

# Connected Partitions via Connected Dominating Sets

Aikaterini Niklanovits 


Hasso Plattner Institute, University of Potsdam, Germany

Kirill Simonov 

Department of Informatics, University of Bergen, Norway

Shaily Verma 

Hasso Plattner Institute, University of Potsdam, Germany

Ziena Zeif 

Hasso Plattner Institute, University of Potsdam, Germany

---

## Abstract

The classical theorem due to Győri and Lovász states that any  $k$ -connected graph  $G$  admits a partition into  $k$  connected subgraphs, where each subgraph has a prescribed size and contains a prescribed vertex, as long as the total size of target subgraphs is equal to the size of  $G$ . However, this result is notoriously evasive in terms of efficient constructions, and it is still unknown whether such a partition can be computed in polynomial time, even for  $k = 5$ .

We make progress towards an efficient constructive version of the Győri–Lovász theorem by considering a natural strengthening of the  $k$ -connectivity requirement. Specifically, we show that the desired connected partition can be found in polynomial time, if  $G$  contains  $k$  disjoint connected dominating sets. As a consequence of this result, we give several efficient approximate and exact constructive versions of the original Győri–Lovász theorem:

- On general graphs, a Győri–Lovász partition with  $k$  parts can be computed in polynomial time when the input graph has connectivity  $\Omega(k \cdot \log^2 n)$ ;
- On convex bipartite graphs, connectivity of  $4k$  is sufficient;
- On biconvex graphs and interval graphs, connectivity of  $k$  is sufficient, meaning that our algorithm gives a “true” constructive version of the theorem on these graph classes.

**2012 ACM Subject Classification** Theory of computation → Graph algorithms analysis

**Keywords and phrases** Győri–Lovász theorem, connected dominating sets, graph classes

**Digital Object Identifier** 10.4230/LIPIcs.ESA.2025.10

**Related Version** *Full Version:* <https://arxiv.org/abs/2503.13112> [33]

## 1 Introduction

Partitioning a graph into connected subgraphs is a problem that naturally arises in various application areas such as image processing, road network decomposition and parallel graph systems for supporting computations on large-scale graphs [30, 31, 18, 21]. Usually in such applications, there is a certain number of parts  $k$  that the partition should have, as well as certain constraints on the size of each part. In particular, a prominent objective is for the size of the parts to be as equal as possible and, therefore, for the partition to be as balanced as possible, known as Balanced Connected Partition problem ( $BCP_k$ ). This problem is proven to be NP-complete [19] and even hard to approximate [14] and, hence, it has been studied in restricted graph classes such as grid graphs for which it was again proven to be NP-hard [3].

From a theoretical point of view, partitioning a graph into connected components each of a certain, chosen in advance, size is of independent interest as it gives us an insight into the structure of the graph. With the simple example of a star graph, one can observe that it



© Aikaterini Niklanovits, Kirill Simonov, Shaily Verma, and Ziena Zeif;  
licensed under Creative Commons License CC-BY 4.0

33rd Annual European Symposium on Algorithms (ESA 2025).

Editors: Anne Benoit, Haim Kaplan, Sebastian Wild, and Grzegorz Herman; Article No. 10; pp. 10:1–10:14

Leibniz International Proceedings in Informatics



LIPICs Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

is not possible to partition it into connected sets where more than one of them has size 2 or more. In general, if there exists a set of  $t$  vertices whose removal disconnects the graph (called a *separator*), and if the number of parts  $k$  is greater than  $t$ , then the  $k$  parts of a connected partition might not necessarily be of arbitrary size. Graphs that do not contain a separator of size strictly less than  $t$  are called  $t$ -connected.

The essence of this observation was captured in 1976 by Győri [22] and Lovász [29] who independently proved that the  $k$ -connectivity of a graph is a necessary and sufficient condition for any partition of a specific size to be possible, even when each part is additionally required to contain a specified vertex called a *terminal*. This was later generalized to weighted graphs by Chandran et al. [12], Chen et al. [13] and Hoyer [24]. Formally, we define the following computational problem.

► **Definition 1.** *In the GL-PARTITION problem, the input is a graph  $G = (V, E)$ , an integer  $k \geq 1$ , a set of  $k$  distinct vertices  $C = \{c_1, \dots, c_k\}$  of  $G$ , and  $k$  integers  $N = \{n_1, \dots, n_k\}$  such that  $\sum_{i=1}^k n_i = |V|$ , in short denoted as  $(G, k, C, N)$ . The task is to compute a partition of  $V$  into  $k$  disjoint subsets  $V_1, \dots, V_k$  such that  $|V_i| = n_i$ ,  $c_i \in V_i$ , and the induced subgraph  $G[V_i]$  on  $V_i$  is connected, for all  $i \in [k]$ .*

We call the partition satisfying the property above a *Győri–Lovász partition*, or a *GL-partition* for short. The result of Győri and Lovász can then be stated as follows.

► **Theorem (Győri–Lovász Theorem).** *Given an instance of GL-PARTITION where the graph is  $k$ -connected, a GL-partition always exists.*

Although the proof of Győri [22] is constructive, the resulting algorithm runs in exponential time. To this day, it remains open whether finding a GL-partition of a given  $k$ -connected graph could be done in polynomial time in general. On the other hand, a few polynomial algorithms have been developed for special cases. The original research direction is concerned with smaller values of  $k$ . Specifically, in 1990, Suzuki et al. [37, 38] provided a polynomial algorithm for the cases  $k = 2$  and  $k = 3$ . Later, in 1993, Wada et al. [39] extended the result for  $k = 3$ , and, in 1997, Nakano et al. [32] showed a linear-time algorithm for the case where  $k = 4$  and the graph is planar with all the terminals located on the same face of the planar embedding. More recently, in 2016, Hoyer and Thomas [24] presented a polynomial-time algorithm for the case  $k = 4$  without additional restrictions on the graph structure. It is still open whether a polynomial-time algorithm exists for any fixed value of  $k$  starting from 5. Another research direction focuses on polynomial-time constructions for the Győri–Lovász theorem on specific graph classes, for all values of  $k$ . Recently, Casel et al. [9] developed a polynomial-time algorithm for chordal graphs and (with a small additive violation of the size constraint) for  $\text{HHI}_4^2$ -free graphs. Additionally, Borndörfer et al. [6] have shown an approximate version of the Győri–Lovász theorem on general graphs, where the size of each part is within a factor of 3 from the given target.

In the light of the challenges above, it is natural to ask whether efficient constructions exist for more restrictive versions of the Győri–Lovász theorem. Censor-Hillel, Ghaffari, and Kuhn [11] pioneered the notion of a connected dominating set (CDS) partition as a proxy for vertex connectivity of the graph. Formally, a graph admits a CDS partition of size  $k$ , if there exists a partition of the vertices into  $k$  sets, such that each set forms a dominating set of the graph, and induces a connected subgraph. It is easy to see that any graph that admits a CDS partition of size  $k$ , is also  $k$ -connected. The converse, unfortunately, does not hold; however, Censor-Hillel, Ghaffari, and Kuhn show that there is only a polylogarithmic gap between the connectivity and the size of the maximum CDS partition. Moreover, a suitable CDS partition can be efficiently computed. The bound was later improved by Censor-Hillel et al. [10].

