The Local Hamiltonian Problem for Quasi-Quantum States: A Toy Model for the Quantum PCP Conjecture

Itai Arad
Centre for Quantum Technologies, Singapore

Miklos Santha
Centre for Quantum Technologies, Singapore

Abstract

In this work we define a new classical constraint satisfaction problem that shares many of the properties of the quantum local Hamiltonian problem, distinguishing it from the usual classical k-SAT problem. The problem consists of minimizing the number of violated local constraints over a restricted set of *distributions of assignments*. We show that these distributions can be 1-to-1 mapped to a superset of the quantum states, which we call *k-local quasi-quantum states*. Nevertheless, we claim that our optimization problem is essentially classical, by proving that it is an NP-complete problem. Interestingly, the optimal distribution shares many of the properties of quantum states. In particular, it is not determined straightforwardly by its local marginals, and consequently, it can be used as a classical toy model to study several aspects of Hamiltonian complexity that are different from their classical counter parts. These include the complexity of 1D systems (which is in P for classical CSPs, but is QMA-hard for quantum systems), and the lack of an easy search-to-decision reduction. Finally, we believe that our model can be used to gain insights into the quantum PCP conjecture. Indeed, while we have shown that approximating the minimal number of unsatisfiable constraints to within an $\Theta(1)$ is NP-hard, it is not clear if the problem remains hard if we want to approximate the minimal *fraction* of unsatisfiable constraints to within an $\Theta(1)$; as in the quantum PCP conjecture, naive quantization of the classical proofs does not seem to work.

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