Improved Algorithms for Quantum MaxCut via Partially Entangled Matchings

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Abstract

We introduce a 0.611-approximation algorithm for Quantum MaxCut and a $\frac{1+\sqrt{5}}{4} \approx 0.809$-approximation algorithm for the EPR Hamiltonian of [12]. A novel ingredient in both of these algorithms is to *partially* entangle pairs of qubits associated to edges in a matching, while preserving the direction of their single-qubit Bloch vectors. This allows us to interpolate between product states and matching-based states with a tunable parameter.

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1 Introduction

Given a graph $G(V, E, w)$ with positive edge weights $w : E \to \mathbb{R}_+$ and a 2-local Hamiltonian term h, define the n-qubit Hamiltonian

$$H_G \stackrel{\text{def}}{=} \sum_{(i,j)\in E(G)} w_{ij} \cdot h_{ij} \,,$$

where h_{ij} is the local term h applied on qubits (i, j). For two particular local terms h, we are interested in the problem of computing the maximum energy of H_G, which we denote as $\lambda_{max}(H_G)$, for any G. This is not an easy task in general: deciding if $\lambda_{max}(H_G)$ is above some threshold with inverse polynomial accuracy is known to be QMA-hard [18].

In the first problem, we choose the local term

$$h_{ij}^{QMC} \stackrel{\text{def}}{=} \frac{1}{2}\left(I_iI_j - X_iX_j - Y_iY_j - Z_iZ_j\right) = 2\left|\psi_-\right\rangle_{ij}\left\langle\psi_-\right|_{ij} \,,$$

where $|\psi_-\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$ is the singlet state. This problem has recently been studied under the name *Quantum MaxCut* (QMC) [7]. In the statistical mechanics literature, Hamiltonians H_G defined by h^{QMC} are instances of the zero-field quantum Heisenberg $\mathrm{XXX}_{1/2}$ model. The decision version of QMC is QMA-hard [18].

In the second problem, we choose the local term

$$h_{ij}^{EPR} \stackrel{\text{def}}{=} \frac{1}{2}\left(I_iI_j + X_iX_j - Y_iY_j + Z_iZ_j\right) = 2\left|\phi_+\right\rangle_{ij}\left\langle\phi_+\right|_{ij} \,,$$

where $|\phi_+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$. This problem, named *EPR* by [12], is thought to be easier than QMC, since Hamiltonians H_G defined by h^{EPR} are *stoquastic* (sign-problem free) [18, 5]. In fact, it is not yet clear if EPR can be solved in polynomial time.

In lieu of exactly computing $\lambda_{max}(H_G)$, we may try to *approximate* this value. In both problems, the local term h is positive semidefinite, so $\lambda_{max}(H_G) \geq 0$. We judge an approximation by its *approximation ratio*. Suppose we can find efficiently computable functions ℓ, u such that for all graphs G,

$$0 \leq \ell(G) \leq \lambda_{max}(H_G) \leq u(G) \,.$$

Then, the approximation ratio α is at least

$$\alpha \geq \min_G \frac{\ell(G)}{\lambda_{max}(H_G)} \geq \min_G \frac{\ell(G)}{u(G)} \,.$$

In most works, the upper bound u is determined by solving a semidefinite programming (SDP) relaxation of the maximization problem [3, 7, 16, 19, 20, 9]. The source of such SDP relaxations are generally hierarchies of SDPs that provide increasingly better upper bounds u at the cost of solving larger-sized SDPs. The quantum moment-SOS hierarchy, typically based on Pauli operators, is widely employed for quantum local Hamiltonian problems. This hierarchy is an instance of the NPA hierarchy [15] and is also known as the quantum Lasserre hierarchy [16]. The second level of the quantum moment-SOS hierarchy is necessary for good approximations, since critical monogamy-of-entanglement properties begin to emerge at that level [16, 17].

The lower bound ℓ is usually determined by an algorithm that prepares a state with this energy. Several approximation algorithms have been proposed for QMC and EPR, using product states [7, 16, 17, 12], matchings [1, 16, 17, 14, 10, 8], and short variational

circuits [1, 13, 12, 11, 8]. During the preparation of this manuscript, the best known approximation ratios for these problems were improved to $\alpha \geq 0.603$ for QMC [8] and $\alpha \geq \frac{1+\sqrt{5}}{4} \approx 0.809$ for EPR [11].