► **Theorem 2** (Corollary 1.6, [10]). *Every  $k$ -connected  $n$ -vertex graph has a CDS partition of size  $\Omega(\frac{k}{\log^2 n})$ , and such a partition can be computed in polynomial time.*

On the other hand, Censor-Hillel, Ghaffari, and Kuhn [11] show that a logarithmic gap between the connectivity and the size of a CDS partition is necessary.

► **Theorem 3** (Theorem 1.3, [11]). *For any sufficiently large  $n$ , and any  $k \geq 1$ , there exist  $k$ -connected  $n$ -vertex graphs, where the maximum CDS partition size is  $O(\frac{k}{\log n})$ .*

More recently, Draganić and Krivelevich [17] obtained a tighter bound of  $(1 + o(1))d / \ln d$  for CDS partitions in  $d$ -regular pseudorandom graphs.

### Our contribution

The core result of our work is an efficient version of the Győri–Lovász theorem that is based on the existence of a CDS partition of size  $k$  in the given graph, which strengthens the original requirement of  $k$ -connectivity.

► **Theorem 4.** *There is an algorithm that, given an instance of GL-PARTITION together with a CDS partition of the graph of size  $k$ , computes a GL-partition in polynomial time.*

However, on its own Theorem 4 would have a notable limitation, that computing an adequate CDS partition may in itself be an obstacle – as opposed to connectivity, which can always be computed exactly in polynomial time. Fortunately, it turns out that a sufficiently large CDS partition can be found efficiently in a range of scenarios. As the first example, Theorem 2 due to Censor-Hillel et al. [10] provides such a construction for general graphs. Combining it with our algorithm, we obtain the following efficient version of the Győri–Lovász theorem, which requires higher connectivity, but puts no additional restrictions on the graph.

► **Corollary 5.** *GL-PARTITION is poly-time solvable on  $\Omega(k \cdot \log^2 n)$ -connected graphs.*

Furthermore, we investigate the cases where Theorem 4 can be applied to obtain a tighter connectivity gap. Following the results of Censor-Hillel, Ghaffari, and Kuhn [11], the case of general graphs cannot be improved beyond a logarithmic factor, as there exist  $\Omega(k \cdot \log n)$ -connected graphs that do not admit CDS partitions of size  $k$ , see Theorem 3. Therefore, putting additional restrictions on the input graph is necessary.

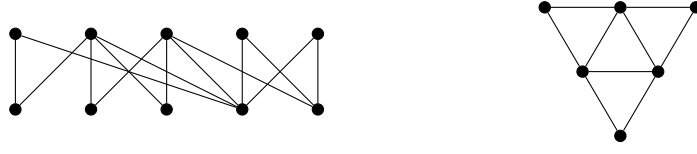
The first natural target here would be efficient constructions for the Győri–Lovász theorem on certain graph classes in its exact version, that is, requiring connectivity exactly  $k$ . In the light of Theorem 4, it is sufficient to show a statement of the following form, that any  $k$ -connected graph in the class  $\mathcal{G}$  admits a CDS partition of size  $k$ . This would imply that the Győri–Lovász theorem admits an efficient construction on graphs in  $\mathcal{G}$ . We identify two such cases, biconvex graphs and interval graphs.

► **Theorem 6.** *Let  $G$  be a  $k$ -connected biconvex graph. A CDS partition of  $G$  of size  $k$  exists, and can be computed in polynomial time.*

► **Theorem 7.** *Let  $G$  be a  $k$ -connected interval graph. A CDS partition of  $G$  of size  $k$  exists, and can be computed in polynomial time.*

Coupled with the algorithm of Theorem 4, this gives an efficient construction for the Győri–Lovász theorem on biconvex graphs and interval graphs.

► **Corollary 8.** *GL-PARTITION is poly-time solvable on  $k$ -connected biconvex graphs.*



■ **Figure 1** Examples of a 2-connected convex bipartite graph and a 2-connected chordal graph that do not admit a CDS partition of size 2.

We note that the efficient construction for chordal graphs was identified by Casel et al. [9], implying the same result for interval graphs, since interval graphs form a subclass of chordal graphs. Nonetheless, we state the algorithmic consequence of Theorem 7 next for completeness.

► **Corollary 9.** *GL-PARTITION is poly-time solvable on  $k$ -connected interval graphs.*

Unfortunately, there is a limit to the results of the form above, as already graph classes such as chordal graphs and convex bipartite graphs do not possess the property that  $k$ -connectivity implies the existence of a CDS partition of size  $k$  (see Figure 1 for examples). On the positive side, we are able to retain the productivity of Theorem 4 even in such cases. We focus on the class of convex bipartite graphs and show that  $4k$ -connected convex bipartite graphs always admit a CDS partition of size  $k$ .

► **Theorem 10.** *Let  $G$  be a  $4k$ -connected convex bipartite graph. A CDS partition of  $G$  of size  $k$  exists, and can be computed in polynomial time.*

This allows us to explore a trade-off between efficiency of the construction and the connectivity requirement for the class of convex bipartite graphs. As before, combined with the algorithm of Theorem 4, Theorem 10 gives us a “4-approximate” (in terms of connectivity) efficient version of the Győri–Lovász theorem on this graph class, considerably improving the connectivity requirement of the general case.

► **Corollary 11.** *GL-PARTITION is solvable in polynomial time on  $4k$ -connected convex bipartite graphs.*

## Related work

The problem of computing the largest CDS partition of a graph was first introduced by Hedetniemi and Laskar [23], where the maximum size of a CDS partition is referred to as the connected domatic number of the graph. A number of bounds on the size of a CDS partition are due to Zelinka [41], including the fact that it is upper-bounded by the connectivity of the graph. Connected dominating sets have many practical applications for tasks such as routing and load-balancing in networks, for a detailed overview we refer to a survey due to Yu et al. [40]. Connections between CDS partitions and graph rigidity were explored by Krivelevich, Lew and Michaeli [26].

Chordal graphs are graphs that do not admit induced cycles of length more than 3. The class of chordal graphs is well-studied, and is known to possess many useful properties; in particular, many problems that are NP-hard on general graphs turn out to be polynomial-time solvable on chordal graphs, such as computing chromatic number, independence number, treewidth [34]. Interval graphs are intersection graphs of intervals on a real line, and form a subclass of chordal graphs [27].

Convex bipartite graphs<sup>1</sup> were introduced in 1967 by Glover [20] as they naturally arise in various industrial and scheduling applications. Formally, a bipartite graph  $G = (A \cup B, E)$  is called *convex* if there exists an ordering  $\sigma$  of the vertices in  $A$  such that the neighborhood of any vertex of  $B$  is consecutive under  $\sigma$ . Restricting to the class of convex bipartite graphs has been proven fruitful for many problems such as maximum independent set, minimum feedback set, (dynamic) maximum matching and independent dominating set [35, 8, 28, 25, 36, 15]. On the other hand, several natural problems are known to be NP-complete on convex bipartite graphs. Most relevant to our work is the example of the  $k$ -path partition problem, proven to be NP-complete on convex bipartite graphs by Asdre and Nikolopoulos [2]. Belmonte and Vatshelle [4] show that convex bipartite graphs have bounded mim-width, which provides a unified perspective on polynomial-time algorithms for various classes. Bonomo-Braberman et al. [5] generalize this perspective further to extended notions of convexity.

