suppose that $R(x, p, y, c_1, c) \cap \mathscr{V} = \emptyset$. In this case, let $\mathcal{T}$ be a set of tuples of the form $(x, p, z, c_1, c', i-1)$, where $z \in \mathcal{P}$, and $c' \in \mathcal{C}$ such that the following conditions are satisfied. (1) $c' \notin \{c, c_1\}$, (2) The minor arc $\mathsf{arc}(p, z, c')$ exists, and (3) When traversing along this arc from z to p, the arc $\mathsf{arc}(p, y, c)$ is a "right turn". Formally, consider the tangents $\ell_c, \ell_{c'}$ to the disks $D(c)$ and $D(c')$ at point p respectively. Let H_c (resp. $H_{c'}$) be the closed halfplane defined by the line ℓ_c ($\ell_{c'}$) that contains $D(c)$ ($D(c')$). Then, arcs $\mathsf{arc}(p, z, c')$ and $\mathsf{arc}(y, p, c)$ must belong to $H_c \cap H_{c'}$. See Figure 1(c). Then,

$$A[x, y, p, c_1, c, i] = \bigvee_{(x,p,z,c_1,c',i-1) \in \mathcal{T}} A[x, p, z, c_1, c', i-1].$$

Since we take an or over at most $|\mathcal{C}| \times |\mathscr{V}|$ many entries, each such entry can be computed in polynomial time. Furthermore, since the number of entries is polynomial in $|\mathcal{C}|$ and $|\mathscr{V}|$, the entire table can be populated in polynomial time.

Now, we iterate over all entries $A[x, y, p, c_1, c, i]$ such that the following conditions hold.

- $A[x, y, p, c_1, c, i] =$ `true`,
- There exists some $c'' \in \mathcal{C} \setminus \{c, c_1\}$ such that $\mathsf{arc}(x, y, c'')$ exists, and
- The region bounded by $\mathsf{arc}(x, y, c'')$ and segment $\overline{xy}$ does not contain any point from $\mathcal{C}$. The meaning here is that $\mathsf{arc}(x, y, c'')$ is the last arc bounding the required region.

If such an entry exists with $i \leq k-1$, then we conclude that the given instance of Obnox-Egal-Median-CS is a yes-instance. Otherwise, it is a no-instance. Finally, using standard backtracking strategy in dynamic programming, it actually computes a set $S \subseteq \mathcal{C}$ such that $I(S) \cap \mathscr{V} = \emptyset$. Next, we establish the proof of correctness of this dynamic program.

A Proof of Correctness.

▶ **Lemma 3 (♣).**[1] *Consider an entry $A[x,y,p,c_1,c,i]$, corresponding to some $x,y,p \in \mathcal{P}, c_1, c \in \mathcal{C}$. Then, $A[x,y,p,c_1,c,i] =$ `true` if and only if the corresponding region R, as in the definition of the table entry, contains no point of $\mathscr{V}$.*

▶ **Lemma 4.** *This algorithm correctly decides whether the given instance of* Obnox-Egal-Median-CS *is a* yes-*instance.*

Proof. First, it is easy to see that the preprocessing step correctly finds a minimum-size subset of at most 3 whose intersection contains no point of $\mathscr{V}$, if such a subset exists. Thus, we now assume that the preprocessing step does not find a solution, and the algorithm proceeds to the dynamic programming part.

Recall that due to Proposition 1, if S is a minimal subset of centers, such that $I(S) \cap \mathscr{V} = \emptyset$ (if any), then $\mathcal{A}(S)$ contains exactly one minor arc that is part of the circle centered at each $c \in S$. In particular, this holds for the optimal set S^* of centers (if any), and let $i = |S^*|$. Pick an arbitrary arc in $\mathcal{A}(S^*)$, and let c_1 be the center of this arc, and x be one of the endpoints of this arc. By traversing the arcs in $\mathcal{A}(S^*)$ in clockwise manner, let the last two arcs be $\mathsf{arc}(y,p,c)$, and $\mathsf{arc}(y,x,c')$. Then, by Lemma 3, it follows that $A[x,y,p,c_1,c',i-1] =$ `true`, and the region bounded by $\overline{xy}$ and $\mathsf{arc}(y,x,c')$ does not contain a point from $\mathcal{C}$. Thus, the algorithm outputs the correct solution corresponding to the entry $A[x,y,p,c_1,c',i-1]$.

In the other direction, if the algorithm finds an entry $A[x,y,p,c_1,c',i-1] =$ `true`, such that (1) $\mathsf{arc}(x,y,c)$ exists, (2) $c \notin \{c', c_1\}$, and (3) the region bounded by $\mathsf{arc}(x,y,c)$ and $\overline{xy}$ does not contain a point from $\mathcal{C}$, then using Lemma 3, we can find a set of i disks whose intersection does not contain a point from $\mathcal{C}$. Therefore, if for all entries it holds that at least one of the conditions does not hold, then the algorithm correctly concludes that the given instance is a no-instance. ◀

The algorithm as it is does not work for $\lambda > 1$. We do not know whether the problem is polynomial time solvable for $\lambda > 1$.

2.2 Hardness in Graph Metric

In this section, we show the intractability of the problem when the voters and candidates are embedded in the graph metric space, which implies the intractability in the general metric space. The metric space defined by the vertex set of a graph as points and distance between two points as the shortest distance between the corresponding vertices in the graph is called the *graph metric space*.

We present a reduction from the Hitting Set problem, defined below, which is known to be NP-hard [42] and W[2]-parameterized by k [23].

[1] The proofs of the statements marked with ♣ can be found in the full version [36].

> Hitting Set
> **Input:** Set system $(\mathcal{U}, \mathcal{F})$, where $\mathcal{U}$ is the ground set of n elements, $\mathcal{F}$ is a family of subsets of $\mathcal{U}$, and a positive integer k
> **Question:** Does there exist $H \subseteq \mathcal{U}$ of size k such that for any $S \in \mathcal{F}$, $H \cap S \neq \emptyset$?

Reduction. Define a graph G with vertex set $\mathscr{V} \cup \mathcal{C}$ as follows. For every element $e \in \mathcal{U}$, we add a candidate c_e to $\mathcal{C}$, and for every set $S \in \mathcal{F}$, we add a voter v_S to $\mathscr{V}$. We add an edge (c_e, v_S) in G if and only if $e \notin S$. The weight of all such edges is equal to 1. Also, for any $c_e, c'_e \in \mathcal{C}$, we add an edge of weight 2. The distance function $d : (\mathscr{V} \cup \mathcal{C}) \to \mathbb{R}^+$ is given by the shortest path distances in G.