We provide a new algorithm for both problems. For each algorithm, we start from a good product state. We then choose a matching in G and *partially* rotate matched qubits towards an entangled state. Our novel contribution is to perform this rotation such that individual single-qubit Bloch vectors are preserved, up to a rescaling of magnitude. This allows us to evaluate the energy of our entangled state in terms of the energy of the product state. For EPR, we use a *fractional* matching and provide a circuit-based algorithm. For QMC, we explicitly describe a tensor-product of single and two-qubit states, which interpolates between the product state and an *integer* matching-based state with a single tunable parameter. For QMC, we additionally introduce the technique of choosing a maximum-weight matching with respect to *rescaled* edge weights. One interpretation of this approach is that we widen the search space of our algorithm to include matchings other than the maximum-weight matching of G. Combining these techniques allows us to achieve state-of-the-art approximation ratios on each problem:

▶ **Theorem 1.** *For EPR,* $\alpha \geq \frac{\varphi}{2} \approx 0.809$*, where* $\varphi \stackrel{\text{def}}{=} \frac{1+\sqrt{5}}{2} \approx 1.618$ *is the golden ratio.*

▶ **Theorem 2.** *For QMC,* $\alpha \geq 0.611$*.*

▶ Remark 3. Theorem 1 was shown simultaneously and independently by [11] using a different algorithm. The algorithm of [11] also *partially* entangles edges while preserving single-qubit marginals. However, we believe both our algorithm and analysis are simpler.

▶ Remark 4. Theorem 2 requires an improved upper bound in addition to new algorithmic techniques. The technique of finding a matching with respect to rescaled edge weights was simultaneously and independently used by [8]; however, they use this as part of an improved approximation algorithm for QMC on triangle-free graphs rather than general graphs. Our QMC approximation does rely on a strengthening by [8] of a class of upper bounds first used in the work [14]. Without this strengthened bound, we still achieve an approximation ratio $\alpha \geq 0.610$.

Our approximation algorithm finds a quantum state ρ_G such that $\mathrm{Tr}\left(H_G \rho_G\right) \geq \ell(G)$. For the EPR Hamiltonian, the analysis of our algorithm is optimal: there exist graphs G (such as the single-edge graph) where

$$\frac{\ell(G)}{u(G)} \leq \frac{\langle \psi_G | H_G | \psi_G \rangle}{\lambda_{max}(G)} = \frac{1+\sqrt{5}}{4}.$$

For QMC, however, it is possible that a better analysis of this algorithm (particularly of $u(G)$) would give a larger approximation ratio.

We stress the novelty of this work is *new algorithmic techniques* for QMC and EPR. We use these techniques to immediately improve the approximation ratios for each algorithm, however we believe that these techniques, potentially combined with previous methods, may lead to further improvements in approximation ratios for both algorithms. In the remainder of this work, we introduce necessary notation, describe each algorithm, and prove Theorems 1 and 2. We defer some technical lemmas to the appendix, which can be found in the full version of this paper [2].

2 Preliminaries

2.1 Graph theory

Let $G(V, E, w)$ denote a graph with vertex set V, edge set E, and positive edge weights $w : E \to \mathbb{R}_+$. In this work, we always take $V = [n] \stackrel{\text{def}}{=} \{1, 2, \ldots, n\}$. We let $W_G \stackrel{\text{def}}{=} \sum_{(i,j)\in E} w_{ij}$. When G is inferred by context, we simply write W. Define $N(v)$ as the set of neighbors of a vertex $v \in V$. For convenience, we index edges by either e or (i, j), or simply ij in subscripts.

A fractional *matching* of a graph $G(V, E, w)$ is a function $m : E(G) \to [0, 1]$ that assigns a value to each edge in G, such that the sum over values $\sum_{j\in N(i)} m_{ij}$ is at most 1 for any vertex i. If $m_{ij} \in \{0, 1\}$ for all edges (i, j), we say that m is an *integral* matching. For an integral matching m, we say an edge (i, j) is in the matching if $m_{ij} = 1$. The *weight* $\mathrm{Wt}(m)$ of a matching m is defined as $\mathrm{Wt}(m) \stackrel{\text{def}}{=} \sum_{(i,j)\in E(G)} w_{ij}\, m_{ij}$. We define FM_G (or M_G) to be the maximum total weight of any fractional (or integral) matching of G. For convenience, we sometimes let M_G also denote the set of edges in the maximum-weight integral matching of G. When the graph is clear from context we drop the subscript G. Optimal integral and fractional matchings can be computed in polynomial time, for example with linear programming [6].