Bipartite graphs with an ordering satisfying the convexity condition for both parts, are called biconvex. Biconvex graphs are thus a subclass of convex bipartite graphs. Abbas and Stewart studied properties of vertex orderings in biconvex graphs, yielding in particular that the vertex ranking problem is polynomial-time solvable on biconvex graphs [1]. Apart from their structural and algorithmic interest, biconvex graphs and convex bipartite graphs have been proven useful in practice, as they model well situations such as task allocation problems [20, 25]. In terms of other well-studied graph classes, convex bipartite graphs and biconvex graphs form a subclass of chordal bipartite graphs, which have a natural analogue of the chordal property among the bipartite graphs. For a more detailed introduction to the graph classes, their properties and relations between the classes, we refer to a survey by Brandstädt, Bang Le and Spinrad [7].

**Paper organization.** In Section 2, we introduce preliminaries that we use throughout the paper. Section 3 is dedicated to the main case of the algorithm constructing a Györi–Lovász partition on the basis of a CDS partition, and Section 4 completes the algorithm. We conclude with an overview and selected open problems in Section 5. Due to space restrictions, the results for biconvex, interval, and convex bipartite graphs, as well as details from Section 4 and selected proofs from Section 3 are omitted and we refer the reader to the full version of the paper [33]. Such results are marked by ( $\star$ ).

## 2 Preliminaries

All graphs mentioned in this paper are finite, undirected and simple. We use standard graph theory notation, for reference see the full version of the paper [33] and the book of Diestel [16]. We call a graph  $G = (V, E)$  a *path* if  $V = \{v_1, \dots, v_n\}$  and  $E = \{v_i v_{i+1} | \text{for } i \in [n-1]\}$ . We usually denote a path by  $P$  and we call its order, the *length* of  $P$ . We call a graph  $G = (V, E)$  a *cycle* if  $V = \{v_1, \dots, v_n\}$  and  $E = \{v_i v_{i+1}, v_1 v_n | \text{for } i \in [n-1]\}$ . We call a graph  $G = (V, E)$  a *tree* if it does not contain any cycle as a subgraph. Throughout this paper a vertex set of a tree is usually denoted by  $T$ , while its edge set is denoted by  $E(T)$ . Given two graphs  $H, G$  we say that  $G$  is *H-free* if  $G$  does not contain  $H$  as an induced subgraph. Given two graphs  $G$  and  $T$ , we say that  $T$  is a *spanning tree* of  $G$  if  $V(G) = V(T)$  and  $G[V(T)]$  induces a tree.

Given two vertex sets  $A, B \in V(G)$  we say that  $A$  is *dominating* to  $B$  if every vertex in  $B$  is adjacent to some vertex in  $A$ . Also, when we say that a vertex set is dominating to some graph  $G$ , we mean that it is dominating to its vertex set  $V(G)$ . A vertex set  $A$  is called

<sup>1</sup> Often referred to simply as *convex graphs* in the literature.

a *connected dominating set* (CDS) if and only if it is a dominating set and  $G[A]$  is connected. Given a graph  $G$ , a collection of vertex sets  $\mathcal{V} = \{V_1, \dots, V_l\}$  such that  $V_i \subseteq V(G)$  is called a *connected dominating set partition* (CDS partition) if and only if the sets in  $\mathcal{V}$  are pairwise disjoint and  $\cup_{U \in \mathcal{V}} U = V(G)$ , and each of them is a connected dominating set of  $G$ . We call  $|\mathcal{V}|$  the *size* of the CDS partition  $\mathcal{V}$ . Throughout the paper, we also use an equivalent notion of vertex-disjoint dominating trees. Given a CDS partition of a graph it is immediate to get a set of vertex-disjoint dominating trees of the same size by considering the spanning tree of each set in the CDS partition. Moreover, given  $k$  disjoint dominating trees, one may obtain a CDS partition of size  $k$  by simply adding the remaining vertices arbitrarily to those trees.

### 3 From Simple CDS Partition to GL-partition

Before we move on to the general algorithm of Theorem 4 that computes a GL-partition from a CDS partition of size  $k$ , we cover in this section the main subroutine of this algorithm. Specifically, we present an algorithm (Algorithm 3) that, given as input an instance of GL-PARTITION and  $k$  dominating trees such that all terminals are contained in the same dominating tree, returns a partial GL-partition with  $l$  sets, for some  $l \in [k]$ , such that the remaining vertices admit a CDS partition of size  $k - l$ . The general algorithm, described in the next section, then uses this result as a subprocedure. We first describe the intuition behind the main steps of Algorithm 3, then provide a more detailed description of these steps, and finally define a formal procedure for each step and show its correctness.

**Intuition of Algorithm 3.** Let  $T_1$  be the dominating tree that contains  $k$  terminal vertices. First we add all the vertices of  $T_1$  to one of the  $k$  vertex sets while maintaining the connectivity of each set. There exist  $k - 1$  dominating trees that have no terminal vertex. Now, we map these  $k$  vertex sets to one of these  $k - 1$  dominating trees. We aim to fulfill the vertex set to its demand while using the corresponding dominating tree.

We use an iterative procedure that first assigns (but does not add) neighbors for each vertex set to these  $k - 1$  dominating trees, and then we add assigned vertices to each vertex set whose demand is not met by the assignment. Now, to fulfill the demand of each such vertex set, we use its corresponding dominating tree while preserving the connectivity of the vertex set. At the end of this procedure, each part is either full or contains a whole dominating tree (apart from the one that was not assigned to any tree). The remaining vertices are arbitrarily added to the non-full sets, as each contains a dominating tree, apart from the one that has not been assigned a tree, for which we have ensured that there are enough adjacent vertices left.

**Description of Algorithm 3.** The algorithm receives an instance of GL-PARTITION,  $(G, k, C, N)$ , where  $C = \{c_1, \dots, c_k\}$ ,  $N = \{n_1, \dots, n_k\}$  and a set of dominating trees  $\mathcal{T} = \{T_1, \dots, T_k\}$  as an input such that  $G$  is  $k$ -connected graph and  $C \subseteq T_1$ .

It is important to note that during this description of the algorithm, every time we add a vertex to a vertex set, it is implied that we also check whether the condition of having  $l$  full sets say  $V_1, \dots, V_l$ , intersecting exactly  $l$  of the dominating trees, say  $T_1, \dots, T_l$  is met. If yes, then we output those  $l$  full sets and return an instance of the original problem,  $G - \cup_{i=1}^l V_i$  and  $k - l$ ,  $\mathcal{T} = \mathcal{T} \setminus \{T_{k-l}, \dots, T_k\}$ ,  $C = C \setminus \{c_{k-l}, \dots, c_k\}$  and  $N = N \setminus \{n_{k-l}, \dots, n_k\}$ . We moreover assume that every time we add a vertex we also check if a set meets its size demand and if so we add it to the set **Full** that includes all the connected vertex sets that satisfy their size demands. We denote this procedure of adding a vertex  $u$  to set  $V$  by  $\text{Add}(V, u)$ .



(similarly  $\text{Add}(V, U)$  when we have a vertex set  $U$  instead of singleton  $\{u\}$ ). Similarly every time we remove a vertex from some set we check if it is in **Full** and if yes, we remove the set from **Full**. We denote this procedure of removing vertex  $u$  from the set  $V$  by  $\text{Remove}(V, u)$ .