▶ **Observation 5.** *For any $e \in \mathcal{U}$, and $S \in \mathcal{F}$, $d(c_e, v_S) = 1$ if and only if $e \notin S$. Otherwise $d(c_e, v_S) = 3$ if and only if $e \in S$.*

▶ **Lemma 6 (♣).** *$(\mathcal{U}, \mathcal{F})$ admits a hitting set of size k if and only if there exists a set $H \subseteq \mathcal{C}$ of size k such that for any $v_S \in \mathscr{V}$, $\max_{c_e \in H} d(c_e, v_S) = 3$.*

In fact, this construction shows that it is NP-hard to approximate the problem within a factor of $1/3 + \epsilon$ for any $\epsilon > 0$. Indeed, suppose there existed such a $\beta = (1/3 + \epsilon)$-approximation for some $\epsilon > 0$. Then, if $(\mathcal{U}, \mathcal{F})$ is a yes-instance of Hitting Set, then Lemma 6 implies that $\mathsf{OPT} = 3$ – here OPT denotes the largest value of t for which we have a yes-instance for the decision version. In this case, the β-approximation returns a solution S of size k and of cost at least $\beta \cdot 3 = 1 + 3\epsilon > 1$. This implies that for each $v_S \in \mathscr{V}$, there exists some $c_e \in S$ with $d(v_S, c_e) > 1$. However, such a c_e must correspond to an element $e \in S$ – otherwise $d(v_S, c_e) = 1$ by construction. Therefore, the solution S corresponds to a hitting set of size k. Alternatively, if $(\mathcal{U}, \mathcal{F})$ is a no-instance of Hitting Set, then Lemma 6 implies that the there is no solution of size k with cost 3. Thus, a β-approximation can be used to distinguish between yes- and no-instances of Hitting Set. Hence, we have the following result.

▶ **Theorem 7.** Obnoxious-Egal-CC *is* NP*-hard. Furthermore, for any $\alpha > 1/3$,* Obnoxious-Egal-CC *does not admit a polynomial time α-approximation algorithm, unless* $\mathsf{P} = \mathsf{NP}$*. Furthermore,* Obnoxious-Egal-CC *does not admit an* FPT*-approximation algorithm parameterized by k with an approximation guarantee of $\alpha > 1/3$, unless* $\mathsf{FPT} = W[2]$.

2.3 Approximation Algorithm in General Metric Space

In this section, we design a polynomial time 1/4-approximation algorithm when voters and candidates are embedded in a general metric space. Since the problem is trivial for $k = 1$ (we can simply iterate over all solutions of size 1, i.e., all $c \in \mathcal{C}$, and check whether it forms a solution), we assume in the rest of the section that $k > 1$.

We first guess a voter $p' \in \mathscr{V}$ and a candidate $c' \in \mathcal{C}$ such that c' is the farthest candidate from p' in an optimal solution S^*. Let $t = d(p', c')$. We know that all candidates in S^* are within a distance t from p', i.e., $S^* \subseteq B$, where $B = B(p', t) \cap \mathcal{C}$. Let M be a $(t/2)$-net of B, i.e., M is a *maximal* set of candidates with the following properties: (1) $d(c_i, c_j) > t/2$ for any distinct $c_i, c_j \in M$, and (2) for any $c \notin M$, there exists some $c' \in M$ such that $d(c, c') \leq t/2$. Note that such a $(t/2)$-net can be found in polynomial-time using a simple greedy algorithm.

Now there are two cases. (1) If $|M| \geq k$, let M' be an arbitrary subset of M of size exactly k. (2) If $|M| < k$, then let $M' = M \cup Q$ where Q is an arbitrary subset of candidates from $B \setminus M$ such that $|M'| = k$.

▶ **Lemma 8.** *Fix some $v \in \mathscr{V}$, and let $c \in \mathcal{C}$ be the farthest center from v in M'. Then, $d(v, c) \geq t/4$.*

Proof. We consider two cases based on the size of M, and the way we obtain M' from M.

Case 1: $|M| \geq k$, and M' is an arbitrary subset of M. Suppose for contradiction $d(v, c) < t/4$. Since c is the farthest candidate from v, the same is true for any candidate $c' \in M'$. Then, $d(p, c) < t/4$ and $d(v, c') < t/4$, which implies that $d(c, c') \leq d(v, c) + d(v, c') < t/4 + t/4 = t/2$, which contradicts property 1 of M.

Case 2: $|M| < k$ and M' is obtained by adding arbitrary candidates to M. If $d(v, c) \geq t/2 \geq t/4$, we are done. So assume that $d(v, c) < t/2$. Let c^* be the farthest center from v in an optimal solution. Then, $d(v, c^*) \geq t$. Also, $c^* \notin M \subseteq M'$, otherwise $d(v, c) \geq d(v, c^*) \geq t$, since c is the farthest candidate from v. Therefore, by property 2, there exists some $c' \in M$ such that $d(c', c^*) \leq t/2$. Again, $d(v, c') \leq d(v, c) < t/2$. Then, $d(v, c^*) \leq d(v, c') + d(c', c^*) < t/2 + t/2 = t$. This contradicts $d(v, c^*) \geq t$. ◀

Thus, we conclude with the following theorem.

▶ **Theorem 9.** *There is a polynomial-time 1/4-approximation algorithm for the optimization variant of* Obnoxious-Egal-CC *when the voters and candidates belong to an arbitrary metric space $\mathscr{M}$.*

3 Obnoxious Egalitarian Median Committee Selection for $\lambda > 1$

In this section, we move our study to the case when $\lambda > 1$. We first show that $\lambda = k - 1$ is NP-hard in $\mathbb{R}^2$. But the extreme cases of $\lambda = 1$ or $\lambda = k$ are tractable: In fact, the $\lambda = 1$ case is polynomial-time solvable for $\mathbb{R}^2$ but the $\lambda = k$ is polynomial-time solvable even in a general metric space. Furthermore, we show that similar to $\lambda = 1$, the problem is hard to approximate for any value of λ in graph metric. Finally, contrary to Theorem 13 we give an FPT-approximation scheme for arbitrary value of λ in $\mathbb{R}^d$.

3.1 Hardness

In this section, we present results pertaining to NP-hardness and approximation hardness.

NP-hardness for $\lambda = k - 1$ in $\mathbb{R}^2$. To exhibit this we give a reduction from the 2-Independent Set problem in unit disk graphs (UDGs). We give a formal definition of UDGs below, followed by the definition of the aforementioned problem.