2.2 Quantum computation

We refer to $\vec{\sigma} = (X, Y, Z)$ as the canonical Pauli matrices. As such, we can define the *Bloch vector* $\vec{B}(\rho)$ of any 1-qubit density matrix ρ as the vector (b_x, b_y, b_z) in the unit sphere S^2 such that

$$\rho = \frac{1}{2}(I + \vec{B}(\rho) \cdot \vec{\sigma}) \stackrel{\text{def}}{=} \frac{1}{2}(I + b_x X + b_y Y + b_z Z)\,.$$

Pure states correspond to unit Bloch vectors. Let $|\psi_-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ be the *singlet state*, and $|\phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ be the *EPR state*.

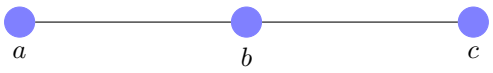
2.3 Previous algorithms

In our work, we use two previously known algorithms as subroutines:

1. The algorithm PROD takes a graph $G(V, E, w)$ as an input, and outputs a good product state ρ_{PROD}. For EPR, we define $\rho_{PROD} \stackrel{\text{def}}{=} |0\rangle^{\otimes n} \langle 0|^{\otimes n}$. For QMC, we define PROD to be the output of the [7] rounding algorithm applied to the some fixed constant level k of the quantum moment-SOS hierarchy, as in [17] and [14]. This is the only part of our algorithm for QMC that relies on an SDP. As described in [17, Sec 2.2], the output from the SDP can be interpreted as a *pseudo-density matrix*. It is called a *pseudo*-density matrix because it satisfies only some of the constraints of a valid density matrix. Furthermore, the SDP at any constant ℓ can be solved in polynomial time.
2. The algorithm MATCH takes a graph $G(V, E, w)$ as an input, and outputs a product of 2-qubit states ρ_{MATCH}. It was first formally proposed in [14]. The algorithm first finds a maximum-weight integral matching M_G. For EPR (and for QMC), it outputs the tensor product of $|\phi_+\rangle_{ij} \langle\phi_+|_{ij}$ (and $|\psi_-\rangle_{ij} \langle\psi_-|_{ij}$ for QMC) for every pair of vertices (i, j) in the matching, and the maximally mixed state for every vertex i not in the matching. This obtains energy 2 on matched edges and 1/2 on unmatched edges, for a total of $(3\mathrm{M}_G + W_G)/2$.

3 Our Algorithms

To motivate our algorithms, consider the following simple graph G:



This graph is bipartite, so QMC and EPR are equivalent under local rotations [12]. We thus focus on EPR for simplicity. It is easy to compute that $\lambda_{max}(H_G) = 3$. The optimal product state ρ_{PROD} is $\rho_0^{\otimes 3}$, where $\rho_0 \stackrel{\text{def}}{=} |0\rangle\langle 0|$, and achieves energy 2. The algorithm MATCH computes the maximum matching $\mathrm{M}_G = \{(a,b)\}$ and returns the state ρ_{MATCH} as described in Section 2. This state gains energy 2 on edge (i,j) and 1/2 on edge (b,c), achieving total energy 5/2. Thus, the algorithm of [14], which returns the better of ρ_{PROD} and ρ_{MATCH}, achieves $5/6 \approx 0.833$ of the optimal energy.