**Step 1** We initialize each set  $V_i$  to contain the corresponding terminal vertex  $c_i$  for all  $i \in [k]$  and then add the vertices of  $T_1 \setminus C$  one by one to a vertex set that it is adjacent to (based only on the adjacencies occurring by  $T_1$ ). Next, we add the vertices of  $V(G) \setminus V(\mathcal{T})$  to an adjacent set one by one.

**Step 2** Each vertex of  $\mathcal{T} \setminus T_1$  is assigned through a function **Vlabel** to an adjacent vertex set. Depending on whether the amount of vertices assigned to some vertex set is less than the vertices needed to be added for the set to satisfy its demand or not, the set is categorized in the set **Under** or **Over** respectively (Algorithm 1).

**Step 3** Through the function **Tlabel** each set in **Under** is assigned one of the trees in  $\mathcal{T} \setminus T_1$ . Then, one vertex of each such tree adjacent to the corresponding set is added. Priority is given to the vertices assigned to this set through the function **Vlabel**. However, if no such vertex exists, another adjacent vertex is arbitrarily chosen to be added, and we change the value of this vertex in the function **Vlabel** accordingly. If this results in a set moving from **Over** to **Under**, we again assign a tree to this set (that has not been assigned to any other set) through the function **Tlabel** (Algorithm 1).

**Step 4** For each tree assigned to some set, we consider its vertices not belonging in that set one by one and add them to that set as long as the set connectivity is preserved. Again, if, during this procedure, a set moves from **Over** to **Under**, go to the previous Step. Repeat this procedure until each set in **Under** is either full or contains a whole tree (Algorithm 2).

**Step 5** Add each of the remaining vertices, the ones not added to any set, to an adjacent non full set one by one (Algorithm 2).

**Step 6** Output the  $k$  sets created.

As Step 1 works in a straightforward way, we do not provide a pseudocode for it. We describe in the proof of the lemma below how this can be done in polynomial time. In the main algorithm, we refer to this procedure as  $\text{AddTrees}(G, C, \mathcal{T})$  that returns the sets  $V_1, \dots, V_k$ .

► **Lemma 12** ( $\star$ ). *Given an instance of GL-PARTITION  $(G, k, C, N)$  and  $k$  vertex disjoint dominating trees  $T_1, \dots, T_k$  such that  $C \subseteq T_1$ , we construct  $k$  vertex connected sets  $V_1, \dots, V_k$  such that  $c_i \in V_i$  for all  $i \in [k]$  and  $T_1 \subseteq \cup_{i=1}^k V_i$ , in polynomial time.*

Algorithm 1 realizes the two next steps of Algorithm 3. In particular, first each remaining vertex of the graph is assigned to some set through **Vlabel** and then each set is categorized in **Over** or **Under** depending on the number of vertices assigned to it. Then, for each set in **Under** the assigned vertices are added and through **Tlabel** we assign a distinct tree each such set. Afterwards we ensure that each set in **Under** contains a vertex from its assigned tree.

It is easy to notice that Step 2 runs in polynomial time. In order to prove the correctness of Step 3 in Lemma 13, it suffices to show that there are at most  $k - 1$  sets in **Under** and this property is not affected while ensuring that each vertex set contains a vertex from the corresponding through **Tlabel** tree. This part of the algorithm also runs in polynomial time because each set in **Under** is considered at most once and a set from **Over** might move to **Under** but once a set is in **Under** it will never move to **Over**.

► **Lemma 13**. *Let  $(G, k, C, N)$  be an instance of GL-PARTITION, where  $C = \{c_1, \dots, c_k\}$ ,  $\{V_1, \dots, V_k\}$  are  $k$  connected vertex sets such that  $c_i \in V_i$  for all  $i \in [k]$ , and a set of  $k$  vertex disjoint dominating trees  $\mathcal{T} = \{T_1, \dots, T_k\}$  such that  $C \subseteq T_1$ , are given as input to Algorithm 1. Then, after the loop in line 17 (Step 3) each vertex set is either in **Over**, or is assigned to a tree through **Tlabel** and contains a vertex from that tree. Moreover,  $G[V_i \cup \text{Vlabel}(V_i)]$  is connected for each  $i \in [k]$  and at least one set is in **Over**.*

**Proof.** First notice that  $G[V_i \cup \text{Vlabel}(V_i)]$  is connected for all  $i \in [k]$ , by the definition of the function  $\text{Vlabel}$ . We now show that there is at least one vertex set in **Over**. Assume that there is no set in **Over** and hence  $n_i - |V_i| > |\text{Vlabel}(V_i)|$  for all  $i \in [k]$ . Then  $\sum_{i=1}^k n_i - |V_i| > \sum_{i=1}^k |\text{Vlabel}(V_i)| \Rightarrow n > \sum_{i=1}^k |\text{Vlabel}(V_i)| + \sum_{i=1}^k |V_i| = n$  which is a contradiction. This implies that regardless of whether a set moves from **Over** to **Under**, as long as all vertices of  $G$  are assigned to the sets  $V_1, \dots, V_k$ , one of these sets will be in **Over**.

In the loop in line 17, we consider each tree assigned to some vertex set and first check whether this vertex set already contains some vertex of the tree. If yes, our lemma is satisfied. Otherwise, since the tree is dominating, there exists at least one vertex adjacent to the part of the vertex set intersecting  $T_1$ . This vertex is already added to some vertex set in **Under** or assigned to one in **Over**. If it belongs in some set in **Over**, then we simply add it to the vertex set. Then we remove this vertex from the set that occurs from the assignment  $\text{Vlabel}$  for the set previously in **Over**, and we check to see if it now meets the condition for this set belonging to **Under**. If yes, we go to line 12 to deal with this set similarly. Repeating this procedure for the other sets in **Under** does not affect them as every vertex set currently in **Under** already contains all the vertices assigned to it through the function  $\text{Vlabel}$ . Moreover, since in this step, we add a vertex to a vertex set, we also check if this set becomes full and intersects exactly one dominating tree. If that is true, we restart the algorithm for the adjusted values of  $k$ ,  $G$ ,  $\mathcal{T}$ , and  $\mathcal{C}$ .

Otherwise, if the adjacent vertex is part of some other set in **Under**, we add it to our current set and remove it from the previous one (again checking if the condition for restarting the algorithm is met). Moreover, we remove it from the set created through the function  $\text{Vlabel}$  to avoid this vertex from existing in two sets (in case we go to line 12) in some other iteration of this loop. Note that sets initially in **Under** might be added in **Full** or removed from it depending on whether we add or remove vertices from them, but they will never go from **Under** to **Over** as we need to maintain the property that each set in **Over** contains no vertices from  $T_2, \dots, T_k$ . ◀

Algorithm 2 realizes essentially the last two steps of the algorithm and either returns the desired partition of size  $k$  or a smaller instance of a more general problem. First we ensure that each set in **Under** is either full or contains the dominating tree assigned to it through  $\text{Tlabel}$ . This is done by considering each set in **Under** separately and then adding the neighboring vertices of the corresponding tree one by one. If during this procedure we “steal” a vertex assigned in some set in **Over** and this causes it to move to **Under** we run Algorithm 1 starting from line 13 (only running the loop of line 12 once, for the set that moved to **Under**). This happens at most once for each set hence this part of Algorithm 2 runs in polynomial time. In the last part of this algorithm we simply add the remaining vertices to some adjacent non full set, first based on their assignment through  $\text{Vlabel}$  and then based on the adjacencies in  $G$ .