▶ **Definition 10.** *Given a set $\mathscr{P} = \{p_1, p_2, \ldots, p_n\}$ of points in the plane, a unit disk graph (UDG, in short) corresponding to the set $\mathscr{P}$ is a graph $G = (\mathscr{P}, E)$ satisfying $E = \{(p_i, p_j) | d(p_i, p_j) \leq 1\}$, where $d(p_i, p_j)$ denotes the Euclidean distance between p_i and p_j.*

> 2-Independent Set In Unit Disk Graph
> **Input:** Given a set $V \subset \mathbb{R}^2$ of n points, and a positive integer k.
> **Question:** Let $G = (V, E)$ be a unit disk graph defined on V. Does there exist a subset $S \subseteq V$ such that $|S| = k$, and for any distinct $u, w \in S$, $d_G(u, w) > 2$?

This problem is shown to be NP-hard in [40].

▶ **Theorem 11.** Obnox-Egal-Median-CS *is* NP*-hard when $\lambda = k - 1$, even in the special case where $\mathscr{V} \cup \mathcal{C} \subset \mathbb{R}^2$ and the distances are given by standard Euclidean distances.*

Proof. Let (V, k) be the given instance of 2-Independent Set In Unit Disk Graph, where $V \subset \mathbb{R}^2$. We create an instance of Obnox-Egal-Median-CS as follows. For every point $p \in V$, we add a voter and a candidate co-located at the point in $\mathbb{R}^2$ at the point p. Let $\mathscr{V}$ and $\mathcal{C}$ be the resulting sets of n voters and n candidates, and the value of k remains unchanged. Without loss of generality, we assume that $k \geq 2$. We set $\lambda = k-1$. We prove the following lemma. Note that due to the strict inequality, this does not quite fit the definition of Obnox-Egal-Median-CS. Subsequently, we discuss how to modify the construction so that this issue is alleviated. In the following proof, we use $d_e(\cdot, \cdot)$ to denote the Euclidean distance and $d_G(\cdot, \cdot)$ to denote the shortest-path distance in the unit disk graph G.

▶ **Lemma 12.** *S is a 2-independent set of size k in G if and only if for the corresponding set $S' \subseteq \mathcal{C}$, it holds that, for every voter $v \in \mathscr{V}$, $d_e^\lambda(v, S') > 2$.*

Proof. In the forward direction, let S be a 2-independent set of size k in G, and let S' be as defined above. Suppose for the contradiction that there exists a voter v for which $d_e^\lambda(v, S') \leq 2$. That is, there exists two distinct candidates $c_1, c_2 \in S'$ such that $d_e(v, c_1) \leq 2$, and $d_e(v, c_2) \leq 2$. We consider two cases, depending on whether v is co-located with either of c_1 or c_2, or not. Suppose v is co-located with c_1 (w.l.o.g., the c_2 case is symmetric). Then, let p_1 and p_2 be the points in $S \subseteq P$ corresponding to v and c_2 respectively. However, since $d_e(p_1, p_2) \leq 2$, (p_1, p_2) is an edge in G, which contradicts the 2-independence of S. In the second case, v is not co-located with c_1 as well as c_2. Even in this case, let q, p_1, p_2 be the points in P corresponding to v, c_1, and c_2 respectively. Note that q, p_1, p_2 are distinct, and $p_1, p_2 \in S$. However, since $d_e(q, p_1) \leq 2, d_e(q, p_2) \leq 2$, (q, p_1) and (q, p_2) are edges in G, which again contradicts the 2-independence of S, as $d_G(p_1, p_2) = 2$.

In the reverse direction, let $S' \subseteq \mathcal{C}$ be a subset of candidates such that for each voter $v \in \mathscr{V}$, $d_e^\lambda(v, S') > 2$. Let $S \subseteq P$ be the corresponding points of S', and suppose for contradiction that S is not a 2-independent set in G, which implies that there exist two distinct $p_1, p_2 \in S$ such that $d_G(p_1, p_2) \leq 2$. Let c_1, c_2 be the candidates in S' corresponding to p_1 and p_2 respectively. Again, we consider two cases. First, suppose that $d_G(p_1, p_2) = 1$, i.e., (p_1, p_2) is an edge in G. Then, let v_1 be the voter co-located at p_1. Then, for v_1, $d_e(v_1, c_1) = 0$, and $d_e(v_1, c_2) \leq 2$, since (p_1, p_2) is an edge. This contradicts that $d_e^\lambda(v_1, S') > 2$. In the second case, suppose $d_G(p_1, p_2) = 2$, then let $q \in P$ be a common neighbor of p_1 and p_2 in G, and let $v_q \in \mathscr{V}$ be the voter co-located to q. Again, note that $d_e(v_q, c_1) \leq 2$ and $d_e(v_q, c_2) \leq 2$, which contradicts that $d_e^\lambda(v_q, S') > 2$. ◀

Let $t \coloneqq \min_{p,q \in P : d_e(p,q) > 2} d_e(p, q)$. That is, t is the smallest Euclidean distance between non-neighbors in G. By definition, for any $p', q' \in P$ such that $d_e(p', q') > 2$, it holds that $d_e(p', q') \geq t$. Now, we observe that the proof of Lemma 12 also works after changing the condition $d_e^\lambda(v, S') > 2$ to $d_e^\lambda(v, S') \geq t$. Note that there exists points p, q such that $d_G(p, q) > 2$, and hence $d_e(p, q) > 2$, otherwise, it is a trivial no-instance of 2-Independent Set In Unit Disk Graph. ◀

Approximation Hardness in Graph Metric. The reduction is same as in Section 2.2. Here, instead of Hitting Set, we give a reduction from the Multi-Hitting Set problem, where each set needs to be hit at least $\lambda \geq 1$ times for some constant λ. It can be easily seen that this is a generalization of Hitting Set and is also NP-complete [57] (for an easy reduction from Hitting Set, simply add $\lambda - 1$ "effectively dummy" sets that contain all the original elements) Thus, we have the following result.

▶ **Theorem 13.** *For any fixed $1 \leq \lambda < k$,* Obnox-Egal-Median-CS *is* NP*-hard. Furthermore, for any fixed $1 \leq \lambda < k$, and for any $\alpha \geq 1/3$,* Obnox-Egal-Median-CS *does not admit a polynomial time α-approximation algorithm, unless* P = NP*. Furthermore,* Obnox-Egal-Median-CS *does not admit an FPT-approximation algorithm parameterized by k with an approximation guarantee of $\alpha \geq 1/3$, unless $FPT = W[2]$.*

3.2 FPT-AS in Euclidean and Doubling Spaces

In this section, we design an FPT approximation scheme for the inputs in $\mathbb{R}^d$, parameterized by λ, d, and ϵ. In fact, the same arguments can be extended to metric spaces of doubling dimension d. However, we focus on $\mathbb{R}^d$ for the ease of exposition, and discuss the case of doubling spaces at the end.