Our algorithms do better by interpolating between ρ_{PROD} and ρ_{MATCH}, rather than taking the better of the two. For our example G, the unitary

$$U = e^{i\theta\left(\frac{X_a - Y_a}{\sqrt{2}} \otimes \frac{X_b - Y_b}{\sqrt{2}}\right)},$$

takes ρ_{PROD} to ρ', a tensor product of a two-qubit state ρ_{ab} and a single-qubit state ρ_c. The parameter θ sets the entanglement for ρ_{ab}: when $\theta = 0$, ρ_{ab} is a product state; when $\theta = \pi/4$, ρ_{ab} is fully rotated into the EPR state. As such, we view this approach as smoothly interpolating between ρ_{PROD} and ρ_{MATCH} in superposition. It can be easily verified that the energy obtained by ρ' on edge (a,b) is $\mathrm{Tr}\left[h_{ab}^{EPR}\rho'\right] = 1 + 2\cos\theta\sin\theta$, and the single qubit marginals of a and b are given by

$$\rho'_a \stackrel{\text{def}}{=} \mathrm{Tr}_{bc}[\rho'] = \cos 2\theta\rho_0 + 2\sin^2\theta I = \rho'_b\,.$$

In particular, note that the single qubit marginals are simply rescaled and shifted by the identity. This fact aids in the analysis: we can still evaluate the energy of (b,c) in terms of ρ_{PROD}, even though the edge is not in our matching:

$$\begin{aligned}
\mathrm{Tr}\left[h_{bc}^{EPR}\rho'\right] &= \mathrm{Tr}\left[h_{bc}^{EPR}\left(\rho'_b \otimes \rho'_c\right)\right] \\
&= \mathrm{Tr}\left[h_{bc}^{EPR}\left(\rho'_b \otimes \rho_0\right)\right] \\
&= \mathrm{Tr}\left[h_{bc}^{EPR}\left(\left(\cos 2\theta\rho_0 + 2\sin^2\theta\ (I/2)\right) \otimes \rho_0\right)\right] \\
&= \mathrm{Tr}\left[h_{bc}^{EPR}\left(\cos 2\theta\rho_{PROD} + \sin^2\theta\, I \otimes \rho_0\right)\right] \\
&= \cos 2\theta + \sin^2\theta = \cos^2\theta\,.
\end{aligned}$$

Thus the total energy is given by

$$\mathrm{Tr}\left[(h_{ab}^{EPR} + h_{bc}^{EPR})\rho'\right] = 1 + 2\cos\theta\sin\theta + \cos^2\theta\,.$$

The parameter θ can be optimized over; in this example, taking $\theta \approx .554$ yields

$$\mathrm{Tr}\left[(h_{ab}^{EPR} + h_{bc}^{EPR})\rho'\right] \approx 2.618,$$

outperforming both ρ_{PROD} and ρ_{MATCH}.

In this example, the analysis simplifies nicely because the initial product state is the symmetric state $\rho_0^{\otimes n}$, and because the graph is bipartite. For EPR on general graphs, we provide a circuit-based algorithm that prepares an entangled state based on a *fractional* matching in the graph. This state still works by *partially* entangling edges according to the

matching while preserving single-qubit marginals, but it does not have a simple interpretation in terms of ρ_{MATCH}. We leave open the possibility that a $(\frac{1+\sqrt{5}}{4})$-approximation algorithm can be achieved with a tensor product of one and two-qubit states that simply interpolates between ρ_{PROD} and ρ_{MATCH}. For QMC, our algorithm does indeed interpolate between ρ_{PROD} and ρ_{MATCH}. However the algorithm and analysis become slightly more complicated. Crucial to our analysis is the following lemma:

▶ **Lemma 5** (Energy obtained by QMC algorithm on matched and unmatched edges). *Given two 1-qubit pure states ρ_i, ρ_j and a real parameter $\theta \in [0, \pi/2]$, there exists a 2-qubit pure state ρ_{ij} such that*

$$\begin{aligned}
\mathrm{Tr}\big[h^{QMC}\rho_{ij}\big] &= \frac{(1+\sin\theta)\left(1-\vec{B}(\rho_i)\cdot\vec{B}(\rho_j)\right)}{2}\,,\\
\vec{B}(\mathrm{Tr}_j[\rho_{ij}]) &= \cos\theta\cdot\vec{B}(\rho_i)\,,\\
\vec{B}(\mathrm{Tr}_i[\rho_{ij}]) &= \cos\theta\cdot\vec{B}(\rho_j)\,.
\end{aligned}$$

The last two constraints in Lemma 5 indicate that the Bloch vectors of the single-qubit marginals of ρ_{ij} are rescaled Bloch vectors of ρ_i and ρ_j, respectively. This is analogous to the argument in our example, where the marginals are rescaled and shifted by the identity. This again allows us to compute the energy of unmatched edges in terms of ρ_{PROD}. The parameter θ again sets the entanglement in ρ_{ij}: when $\theta = 0$, ρ_{ij} is a product state; when $\theta = \pi/2$, ρ_{ij} is maximally entangled. The proof of Lemma 5 is given in the appendix of [2].