► **Lemma 14.** *Let an instance of GL-PARTITION  $(G, k, C = \{c_1, \dots, c_k\}, N)$ ,  $k$  connected vertex sets  $\{V_1, \dots, V_k\}$  such that  $|V_i| < n_i$  and  $c_i \in V_i$  for all  $i \in [k]$ ,  $k$  dominating to  $G$  trees  $\{T_1, \dots, T_k\}$  and a function  $\text{Vlabel} : \{V_1, \dots, V_k\} \rightarrow P(V(G))$ , are given as an input to Algorithm 2. Then, after the loop in line 1 (Step 5), each vertex set is connected and is either full or completely contains a dominating tree or is assigned through  $\text{Vlabel}$  to at least as many vertices as its remaining size demand that do not belong in any other vertex set.*

**Proof.** In order to see that the loop in line 1 terminates, it suffices to notice that it only considers sets in **Under** and the value of  $\sum_{i=2}^k |T'_i \cap \text{Tlabel}^{-1}(T_1)|$ .



■ **Algorithm 1** Labeling (Step 2, Step 3).

---

**Require:** An instance of GL-PARTITION  $(G, k, C = \{c_1, \dots, c_k\}, N = \{n_1, \dots, n_k\})$ , a set of  $k$  vertex disjoint dominating trees  $\mathcal{T} = \{T_1, \dots, T_k\}$  such that  $C \subseteq T_1$ , and  $k$  connected vertex sets  $\{V_1, \dots, V_k\}$  such that  $c_i \in V_i$  for all  $i \in [k]$

- 1: Initialize  $\text{Vlabel} : \{V_1, \dots, V_k\} \rightarrow P(V(G))$  and  $\text{Tlabel} : \{V_1, \dots, V_k\} \rightarrow \mathcal{T}$  such that  $\text{Vlabel}(V_i) \leftarrow \emptyset, \text{Tlabel}(V_i) \leftarrow \emptyset, \forall i \in [k]$ .
- 2:  $\text{Over} \leftarrow \emptyset, \text{Under} \leftarrow \emptyset$
- 3: **for**  $i \leftarrow 2$  to  $k$  **do**
- 4:     **for** every vertex  $v \in T_i$  **do** ▷ Step 2
- 5:         Let  $V_j \in \{V_1, \dots, V_k\}$  such that  $v \in N_G(V_j \cap T_1)$
- 6:          $\text{Vlabel}(V_j) \leftarrow \text{Vlabel}(V_j) \cup v$
- 7: **for**  $i \leftarrow 2$  to  $k$  **do**
- 8:     **if**  $n_i - |V_i| \leq |\text{Vlabel}(V_i)|$  **then**
- 9:          $\text{Over} \leftarrow \text{Over} \cup V_i$
- 10:    **else**
- 11:         $\text{Under} \leftarrow \text{Under} \cup V_i$
- 12: **for** every  $V_i \in \text{Under}$  **do** ▷ Step 3
- 13:      $V_i \leftarrow \text{Add}(V_i, \text{Vlabel}(V_i))$
- 14:     **if**  $\text{Tlabel}(V_i) = \emptyset$  **then**
- 15:         Let  $T_j$  be a tree such that  $T_j \notin \cup_{s=1}^k \text{Tlabel}(V_s)$
- 16:          $\text{Tlabel}(V_i) \leftarrow T_j$
- 17: **for** every tree  $T_i \in \mathcal{T} \setminus T_1$  such that  $\text{Tlabel}^{-1}(T_i) \neq \emptyset$  **do**
- 18:     Let  $V_j$  be the vertex set such that  $\text{Tlabel}(V_j) = T_i$ .
- 19:     **if**  $V_j \cap T_i = \emptyset$  **then**
- 20:         Let  $u \in T_i$  such that  $u \in N(V_j \cap T_1)$  and  $V_s$  such that  $u \in \text{Vlabel}(V_s)$
- 21:         **if**  $V_s \in \text{Over}$  **then**
- 22:              $V_j \leftarrow \text{Add}(V_j, u), \text{Vlabel}(V_s) \leftarrow \text{Remove}(\text{Vlabel}(V_s), u)$ .
- 23:             **if**  $n_s - |V_s| > |\text{Vlabel}(V_s)|$  **then**
- 24:                  $\text{Over} \leftarrow \text{Over} \setminus V_s, \text{Under} \leftarrow \text{Under} \cup V_s$
- 25:                 Go to line 12
- 26:         **else**
- 27:              $V_s \leftarrow \text{Remove}(V_s, u), \text{Vlabel}(V_s) \leftarrow \text{Vlabel}(V_s) \setminus u, V_j \leftarrow \text{Add}(V_j, u)$
- 28: **return**  $\text{Over}, \text{Under}, \text{Vlabel}, \text{Tlabel}$

---

We start by considering the vertex set  $V_j$  such that  $\text{Tlabel}(V_j) = T_i$  for some value of  $i$ . Due to Lemma 13, we know that  $T_i$  is either completely contained in  $V_j$ , in which case we do nothing and we proceed to consider the next tree, or it has a vertex of  $T_i$  that is adjacent to  $V_j \cap T_1$  and not in  $V_j$ . We then add that vertex to  $V_j$  in the same manner as in the loop in line 17 of Algorithm 1 depending on whether this vertex belongs already in some set or not.

Notice that this action preserves the connectivity of the set  $V_j$  due to the way the vertex is chosen and also does not affect the connectivity of other sets as this is based only on the adjacencies of the parts of each vertex set intersecting  $T_1$ , which is not affected. Moreover it increases the value of  $\sum_{i=2}^k |T_i \cap \text{Tlabel}^{-1}(T_1)|$  by 1, which ensures that this loop terminates. Again, every time we add a vertex to a vertex set, it is implied that we also check if that set should meet its size demand and should be added to **Full** and also if the condition to terminate and output  $l$  full sets and a smaller instance of the general problem is met. ◀