In the subsequent discussions, we say that $S \subseteq \mathcal{C}$ is a *solution* if it satisfies the following two properties: (i) $|S| \geq \lambda$, and (ii) for each $v \in \mathscr{V}$, $d^\lambda(v, S) \geq t$. For any given instance of Obnox-Egal-Median-CS, we state the following simple observations.

▶ **Observation 14.** *If there exists $S \subseteq \mathcal{C}$ of size at least λ, such that each $c \in S$ is at distance at least t from each $v \in \mathscr{V}$, then S is a solution.*

▶ **Observation 15 (♣).** *A subset $S \subseteq \mathcal{C}$ of $\lambda + 1$ points that are pairwise $2t$ distance away from each other is a solution.*

First, note that we can assume $\lambda + 1 \leq k$ – otherwise $\lambda = k$ case can be easily solved in polynomial-time using the argument mentioned in the preliminaries. Now, if a set $S \subseteq \mathcal{C}$ with $|S| \geq \lambda + 1$ satisfying the conditions of Observation 15 exists, then we can immediately augment it with arbitrary $k - (\lambda + 1)$ candidates from $\mathcal{C} \setminus S$, yielding a solution of size k. Thus, henceforth, we may assume that any subset $S \subseteq \mathcal{C}$ consisting of candidates that are pairwise $2t$ distance away from each other, has size at most λ.

Let us fix N to be one such maximal subset – note that we can compute N in polynomial time using a greedy algorithm. The following observation follows from the maximality of N.

▶ **Observation 16.** *Any point $p \in \mathcal{C}$ must be in $\bigcup_{c \in N} B(c, 2t)$. In other words, each $p \in \mathcal{C}$ is inside a ball of radius $2t$ centered at one of the points in N.*

This simple observation, combined with the following covering-packing property of the underlying Euclidean space will allow our algorithm to pick points from the vicinity of those chosen by an optimal algorithm.

▶ **Proposition 17 (♣).** *In $\mathbb{R}^d$, for any $0 < r_1 < r_2$, a ball of radius r_2 can be covered by $\alpha_d \cdot (r_2/r_1)^d$ balls of radius r_1. Here, α_d is a constant that depends only on the dimension d.*

Next, for each $c \in N$, we find an "$\epsilon t/4$-net" inside the ball $B(c, 2t)$, i.e., a maximal subset $Q \subseteq B(c, 2t) \cap \mathcal{C}$, such that (i) for any distinct $c_1, c_2 \in Q$, $d(c_1, c_2) > \epsilon t/4$, and (ii) For each $c_1 \in \mathcal{C} \setminus Q$, there exists some $c_2 \in Q$, such that $d(c_1, c_2) \leq \epsilon t/4$. Note that Q can be computed using a greedy algorithm. Next, we iterate over each $c' \in Q$, and mark the $\lambda - 1$ closest unmarked candidates to c' that are not in Q (if any). Let $R_c \coloneqq Q \cup M$, where M denotes the set of marked candidates during the second phase.

▶ **Observation 18 (♣).** *For each $c \in N$, $|R_c| \leq \mathcal{O}_d(\lambda \cdot (1/\epsilon)^d)$, where $\mathcal{O}_d(\cdot)$ hides a constant that depends only on the dimension d.*

Let $S' = \bigcup_{c \in N} R_c$. Finally, let $S \coloneqq N \cup S'$, and note that $|S| \leq \mathcal{O}_d(\lambda^2 \cdot (1/\epsilon)^d)$, where $\mathcal{O}_d(\cdot)$ notation hides constants that depend only on d. Now we consider two cases.

- If $|S| \leq k$, then we augment it with arbitrary $k - |S|$ candidates from $\mathcal{C} \setminus S$, and output the resulting set.
- If $|S| > k$, then try all possible k-sized subsets of S to see if it constitutes a solution. There can be at most $\binom{|S|}{k} < 2^{|S|} = 2^{\mathcal{O}_d(\lambda^2(1/\epsilon)^d)}$ sets to check resulting in time $2^{\mathcal{O}_d(\lambda^2(1/\epsilon)^d)} \cdot (|\mathscr{V}| + |\mathcal{C}|)^{\mathcal{O}(1)}$.

The next lemma completes the proof. We prove it by comparing S to an optimal solution, and show that for every point in the latter there is a point in the vicinity that is present in S.

▶ **Lemma 19.** *If $|S| > k$, then there is a subset $Q \subseteq S$ of size k that constitutes a solution.*

Proof. Suppose that there is an optimal solution, denoted by O, that contains k points and for each point $v \in \mathscr{V}$ there exist at least λ points in O (called *representatives*, $\mathcal{R}(v)$) that are at least t distance away from v. Let $\mathcal{R} = \bigcup_{v \in \mathscr{V}} \mathcal{R}(v)$ denote the set of all representatives.

First, due to Observation 16, each $c \in \mathcal{R}$ is inside some $B(c', 2t)$ for some $c' \in N$. Let $\tilde{c} \in Q$ be the closest (breaking ties arbitrarily) candidate to c from Q. By construction, $d(\tilde{c}, c) \leq t\epsilon/4$. Let $A(\tilde{c}) \subseteq \mathcal{R}$ be the points for which $\tilde{c}$ is the closest point in $R_{c'}$ (breaking ties arbitrarily).

Case 1: $|A(\tilde{c})| \leq \lambda$. In this case, we claim that for each $c_1 \in A(\tilde{c})$, we have added a unique c_2 to $R_{c'} \subseteq S'$ such that $d(c_1, c_2) \leq \epsilon t$. First, if $A(\tilde{c}) \subseteq R_{c'}$, then the claim is trivially true (the required bijection is the identity mapping). Otherwise, there exists some $c_1 \in A(\tilde{c})$ such that $c_1 \notin R_c$. In particular, this means that c_1 was not marked during the iteration of the marking phase corresponding to $\tilde{c} \in Q$. This means that at least $\lambda - 1$ other candidates with distance at most $\epsilon t/4$ from $\tilde{c}$ were marked. For any of these marked candidates c_2, it holds that $d(c_1, c_2) \leq d(c_1, \tilde{c}) + d(\tilde{c}, c_2) \leq \epsilon t/2 \leq \epsilon t$. Accounting for $\tilde{c}$, this implies that, for each $c_1 \in A(\tilde{c})$, there are at least $\lambda \geq |A(\tilde{c})|$ distinct candidates in R_c within distance ϵt. Let $Q(\tilde{c}) \subseteq S'$ denote an arbitrary such subset of size λ in this case.