3.1 EPR

Using the intuition from our example, we introduce the following algorithm for EPR:

▶ **Algorithm 1** (Fractional matching algorithm for EPR). *Given a weighted graph $G(V, E, w)$:*

1. *Define the 1-qubit Hamiltonian*

$$P \stackrel{\text{def}}{=} \frac{1}{\sqrt{2}}(X - Y)\,,$$

and the angle

$$\theta \stackrel{\text{def}}{=} \frac{\ln(\varphi)}{2} \approx 0.240\,, \tag{1}$$

where $\varphi = \frac{1+\sqrt{5}}{2}$ is the golden ratio.

2. *Find a fractional matching $(m_{uv})_{(u,v)\in E}$ of maximum weight (e.g., via linear programming).*
3. *Output the state*

$$|\chi\rangle \stackrel{\text{def}}{=} \prod_{(u,v)\in E} e^{i\gamma_{uv}P_uP_v}\,|0\rangle^{\otimes n}\,, \tag{2}$$

where

$$\gamma_{uv} = \frac{1}{2}\cos^{-1}\exp\left[-\theta\cdot m_{uv}\right] \;\in\; [0, \pi/2]\,. \tag{3}$$

The unitary $e^{i\gamma P_uP_v}$ rotates the state $|00\rangle_{ij}$ towards the Bell state $|\phi^+\rangle$; this circuit was proposed in the approximation algorithm of [12].

3.2 QMC

Our algorithm for QMC relies on the following subroutine, which we call PMATCH since it places partially entangled 2-qubit states on the edges of a matching, while MATCH places maximally entangled states:

▶ **Algorithm 2** (PMATCH). *Given a weighted graph $G(V, E, w)$ and a real parameter $\theta \in [0, \pi/2]$:*

1. *Prepare the state $\rho_{PROD} = \bigotimes_{i\in[n]} \rho_i$ by running the* PROD *algorithm on QMC from Section 2 for some fixed, constant SDP level ℓ.*
2. *For each edge $(i, j) \in E$, compute $t_{ij} \stackrel{\text{def}}{=} \vec{B}(\rho_i) \cdot \vec{B}(\rho_j)$ from ρ_{PROD}.*
3. *Find a maximum-weight integral matching $\widetilde{M} \subseteq E$ on the* reweighted *graph $G(V, E', \widetilde{w})$, where*

$$\widetilde{w}_{ij} = w_{ij} \cdot \left(\frac{\sin\theta \left(1 - t_{ij}\left(1 + \sin\theta\right)\right)}{2} \right)^{+}, \tag{4}$$

   and $E' = \{(i, j) \in E \mid \widetilde{w}_{ij} > 0\}$. Here, we use the notation $(\cdot)^+ \stackrel{\text{def}}{=} \max(0, \cdot)$ from [14].
4. *Let $\rho_{PMATCH}(\theta)$ be the state starting from ρ_{PROD}, but for each edge (i, j) in $\widetilde{M}$, replace $\rho_i \otimes \rho_j$ with the state ρ_{ij} described in Lemma 5, parametrized by ρ_i, ρ_j, θ.*
5. *Output the state $\rho_{PMATCH}(\theta)$.*

Our algorithm for QMC chooses the better of the algorithms PMATCH and MATCH:

▶ **Algorithm 3** (Combined algorithm for QMC). *Given a weighted graph $G(V, E, w)$:*

1. *Prepare the state ρ_{MATCH} by running the* MATCH *algorithm from Section 2.*
2. *Prepare the state $\rho_{PMATCH}(\theta)$ by running* PMATCH *with parameter $\theta = 1.286$.*
3. *Output the state out of ρ_{MATCH} and $\rho_{PMATCH}(\theta)$ obtaining larger energy on the QMC Hamiltonian H_G.*