---

**Algorithm 2** Add Vertices (Step 4, Step 5).

---

**Require:** An instance of GL-PARTITION  $(G, k, C = \{c_1, \dots, c_k\}, N = \{n_1, \dots, n_k\})$ , a set of  $k$  vertex disjoint dominating trees  $\mathcal{T} = \{T_1, \dots, T_k\}$  such that  $C \subseteq T_1$ ,  $k$  connected vertex sets  $\{V_1, \dots, V_k\}$  such that  $c_i \in V_i$  for all  $i \in [k]$ , sets Under, Over, Full, and functions Vlabel, Tlabel.

```

1: while  $\exists V_j \in \text{Under}$  such that  $V_j \not\subseteq \text{Full}$  and  $T_i \not\subseteq V_j \ \forall i \in \{2, \dots, k\}$  do ▷ Step 4
2:   Let  $T_i$  be the tree such that  $\text{Tlabel}(V_j) = T_i$ 
3:   while  $V_j$  doesn't contain vertices only from  $T_i$  and  $T_i \not\subseteq V_j$  and  $V_j \not\subseteq \text{Full}$  do
4:     Let  $v \in T_i$  such that  $v \in N(V_j \cap (T_1 \cup T_i))$ 
5:     if  $v$  is assigned through the function Vlabel to some set  $V_s \in \text{Over}$  then
6:        $V_j \leftarrow \text{Add}(V_j, v)$ ,  $\text{Vlabel}(V_s) \leftarrow \text{Vlabel}(V_s) \setminus v$ 
7:       if  $n_s - |V_s| > |\text{Vlabel}(V_s)|$  then
8:          $\text{Over} \leftarrow \text{Over} \setminus V_s$ ,  $\text{Under} \leftarrow \text{Under} \cup V_s$ 
9:       Run Algorithm 1 with input  $(G, \{T_1, \dots, T_k\}, C, \text{Under}, \text{Over}, \text{Vlabel}, N)$ 
      starting from line 13.
10:    else
11:       $V_s \leftarrow \text{Remove}(V_s, v)$ ,  $\text{Vlabel}(V_s) \leftarrow \text{Vlabel}(V_s) \setminus v$ ,  $V_j \leftarrow \text{Add}(V_j, v)$ 
12:  for every  $V \in \text{Over}$  and  $V \not\subseteq \text{Full}$  do ▷ Step 5
13:    Let  $u$  be a vertex in  $\text{Vlabel}(V)$  and not in  $V$ 
14:     $V \leftarrow \text{Add}(V, u)$ 
15:  for every  $v \in \cup_{i=2}^k T_i \setminus \cup_{i=1}^k V_i$  do
16:    Let  $V$  be a set such that  $V \not\subseteq \text{Full}$  and  $v \in N(V)$ 
17:     $V \leftarrow \text{Add}(V, v)$ 
18: return  $\{V_1, \dots, V_k\}$ 

```

---

Now that we have proved the correctness of Step 4, notice that in the last part of Algorithm 2 each non full set either contains a whole dominating tree or is adjacent to at least as many available vertices as it needs for its size demand to be met. We first start by the second case in which we add the assigned vertices to the set one by one until it is full. Then we are left with the case where each non full set contains a dominating tree, in which we can greedily add adjacent vertices to each such set one by one until all of them meet their size demands.

We now put everything together and provide the main algorithm of this section.

---

**Algorithm 3** Györi–Lovász theorem for special case of graphs with  $k$  dominating trees.

---

**Require:** An instance of GL-PARTITION  $(G, k, C, N)$  and a set of  $k$  vertex disjoint dominating trees  $\mathcal{T} = \{T_1, \dots, T_k\}$  such that  $C \subseteq T_1$ .

```

1:  $(V_1, \dots, V_k) \leftarrow \text{AddTrees}(G, C, \mathcal{T})$ 
2:  $(\text{Over}, \text{Under}, \text{Vlabel}, \text{Tlabel}) \leftarrow \text{Labeling}(G, C, \mathcal{T}, N, V_1, \dots, V_k)$ 
3:  $(V_1, \dots, V_k) \leftarrow \text{Add Vertices}(G, C, \mathcal{T}, N, \text{Over}, \text{Under}, \text{Vlabel}, \text{Tlabel}, \text{Full})$ 
4: Output  $V_1, \dots, V_k$ 

```

---

► **Lemma 15** ( $\star$ ). *Let an instance of GL-PARTITION  $(G, k, C = \{c_1, \dots, c_k\}, N = \{n_1, \dots, n_k\})$  and  $k$  vertex disjoint dominating trees  $T_1, \dots, T_k$  such that  $C \subseteq T_1$  are given as an input to Algorithm 3. Then Algorithm 3 returns  $l$  full sets  $V_1, \dots, V_l$  and a reduced instance,  $(G - \cup_{i=1}^l V_i, k - l, C \setminus \{c_{k-l}, \dots, c_k\}, N \setminus \{n_{k-l}, \dots, n_k\})$  of GL-PARTITION with a set of vertex-disjoint dominating trees of reduced graph  $\mathcal{T} \setminus \{T_{k-l}, \dots, T_k\}$ , for some  $l \in [k]$ , in polynomial time.*

In the next section we tackle the more general instance in which there is a dominating tree that contains strictly less than  $k$  terminals and some other contains at least one by showing that we are able to iteratively find a smaller instance in which all the terminals are contained in some dominating tree and then employ the algorithm developed in this section.

## 4 From General CDS Partition to GL-partition

In this section we provide the algorithm from Theorem 4. The algorithm receives an instance of GL-PARTITION  $(G, k, C, N)$  and  $k$  vertex disjoint dominating trees  $T_1, \dots, T_k$  as input and it outputs a GL-partition  $V_1, \dots, V_k$  of  $V(G)$ .

If all terminal vertices are contained in one dominating tree then we can directly use Algorithm 3 of the previous section. Notice that if a terminal is not contained in any tree we can simply add it to one since each tree is dominating. Hence, we assume without loss of generality that every terminal vertex is contained in some tree. Moreover, we assume that the dominating trees are separated into three sets  $\mathcal{T}_0, \mathcal{T}_1$  and  $\mathcal{T}_{>1}$  depending on whether they contain 0, 1 or more than 1 terminal vertices, respectively. This categorization can be done in polynomial time so for simplicity we assume that this is also part of the input. As in the previous section we first provide an intuitive description of Algorithm 4. The formal descriptions of the steps and the proof of correctness is deferred to the appendix.

**Description of Algorithm 4.** The Algorithm receives as input a GL-PARTITION instance  $(G, k, C, N)$ , where  $C = \{c_1, \dots, c_k\}$ ,  $N = \{n_1, \dots, n_k\}$ , and a set of  $k$  vertex-disjoint dominating trees  $\mathcal{T} = \{T_1, \dots, T_k\}$  separated into three sets  $\mathcal{T}_0, \mathcal{T}_1, \mathcal{T}_{>1}$  depending on how many terminal vertices they contain. It is important to note that, like in the previous section, during this description of the algorithm, every time we add a vertex to a vertex set, it is implied that we also check if the condition of having  $l$  full sets say  $V_1, \dots, V_l$ , intersecting exactly  $l$  of the dominating trees, say  $T_1, \dots, T_l$  is met. If yes, then we output those  $l$  full sets and restart the algorithm for  $G - \bigcup_{i=1}^l V_i$  and  $k-l$ ,  $\mathcal{T} = \mathcal{T} \setminus \{T_{k-l}, \dots, T_k\}$ ,  $C = \{c_{k-l}, \dots, c_l\}$  and  $N = \{n_{k-l}, \dots, n_k\}$ . For simplicity and space constraints, we do not include this in every step of this description nor in the pseudocodes provided in this section. We moreover assume both in pseudocodes of this section and in this description that every time we add a vertex we also check if a set meets its size demand and if so we add it to the set **Full** that includes all the connected vertex sets that satisfy their size demands. Similarly every time we remove a vertex from some set we check if it is in **Full** and if yes, we remove it from **Full**.

**Step 1** We first initialize each vertex set  $V_i$  to contain the corresponding terminal vertex  $c_i$ , where  $i \in [k]$ . Then, for each tree in  $\mathcal{T}_1 \cup \mathcal{T}_{>1}$ , we add its vertices one by one to a vertex set that it is adjacent to (based only on the adjacencies occurring only by the tree considered). Next, we add the vertices of  $V(G) \setminus V(\mathcal{T})$  to an adjacent set one by one.