Case 2: $|A(\tilde{c})| > \lambda$. In this case, let $A'(\tilde{c}) \subset A(\tilde{c})$ be an arbitrary subset of size λ. We claim that $A'(\tilde{c})$ is sufficient for any solution. In particular, consider a $v \in \mathscr{V}$ and $c \in A(\tilde{c}) \setminus A'(\tilde{c})$ such that c is a representative of v. We claim that for all $c' \in A'(\tilde{c})$, $d(\tilde{c}, c') \geq (1 - \epsilon/2)t$, which follows from $d(c, c') \leq \epsilon t/2$. Thus, the λ points of $A'(\tilde{c})$ constitute an approximate set of representatives for v. Now, by using the argument from the previous paragraph w.r.t. $A'(\tilde{c})$, we can obtain a set $Q(\tilde{c})$ of size λ, such that for any voter $v \in V$ such that $\mathcal{R}(v) \cap A(\tilde{c}) \neq \emptyset$, every point in $Q(\tilde{c})$ is at distance at least $(1 - \epsilon)t$ from v.

Finally, let Q denote the union of all sets $Q(\tilde{c})$ defined in this manner (note that $Q(\tilde{c})$ is defined only if $A(\tilde{c}) \neq \emptyset$). First, by construction, for each $v \in \mathscr{V}$, Q contains at least λ points at distance at least $(1 - \epsilon)t$. Next, $Q \subseteq R'$ and $|Q| \leq k$ since for each point in $\mathcal{R}$, we add at most one point to Q. Now, if $|Q| < k$, then we can simply add arbitrary $k - |Q|$ points to obtain the desired set. ◀

In fact, the covering-packing properties of the underlying metric space that are crucial in our algorithm are abstracted in the following well-known notion.

▶ **Definition 20** (Doubling dimension and doubling spaces). *Let $\mathscr{M} = (P, d)$ be a metric space, where P is a set of points and d is the distance function. We say that $\mathscr{M}$ has* doubling dimension *δ, if for any $p \in P$, and any $r \geq 0$, the ball $B(p, r) := \{q \in P : d(p, q) \leq r\}$ can be covered using at most 2^δ balls of radius $r/2$. If the doubling dimension of a metric space $\mathscr{M}$ is a constant, then we say that $\mathscr{M}$ is a* doubling space.

Note that Euclidean space of dimension d has doubling dimension $O(d)$. By a simple repeated application of the above definition, we obtain the following Proposition 21 that is an analogue of Proposition 17.

▶ **Proposition 21.** *Let $\mathscr{M} = (P, d)$ be a metric space of doubling dimension δ. Then, any ball $B(p, r_2)$ can be covered with $\left(\lceil \frac{r_2}{r_1} \rceil\right)^\delta$ balls of radius r_1, where $0 < r_1 \leq r_2$.*

Our algorithm generalizes to metric spaces of doubling dimension δ in a straightforward manner, resulting in the following theorem.

▶ **Theorem 22.** *For any ϵ, $0 < \epsilon < 1$, we have an algorithm that given an instance of* Obnox-Egal-Median-CS *in a metric space of doubling dimension δ, computes a solution of size k such that for every point $v \in \mathscr{V}$ there are at least λ points in the solution that are at distance at least $(1-\epsilon)t$ from v in time* FPT *in $(\epsilon, \lambda, \delta)$. In particular, we obtain this result in Euclidean spaces of dimension d, in time* FPT *in (ϵ, λ, d).*

4 Outlook

In this paper we studied a committee selection problem, where preferences of voters towards candidates was captured via a metric space. In particular, we studied a variant where larger distance corresponds to higher preference for a candidate in comparison to a candidate who is nearer. We showed that our problem is NP-hard in general, and designed some polynomial time algorithms, as well as (parameterized) approximation algorithms. We conclude with some research directions for future study. One of our concrete open question is that Is Obnox-Egal-Median-CS in $\mathbb{R}^2$ for $\lambda > 1$ polynomial-time solvable? In this paper, we considered median scoring rules. It would be interesting to study other scoring rules as well, when the voters and candidates are embedded in a metric space.

Moreover, we note that situations where, for each neighborhood we want exactly λ facilities nearby, and the remaining $k - \lambda$ to be far away, is not handled by this model. This would be the "exact" variant of our problem Obnox-Egal-Median-CS and would be of natural interest.

References

1 Hyung-Chan An and O. Svensson. Recent developments in approximation algorithms for facility location and clustering problems. In *Combinatorial Optimization and Graph Algorithms: Communications of NII Shonan Meetings*, pages 1–19. Springer, 2017.

2 Nima Anari, Moses Charikar, and Prasanna Ramakrishnan. Distortion in metric matching with ordinal preferences. In Kevin Leyton-Brown, Jason D. Hartline, and Larry Samuelson, editors, *Proceedings of the 24th ACM Conference on Economics and Computation, EC 2023, London, United Kingdom, July 9-12, 2023*, pages 90–110. ACM, 2023. doi:10.1145/3580507.3597740.

3 Haris Aziz, Felix Brandt, Edith Elkind, and Piotr Skowron. Computational social choice: The first ten years and beyond. In Bernhard Steffen and Gerhard J. Woeginger, editors, *Computing and Software Science - State of the Art and Perspectives*, volume 10000 of *Lecture Notes in Computer Science*, pages 48–65. Springer, 2019. doi:10.1007/978-3-319-91908-9_4.

4 Haris Aziz, Piotr Faliszewski, Bernard Grofman, Arkadii Slinko, and Nimrod Talmon. Egalitarian committee scoring rules. In Jérôme Lang, editor, *Proceedings of the Twenty-Seventh International Joint Conference on Artificial Intelligence, IJCAI 2018, July 13-19, 2018, Stockholm, Sweden*, pages 56–62. ijcai.org, 2018. doi:10.24963/IJCAI.2018/8.

5 Haris Aziz, Serge Gaspers, Joachim Gudmundsson, Simon Mackenzie, Nicholas Mattei, and Toby Walsh. Computational aspects of multi-winner approval voting. In Gerhard Weiss, Pinar Yolum, Rafael H. Bordini, and Edith Elkind, editors, *Proceedings of the 2015 International Conference on Autonomous Agents and Multiagent Systems, AAMAS 2015, Istanbul, Turkey, May 4-8, 2015*, pages 107–115. ACM, 2015. URL: `http://dl.acm.org/citation.cfm?id=2772896`.

6 Nadja Betzler, Robert Bredereck, Jiehua Chen, and Rolf Niedermeier. Studies in computational aspects of voting - A parameterized complexity perspective. In Hans L. Bodlaender, Rod Downey, Fedor V. Fomin, and Dániel Marx, editors, *The Multivariate Algorithmic Revolution and Beyond - Essays Dedicated to Michael R. Fellows on the Occasion of His 60th Birthday*, volume 7370 of *Lecture Notes in Computer Science*, pages 318–363. Springer, 2012. `doi:10.1007/978-3-642-30891-8_16`.

7 Nadja Betzler, Arkadii Slinko, and Johannes Uhlmann. On the computation of fully proportional representation. *J. Artif. Intell. Res.*, 47:475–519, 2013. `doi:10.1613/JAIR.3896`.

8 Steven J Brams, D Marc Kilgour, and M Remzi Sanver. A minimax procedure for electing committees. *Public Choice*, 132:401–420, 2007.

9 Felix Brandt, Markus Brill, and Bernhard Harrenstein. Tournament Solutions. In F Brandt, V Conitzer, U Endriss, J Lang, and A. D. Procaccia, editors, *Handbook of Computational Social Choice*, pages 453–474. Cambridge University Press, 2016.

10 Robert Bredereck, Piotr Faliszewski, Andrzej Kaczmarczyk, Dusan Knop, and Rolf Niedermeier. Parameterized algorithms for finding a collective set of items. In *The Thirty-Fourth AAAI Conference on Artificial Intelligence, AAAI 2020, The Thirty-Second Innovative Applications of Artificial Intelligence Conference, IAAI 2020, The Tenth AAAI Symposium on Educational Advances in Artificial Intelligence, EAAI 2020, New York, NY, USA, February 7-12, 2020*, pages 1838–1845. AAAI Press, 2020. `doi:10.1609/AAAI.V34I02.5551`.

11 Robert Bredereck, Piotr Faliszewski, Andrzej Kaczmarczyk, Rolf Niedermeier, Piotr Skowron, and Nimrod Talmon. Robustness among multiwinner voting rules. *Artif. Intell.*, 290:103403, 2021. `doi:10.1016/J.ARTINT.2020.103403`.

12 Markus Brill, Piotr Faliszewski, Frank Sommer, and Nimrod Talmon. Approximation algorithms for balancedcc multiwinner rules. In *AAMAS '19*, pages 494–502, 2019. URL: `http://dl.acm.org/citation.cfm?id=3331732`.

13 Laurent Bulteau, Gal Shahaf, Ehud Shapiro, and Nimrod Talmon. Aggregation over metric spaces: Proposing and voting in elections, budgeting, and legislation. *J. Artif. Intell. Res.*, 70:1413–1439, 2021. `doi:10.1613/JAIR.1.12388`.

14 Deeparnab Chakrabarty, Luc Côté, and Ankita Sarkar. Fault-tolerant k-supplier with outliers. In Olaf Beyersdorff, Mamadou Moustapha Kanté, Orna Kupferman, and Daniel Lokshtanov, editors, *41st International Symposium on Theoretical Aspects of Computer Science, STACS 2024, March 12-14, 2024, Clermont-Ferrand, France*, volume 289 of *LIPIcs*, pages 23:1–23:19. Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2024. `doi:10.4230/LIPICS.STACS.2024.23`.

15 Deeparnab Chakrabarty and Chaitanya Swamy. Interpolating between k-median and k-center: Approximation algorithms for ordered k-median. In Ioannis Chatzigiannakis, Christos Kaklamanis, Dániel Marx, and Donald Sannella, editors, *45th International Colloquium on Automata, Languages, and Programming, ICALP 2018, July 9-13, 2018, Prague, Czech Republic*, volume 107 of *LIPIcs*, pages 29:1–29:14. Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2018. `doi:10.4230/LIPICS.ICALP.2018.29`.

16 John R Chamberlin and Paul N Courant. Representative deliberations and representative decisions: Proportional representation and the borda rule. *American Political Science Review*, 77(3):718–733, 1983.

17 Hau Chan, Aris Filos-Ratsikas, Bo Li, Minming Li, and Chenhao Wang. Mechanism design for facility location problems: A survey. In *Proceedings of IJCAI*, pages 4356–4365, 2021. `doi:10.24963/IJCAI.2021/596`.

18 Moses Charikar, Kangning Wang, Prasanna Ramakrishnan, and Hongxun Wu. Breaking the metric voting distortion barrier. In David P. Woodruff, editor, *Proceedings of the 2024 ACM-SIAM Symposium on Discrete Algorithms, SODA 2024, Alexandria, VA, USA, January 7-10, 2024*, pages 1621–1640. SIAM, 2024. doi:10.1137/1.9781611977912.65.

19 Shiri Chechik and David Peleg. The fault-tolerant capacitated k-center problem. *Theor. Comput. Sci.*, 566:12–25, 2015. doi:10.1016/J.TCS.2014.11.017.

20 Jiehua Chen and Sven Grottke. Small one-dimensional euclidean preference profiles. *Soc. Choice Welf.*, 57(1):117–144, 2021. doi:10.1007/S00355-020-01301-Y.

21 Richard L Church and Zvi Drezner. Review of obnoxious facilities location problems. *Comput. Oper. Res.*, 138:105468, 2022. doi:10.1016/J.COR.2021.105468.

22 Vincent Cohen-Addad, Fabrizio Grandoni, Euiwoong Lee, and Chris Schwiegelshohn. Breaching the 2 LMP approximation barrier for facility location with applications to k-median. In *Proceedings of SODA*, pages 940–986, 2023. doi:10.1137/1.9781611977554.CH37.

23 Marek Cygan, Fedor V. Fomin, Lukasz Kowalik, Daniel Lokshtanov, Dániel Marx, Marcin Pilipczuk, Michal Pilipczuk, and Saket Saurabh. *Parameterized Algorithms.* Springer, 2015. doi:10.1007/978-3-319-21275-3.

24 Eva Michelle Deltl, Till Fluschnik, and Robert Bredereck. Algorithmics of egalitarian versus equitable sequences of committees. In *IJCAI 2023*, pages 2651–2658, 2023. doi:10.24963/IJCAI.2023/295.

25 Robert G. Downey and Michael R. Fellows. *Fundamentals of parameterized complexity*, volume 4. Springer, 2013.

26 Herbert Edelsbrunner and Ernst P. Mücke. Simulation of simplicity: a technique to cope with degenerate cases in geometric algorithms. *ACM Trans. Graph.*, 9(1):66–104, 1990. doi:10.1145/77635.77639.