**Step 2** For each tree  $T \in \mathcal{T}_{>1}$  that contains  $l$  terminals, we create a tree set  $\mathcal{T}'$  containing  $T$  and  $l-1$  trees from  $\mathcal{T}_0$ , which we then remove from  $\mathcal{T}_{>1}$  and  $\mathcal{T}_0$  respectively. Among those sets, we choose one that has enough vertices available to satisfy the size demands of all of the  $l$  vertex sets intersecting  $T$ . (Algorithm Choose a Tree Set)

**Step 3** Call Algorithm 3 that has as input those  $l$  dominating trees, the corresponding vertex sets and terminal vertices, while the graph we work on is the one that occurs from the union of those sets and trees. Notice that the sum of the demands of those sets might be less than the size of the graph but this does not affect the correctness of the algorithm.

■ **Algorithm 4** Györi–Lovász theorem for graphs with  $k$  dominating trees.

---

**Require:** A graph  $G$ , three sets of dominating trees  $\mathcal{T}_0, \mathcal{T}_1, \mathcal{T}_{>1}$ , a set of  $k$  vertices  $C = \{c_1, \dots, c_k\}$ , a set of  $k$  natural numbers  $N = \{n_1, \dots, n_k\}$  and  $k$  connected vertex sets  $\{V_1, \dots, V_k\}$  such that  $c_i \in V_i$  for all  $i \in [k]$

- 1:  $(V_1, \dots, V_k) \leftarrow \text{Add Vertices}(\mathcal{T}_0, \mathcal{T}_1, \mathcal{T}_{>1}, V_1, \dots, V_k, C, N, U)$
- 2:  $(T, T_1, \dots, T_{l-1}) \leftarrow \text{Choose a Tree Set}(\mathcal{T}_0, \mathcal{T}_1, \mathcal{T}_{>1}, C, N, V_1, \dots, V_k)$
- 3: Algorithm 3( $\cup_{i=1}^l V_i \cup_{i=1}^{l-1} T'_i \cup T, T, T'_1, \dots, T'_{l-1}, c_1, \dots, c_l, n_1, \dots, n_l$ ).

---

## 5 Conclusion

As we showcase in this work, the algorithm of Theorem 4 opens up a new avenue for efficiently computing connected partitions, in the settings where sufficiently large CDS partitions are available. This motivates further research into improving bounds for CDS partitions beyond the worst-case instances, as well as into parameterized and approximation algorithms for computing a CDS partition of the largest size.

To the best of our knowledge, Corollaries 5 and 11 are the first examples of a Györi–Lovász type of results that allow for a trade-off between efficiency of the construction and the connectivity requirement. It would be interesting to investigate whether a more refined trade-off is possible, interpolating smoothly between the polynomial time for higher connectivity and exponential time for extremal connectivity.

Our work also initiates the study of the gap between connectivity and the size of a CDS packing in structured graph classes. It remains a curious open question to characterize the class of graphs, for which  $k$ -connectivity implies the existence of a  $k$ -CDS partition. So far, our results show that this class contains interval and biconvex graphs (Theorems 7 and 6), but does not contain chordal and convex bipartite graphs.

---

## References

- 1 Nesrine Abbas and Lorna K. Stewart. Biconvex graphs: ordering and algorithms. *Discrete Applied Mathematics*, 103(1):1–19, 2000. doi:10.1016/S0166-218X(99)00217-6.
- 2 Katerina Asdre and Stavros D. Nikolopoulos. Np-completeness results for some problems on subclasses of bipartite and chordal graphs. *Theoretical Computer Science*, 381(1):248–259, 2007. doi:10.1016/j.tcs.2007.05.012.
- 3 Ronald I. Becker, Isabella Lari, Mario Lucertini, and Bruno Simeone. Max-min partitioning of grid graphs into connected components. *Networks*, 32(2):115–125, 1998. doi:10.1002/(SICI)1097-0037(199809)32:2<115::AID-NET4%3E3.0.CO;2-E.
- 4 Rémy Belmonte and Martin Vatshelle. Graph classes with structured neighborhoods and algorithmic applications. *Theoretical Computer Science*, 511:54–65, 2013. Exact and Parameterized Computation. doi:10.1016/j.tcs.2013.01.011.
- 5 Flavia Bonomo-Braberman, Nick Brettell, Andrea Munaro, and Daniël Paulusma. Solving problems on generalized convex graphs via mim-width. *Journal of Computer and System Sciences*, 140:103493, 2024. doi:10.1016/j.jcss.2023.103493.
- 6 Ralf Borndörfer, Katrin Casel, Davis Issac, Aikaterini Niklanovits, Stephan Schwartz, and Ziena Zeif. Connected  $k$ -partition of  $k$ -connected graphs and  $c$ -claw-free graphs. In Mary Wootters and Laura Sanità, editors, *Approximation, Randomization, and Combinatorial Optimization. Algorithms and Techniques, APPROX/RANDOM 2021, August 16-18, 2021, University of Washington, Seattle, Washington, USA (Virtual Conference)*, volume 207 of *LIPICs*, pages 27:1–27:14. Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2021. doi:10.4230/LIPICs.APPROX/RANDOM.2021.27.