27 Edith Elkind, Piotr Faliszewski, Jean-François Laslier, Piotr Skowron, Arkadii Slinko, and Nimrod Talmon. What do multiwinner voting rules do? an experiment over the two-dimensional euclidean domain. In Satinder Singh and Shaul Markovitch, editors, *Proceedings of the Thirty-First AAAI Conference on Artificial Intelligence, February 4-9, 2017, San Francisco, California, USA*, pages 494–501. AAAI Press, 2017. doi:10.1609/AAAI.V31I1.10612.

28 Piotr Faliszewski, Piotr Skowron, Arkadii Slinko, and Nimrod Talmon. Multiwinner rules on paths from k-borda to chamberlin-courant. In Carles Sierra, editor, *Proceedings of the Twenty-Sixth International Joint Conference on Artificial Intelligence, IJCAI 2017, Melbourne, Australia, August 19-25, 2017*, pages 192–198. ijcai.org, 2017. doi:10.24963/IJCAI.2017/28.

29 Piotr Faliszewski, Piotr Skowron, Arkadii Slinko, and Nimrod Talmon. Multiwinner voting: A new challenge for social choice theory. *Trends in computational social choice*, 74(2017):27–47, 2017.

30 Piotr Faliszewski, Piotr Skowron, Arkadii Slinko, and Nimrod Talmon. Multiwinner analogues of the plurality rule: axiomatic and algorithmic perspectives. *Soc. Choice Welf.*, 51(3):513–550, 2018. doi:10.1007/S00355-018-1126-4.

31 Piotr Faliszewski, Piotr Skowron, Arkadii Slinko, and Nimrod Talmon. Committee scoring rules: Axiomatic characterization and hierarchy. *ACM Trans. Economics and Comput.*, 7(1):3:1–3:39, 2019. doi:10.1145/3296672.

32 Piotr Faliszewski, Piotr Skowron, and Nimrod Talmon. Bribery as a measure of candidate success: Complexity results for approval-based multiwinner rules. In Kate Larson, Michael Winikoff, Sanmay Das, and Edmund H. Durfee, editors, *Proceedings of the 16th Conference on Autonomous Agents and MultiAgent Systems, AAMAS 2017, São Paulo, Brazil, May 8-12, 2017*, pages 6–14. ACM, 2017. URL: http://dl.acm.org/citation.cfm?id=3091133.

33 Jörg Flum and Martin Grohe. *Parameterized Complexity Theory.* Springer, 2006. doi:10.1007/3-540-29953-X.

34 Michal Tomasz Godziszewski, Pawel Batko, Piotr Skowron, and Piotr Faliszewski. An analysis of approval-based committee rules for 2d-euclidean elections. In *Proceedings of AAAI*, pages 5448–5455, 2021. doi:10.1609/AAAI.V35I6.16686.

35 Kishen N. Gowda, Thomas W. Pensyl, Aravind Srinivasan, and Khoa Trinh. Improved bi-point rounding algorithms and a golden barrier for k-median. In *Proceedings of SODA*, pages 987–1011, 2023. doi:10.1137/1.9781611977554.CH38.

36 Sushmita Gupta, Tanmay Inamdar, Pallavi Jain, Daniel Lokshtanov, Fahad Panolan, and Saket Saurabh. When far is better: The chamberlin-courant approach to obnoxious committee selection. *CoRR*, abs/2405.15372, 2024. doi:10.48550/arXiv.2405.15372.

37 Sushmita Gupta, Pallavi Jain, Saket Saurabh, and Nimrod Talmon. Even more effort towards improved bounds and fixed-parameter tractability for multiwinner rules. In *Proceedings of IJCAI*, pages 217–223, 2021. doi:10.24963/IJCAI.2021/31.

38 Aviram Imber, Jonas Israel, Markus Brill, Hadas Shachnai, and Benny Kimelfeld. Spatial voting with incomplete voter information. In *AAAI 2024*, pages 9790–9797, 2024. doi:10.1609/AAAI.V38I9.28838.

39 Tanmay Inamdar and Kasturi R. Varadarajan. Fault tolerant clustering with outliers. In Evripidis Bampis and Nicole Megow, editors, *Approximation and Online Algorithms - 17th International Workshop, WAOA 2019, Munich, Germany, September 12-13, 2019, Revised Selected Papers*, volume 11926 of *Lecture Notes in Computer Science*, pages 188–201. Springer, 2019. doi:10.1007/978-3-030-39479-0_13.

40 Sangram Kishor Jena, Ramesh K. Jallu, Gautam K. Das, and Subhas C. Nandy. The maximum distance-d independent set problem on unit disk graphs. In Jianer Chen and Pinyan Lu, editors, *Frontiers in Algorithmics - 12th International Workshop, FAW 2018, Guangzhou, China, May 8-10, 2018, Proceedings*, volume 10823 of *Lecture Notes in Computer Science*, pages 68–80. Springer, 2018. doi:10.1007/978-3-319-78455-7_6.

41 Yusuf Hakan Kalayci, David Kempe, and Vikram Kher. Proportional representation in metric spaces and low-distortion committee selection. In *AAAI 2024*, pages 9815–9823, 2024. doi:10.1609/AAAI.V38I9.28841.

42 Richard M. Karp. Reducibility among combinatorial problems. In Raymond E. Miller and James W. Thatcher, editors, *Proceedings of CCC*, pages 85–103, 1972. doi:10.1007/978-1-4684-2001-2_9.

43 Samir Khuller, Robert Pless, and Yoram J. Sussmann. Fault tolerant k-center problems. *Theor. Comput. Sci.*, 242(1-2):237–245, 2000. doi:10.1016/S0304-3975(98)00222-9.

44 Steven Klein. On the egalitarian value of electoral democracy. *Political Theory*, page 00905917231217133, 2023.

45 Nirman Kumar and Benjamin Raichel. Fault tolerant clustering revisited. In *Proceedings of the 25th Canadian Conference on Computational Geometry, CCCG 2013, Waterloo, Ontario, Canada, August 8-10, 2013*. Carleton University, Ottawa, Canada, 2013. URL: http://cccg.ca/proceedings/2013/papers/paper_36.pdf.

46 Martin Lackner and Piotr Skowron. *Multi-winner voting with approval preferences*. Springer Nature, 2023.

47 Alexander Lam, Haris Aziz, Bo Li, Fahimeh Ramezani, and Toby Walsh. Proportional fairness in obnoxious facility location. In Mehdi Dastani, Jaime Simão Sichman, Natasha Alechina, and Virginia Dignum, editors, *Proceedings of the 23rd International Conference on Autonomous Agents and Multiagent Systems, AAMAS 2024, Auckland, New Zealand, May 6-10, 2024*, pages 1075–1083. International Foundation for Autonomous Agents and Multiagent Systems / ACM, 2024. doi:10.5555/3635637.3662963.

48 Jean-François Laslier. Spatial approval voting. *Political Analysis*, 14(2):160–185, 2006.

49 Hong Liu and Jiong Guo. Parameterized complexity of winner determination in minimax committee elections. In Catholijn M. Jonker, Stacy Marsella, John Thangarajah, and Karl Tuyls, editors, *Proceedings of the 2016 International Conference on Autonomous Agents & Multiagent Systems, Singapore, May 9-13, 2016*, pages 341–349. ACM, 2016. URL: http://dl.acm.org/citation.cfm?id=2936975.

50 Jirí Matousek. *Lectures on discrete geometry*, volume 212 of *Graduate texts in mathematics*. Springer, 2002.

51 Ivan-Aleksandar Mavrov, Kamesh Munagala, and Yiheng Shen. Fair multiwinner elections with allocation constraints. In *Proceedings of EC*, pages 964–990, 2023. doi:10.1145/3580507.3597685.

52 Neeldhara Misra, Arshed Nabeel, and Harman Singh. On the parameterized complexity of minimax approval voting. In Gerhard Weiss, Pinar Yolum, Rafael H. Bordini, and Edith Elkind, editors, *Proceedings of the 2015 International Conference on Autonomous Agents and Multiagent Systems, AAMAS 2015, Istanbul, Turkey, May 4-8, 2015*, pages 97–105. ACM, 2015. URL: http://dl.acm.org/citation.cfm?id=2772895.

53 Burt L Monroe. Fully proportional representation. *American Political Science Review*, 89(4):925–940, 1995.

54 Kamesh Munagala, Yiheng Shen, Kangning Wang, and Zhiyi Wang. Approximate core for committee selection via multilinear extension and market clearing. In *Proceedings of SODA*, pages 2229–2252, 2022. doi:10.1137/1.9781611977073.89.

55 Kamesh Munagala, Zeyu Shen, and Kangning Wang. Optimal algorithms for multiwinner elections and the chamberlin-courant rule. In *EC '21*, pages 697–717. ACM, 2021. doi:10.1145/3465456.3467624.

56 Rolf Niedermeier. *Invitation to Fixed-Parameter Algorithms*. Oxford University Press, 2006. doi:10.1093/ACPROF:OSO/9780198566076.001.0001.

57 Sridhar Rajagopalan and Vijay V. Vazirani. Primal-dual RNC approximation algorithms for set cover and covering integer programs. *SIAM J. Comput.*, 28(2):525–540, 1998. doi:10.1137/S0097539793260763.

58 John Rawls. *A Theory of Justice: Original Edition*. Harvard University Press, 1971. URL: http://www.jstor.org/stable/j.ctvjf9z6v.

59 John Rawls. *Justice as Fairness: Political Not Metaphysical*, pages 145–173. Palgrave Macmillan UK, London, 1991. doi:10.1007/978-1-349-21763-2_10.

60 Piotr Skowron. FPT approximation schemes for maximizing submodular functions. *Inf. Comput.*, 257:65–78, 2017. doi:10.1016/J.IC.2017.10.002.

61 Piotr Skowron and Piotr Faliszewski. Chamberlin-courant rule with approval ballots: Approximating the maxcover problem with bounded frequencies in FPT time. *J. Artif. Intell. Res.*, 60:687–716, 2017. doi:10.1613/JAIR.5628.

62 Piotr Skowron, Piotr Faliszewski, and Jérôme Lang. Finding a collective set of items: From proportional multirepresentation to group recommendation. *Artif. Intell.*, 241:191–216, 2016. doi:10.1016/J.ARTINT.2016.09.003.

63 Piotr Krzysztof Skowron and Edith Elkind. Social choice under metric preferences: Scoring rules and STV. In *Proceedings of AAAI*, pages 706–712. AAAI Press, 2017. doi:10.1609/AAAI.V31I1.10591.

64 Chinmay Sonar, Subhash Suri, and Jie Xue. Multiwinner elections under minimax chamberlin-courant rule in euclidean space. In *Proceedings of IJCAI*, pages 475–481, 2022. doi:10.24963/IJCAI.2022/68.

65 Chinmay Sonar, Subhash Suri, and Jie Xue. Fault tolerance in euclidean committee selection. In *ESA 2023*, volume 274, pages 95:1–95:14, 2023. doi:10.4230/LIPICS.ESA.2023.95.

66 Gogulapati Sreedurga, Mayank Ratan Bhardwaj, and Yadati Narahari. Maxmin participatory budgeting. In *IJCAI 2022*, pages 489–495, 2022. doi:10.24963/IJCAI.2022/70.

67 Arie Tamir. Obnoxious facility location on graphs. *SIAM J. Discret. Math.*, 4(4):550–567, 1991. doi:10.1137/0404048.

68 Thorvald N. Thiele. Om flerfoldsvalg. *Oversigt over det Kongelige Danske Videnskabernes Selskabs Forhandlinger*, pages 415–441, 1895.

69 Yongjie Yang. On the complexity of calculating approval-based winners in candidates-embedded metrics. In *Proceedings of the Thirty-First International Joint Conference on Artificial Intelligence, IJCAI-22*, pages 585–591, 2022. doi:10.24963/IJCAI.2022/83.

70 Yongjie Yang and Jianxin Wang. Parameterized complexity of multi-winner determination: More effort towards fixed-parameter tractability. In Elisabeth André, Sven Koenig, Mehdi Dastani, and Gita Sukthankar, editors, *Proceedings of the 17th International Conference on Autonomous Agents and MultiAgent Systems, AAMAS 2018, Stockholm, Sweden, July 10-15, 2018*, pages 2142–2144. International Foundation for Autonomous Agents and Multiagent Systems Richland, SC, USA / ACM, 2018. URL: http://dl.acm.org/citation.cfm?id=3238099.

71 Vikram Kher Yusuf Hakan Kalayci, David Kempe. Proportional representation in metric spaces and low-distortion committee selection. In *Proceedings of AAAI*, 2024.

72 Aizhong Zhou, Yongjie Yang, and Jiong Guo. Parameterized complexity of committee elections with dichotomous and trichotomous votes. In Edith Elkind, Manuela Veloso, Noa Agmon, and Matthew E. Taylor, editors, *Proceedings of the 18th International Conference on Autonomous Agents and MultiAgent Systems, AAMAS '19, Montreal, QC, Canada, May 13-17, 2019*, pages 503–510. International Foundation for Autonomous Agents and Multiagent Systems, 2019. URL: http://dl.acm.org/citation.cfm?id=3331733.