- 7 Andreas Brandstädt, Van Bang Le, and Jeremy P. Spinrad. *Graph Classes: A Survey*. Society for Industrial and Applied Mathematics, 1999. doi:10.1137/1.9780898719796.
- 8 Gerth Stølting Brodal, Loukas Georgiadis, Kristoffer Arnsfelt Hansen, and Irit Katriel. Dynamic matchings in convex bipartite graphs. In Ludek Kucera and Antonín Kucera, editors, *Mathematical Foundations of Computer Science 2007, 32nd International Symposium, MFCS 2007, Český Krumlov, Czech Republic, August 26-31, 2007, Proceedings*, volume 4708 of *Lecture Notes in Computer Science*, pages 406–417. Springer, 2007. doi:10.1007/978-3-540-74456-6\_37.
- 9 Katrin Casel, Tobias Friedrich, Davis Issac, Aikaterini Niklanovits, and Ziena Zeif. Efficient constructions for the Györi-Lovász theorem on almost chordal graphs. In Daniël Paulusma and Bernard Ries, editors, *Graph-Theoretic Concepts in Computer Science - 49th International Workshop, WG 2023, Fribourg, Switzerland, June 28-30, 2023, Revised Selected Papers*, volume 14093 of *Lecture Notes in Computer Science*, pages 143–156. Springer, 2023. doi:10.1007/978-3-031-43380-1\_11.
- 10 Keren Censor-Hillel, Mohsen Ghaffari, George Giakkoupis, Bernhard Haeupler, and Fabian Kuhn. Tight bounds on vertex connectivity under sampling. *ACM Trans. Algorithms*, 13(2), May 2017. doi:10.1145/3086465.
- 11 Keren Censor-Hillel, Mohsen Ghaffari, and Fabian Kuhn. A new perspective on vertex connectivity. In Chandra Chekuri, editor, *Proceedings of the Twenty-Fifth Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2014, Portland, Oregon, USA, January 5-7, 2014*, pages 546–561. SIAM, 2014. doi:10.1137/1.9781611973402.41.
- 12 L. Sunil Chandran, Yun Kuen Cheung, and Davis Issac. Spanning tree congestion and computation of generalized györi-lovász partition. In Ioannis Chatzigiannakis, Christos Kaklamanis, Dániel Marx, and Donald Sannella, editors, *45th International Colloquium on Automata, Languages, and Programming, ICALP 2018, July 9-13, 2018, Prague, Czech Republic*, volume 107 of *LIPICs*, pages 32:1–32:14. Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2018. doi:10.4230/LIPICs.ICALP.2018.32.
- 13 Jiangzhuo Chen, Robert D. Kleinberg, László Lovász, Rajmohan Rajaraman, Ravi Sundaram, and Adrian Vetta. (almost) tight bounds and existence theorems for single-commodity confluent flows. *J. ACM*, 54(4):16, 2007. doi:10.1145/1255443.1255444.
- 14 Janka Chlebíková. Approximating the maximally balanced connected partition problem in graphs. *Information Processing Letters*, 60(5):225–230, 1996. doi:10.1016/S0020-0190(96)00175-5.
- 15 Peter Damaschke, Haiko Müller, and Dieter Kratsch. Domination in convex and chordal bipartite graphs. *Information Processing Letters*, 36(5):231–236, 1990. doi:10.1016/0020-0190(90)90147-P.
- 16 Reinhard Diestel. *Graph Theory, 4th Edition*, volume 173 of *Graduate texts in mathematics*. Springer, 2012.
- 17 Nemanja Draganić and Michael Krivelevich. Disjoint connected dominating sets in pseudorandom graphs, 2024. arXiv:2410.16072.
- 18 Wenfei Fan, Wenyuan Yu, Jingbo Xu, Jingren Zhou, Xiaojian Luo, Qiang Yin, Ping Lu, Yang Cao, and Ruiqi Xu. Parallelizing sequential graph computations. *ACM Trans. Database Syst.*, 43(4):18:1–18:39, 2018. doi:10.1145/3282488.
- 19 Michael R. Garey and David S. Johnson. A note on “A note on ‘Some simplified NP-complete graph problems’”. *SIGACT News*, 9(4):17, 1978. doi:10.1145/1008369.1008371.
- 20 Fred Glover. Maximum matching in a convex bipartite graph. *Naval research logistics quarterly*, 14(3):313–316, 1967.
- 21 Joseph E. Gonzalez, Yucheng Low, Haijie Gu, Danny Bickson, and Carlos Guestrin. Powergraph: Distributed graph-parallel computation on natural graphs. In Chandu Thekkath and Amin Vahdat, editors, *10th USENIX Symposium on Operating Systems Design and Implementation, OSDI 2012, Hollywood, CA, USA, October 8-10, 2012*, pages 17–30. USENIX Association, 2012. URL: <https://www.usenix.org/conference/osdi12/technical-sessions/presentation/gonzalez>.

- 22 Ervin Györi. On division of graphs to connected subgraphs. *Colloq. Math. Soc. Janos Bolyai*, 18:485–494, 1976.
- 23 S. T. Hedetniemi and R. Laskar. Connected domination in graphs. Technical report, Technical Report 414, Clemson University, SC, USA, 1983.
- 24 Alexander Hoyer. On the independent spanning tree conjectures and related problems. *PhD thesis, Georgia Institute of Technology*, 2016.
- 25 Witold Lipski Jr. and Franco P. Preparata. Efficient algorithms for finding maximum matchings in convex bipartite graphs and related problems. *Acta Informatica*, 15:329–346, 1981. doi:10.1007/BF00264533.
- 26 Michael Krivelevich, Alan Lew, and Peleg Michaeli. Rigid partitions: from high connectivity to random graphs, 2023. arXiv:2311.14451.
- 27 C. Lekkerkerker and J. Boland. Representation of a finite graph by a set of intervals on the real line. *Fundamenta Mathematicae*, 51(1):45–64, 1962. URL: <http://eudml.org/doc/213681>.
- 28 Y. Daniel Liang and Maw-Shang Chang. Minimum feedback vertex sets in cocomparability graphs and convex bipartite graphs. *Acta Informatica*, 34(5):337–346, 1997. doi:10.1007/S002360050088.
- 29 Lázló Lovász. A homology theory for spanning trees of a graph. *Acta Math. Acad. Sci. Hungaricae*, 30(3-4):241–251, 1977.
- 30 Mario Lucertini, Yehoshua Perl, and Bruno Simeone. Most uniform path partitioning and its use in image processing. *Discret. Appl. Math.*, 42(2):227–256, 1993. doi:10.1016/0166-218X(93)90048-S.
- 31 Rolf H. Möhring, Heiko Schilling, Birk Schütz, Dorothea Wagner, and Thomas Willhalm. Partitioning graphs to speedup dijkstra’s algorithm. *ACM J. Exp. Algorithmics*, 11, 2006. doi:10.1145/1187436.1216585.
- 32 Shin-Ichi Nakano, Md. Saidur Rahman, and Takao Nishizeki. A linear-time algorithm for four-partitioning four-connected planar graphs. *Inf. Process. Lett.*, 62(6):315–322, 1997. doi:10.1016/S0020-0190(97)00083-5.
- 33 Aikaterini Niklanovits, Kirill Simonov, Shaily Verma, and Ziena Zeif. Connected partitions via connected dominating sets, 2025. doi:10.48550/arXiv.2503.13112.
- 34 Donald J. Rose, R. Endre Tarjan, and George S. Lueker. Algorithmic aspects of vertex elimination on graphs. *SIAM Journal on Computing*, 5(2):266–283, 1976. doi:10.1137/0205021.
- 35 José Soares and Marco Aurelio Stefanos. Algorithms for maximum independent set in convex bipartite graphs. *Algorithmica*, 53(1):35–49, 2009. doi:10.1007/S00453-007-9006-9.
- 36 George Steiner and Julian Scott Yeomans. A linear time algorithm for maximum matchings in convex, bipartite graphs. *Computers & Mathematics with Applications*, 31(12):91–96, 1996.
- 37 Hitoshi Suzuki, Naomi Takahashi, and Takao Nishizeki. A linear algorithm for bipartition of biconnected graphs. *Inf. Process. Lett.*, 33(5):227–231, 1990. doi:10.1016/0020-0190(90)90189-5.
- 38 Hitoshi Suzuki, Naomi Takahashi, Takao Nishizeki, H Miyano, and S Ueno. An algorithm for tripartitioning 3-connected graphs. *Journal of Information Processing Society of Japan*, 31(5):584–592, 1990.
- 39 Koichi Wada and Kimio Kawaguchi. Efficient algorithms for tripartitioning triconnected graphs and 3-edge-connected graphs. In Jan van Leeuwen, editor, *Graph-Theoretic Concepts in Computer Science, 19th International Workshop, WG ’93, Utrecht, The Netherlands, June 16-18, 1993, Proceedings*, volume 790 of *Lecture Notes in Computer Science*, pages 132–143. Springer, 1993. doi:10.1007/3-540-57899-4\_47.
- 40 Jiguo Yu, Nannan Wang, Guanghui Wang, and Dongxiao Yu. Connected dominating sets in wireless ad hoc and sensor networks – a comprehensive survey. *Computer Communications*, 36(2):121–134, 2013. doi:10.1016/j.comcom.2012.10.005.
- 41 Bohdan Zelinka. Connected domatic number of a graph. *Mathematica Slovaca*, 36(4):387–392, 1986. URL: <http://eudml.org/doc/34243>.