4 Analysis

We upper-bound the maximum energy $\lambda_{max}(H_G)$ of QMC and EPR Hamiltonians using a quantifiable *monogamy of entanglement*:

▶ **Lemma 6** (Monogamy of Entanglement, e.g. [14, Lemma 1]). *For both EPR and QMC, and for all graphs $G(V, E, w)$, we have*

$$\lambda_{max}(H_G) \leq W_G + \mathrm{FM}_G\,.$$

For QMC, our analysis improves with better upper bounds on $\lambda_{max}(H_G)$. We use an inequality of the form $W_G + \mathrm{M}_G/d$; this first appears in [14, Lemma 4] with the constant $d = 4/5$. By a detailed analysis and numerical verification on graphs with up to 13 vertices, this constant was recently improved to $14/15$ in [8]:

▶ **Lemma 7** (Strengthened monogamy of entanglement, [8, Lemma 3.10]). *For QMC, and for all graphs $G(V, E, w)$, we have*

$$\lambda_{max}(H_G) \leq W_G + \frac{\mathrm{M}_G}{d}\,,$$

where $d = \frac{14}{15}$. Additionally, the bound holds for the optimal value of the 13-th level of the quantum moment SoS hierarchy.

4.1 EPR

We now prove Theorem 1. Given the state in Equation (2)

$$|\chi\rangle \stackrel{\text{def}}{=} \prod_{(i,j)\in E} e^{i\gamma_{ij} P_i P_j} |0\rangle^{\otimes n} , \tag{5}$$

[12, Lemma 9] showed that the energy on the local term h_{ij}^{EPR} is at least

$$\langle\chi| h_{ij}^{EPR} |\chi\rangle \geq \frac{1}{2} \left(1 + A_{ij} B_{ij} + \sin 2\gamma_{ij} \cdot (A_{ij} + B_{ij})\right) , \tag{6}$$

where

$$A_{ij} \stackrel{\text{def}}{=} \prod_{k\in N(i)\setminus\{j\}} \cos 2\gamma_{ik} ,$$
$$B_{ij} \stackrel{\text{def}}{=} \prod_{k\in N(j)\setminus\{i\}} \cos 2\gamma_{jk} .$$

In Algorithm 1, the output state $|\chi\rangle$ is in the form of Equation (5). Using the angles γ_{ij} specified by Equation (3), we have

$$A_{ij} = \exp\left[-\theta \sum_{k\in N(i)\setminus\{j\}} m_{ik}\right] \geq \exp\left[-\theta(1 - m_{ij})\right] , \tag{7}$$

$$B_{ij} = \exp\left[-\theta \sum_{k\in N(j)\setminus\{i\}} m_{jk}\right] \geq \exp\left[-\theta(1 - m_{ij})\right] , \tag{8}$$

$$\sin 2\gamma_{ij} = \sqrt{1 - \cos^2 2\gamma_{ij}} = \sqrt{1 - \exp\left[-2\theta m_{ij}\right]} . \tag{9}$$

The inequalities in the first two lines follow because m is a matching. For example, we have $m_{ij} + \sum_{k\in N(i)\setminus\{j\}} m_{ik} \leq 1$. Using Equation (6), the energy of $|\chi\rangle$ on h_{ij}^{QMC} is at least

$$T(\theta, m_{ij}) \stackrel{\text{def}}{=} \frac{1}{2}\left(1 + \exp\left[-2\theta(1 - m_{ij})\right] + 2\sqrt{1 - \exp\left[-2\theta m_{ij}\right]} \exp\left[-\theta(1 - m_{ij})\right]\right) . \tag{10}$$

We combine Equation (10) with Lemma 6 to bound the approximation ratio on any graph $G(V, E, w)$:

$$\frac{\langle\chi| H_G |\chi\rangle}{\lambda_{max}(H_G)} \geq \frac{\sum_{(i,j)\in E} w_{ij} \cdot T(\theta, m_{ij})}{\sum_{(i,j)\in E} w_{ij} (1 + m_{ij})} .$$

Each term in the numerator and denominator is positive, so the approximation ratio is at least the approximation ratio of the worst edge

$$\min_{(i,j)\in E} \frac{T(\theta, m_{ij})}{1 + m_{ij}} \geq \min_{x\in[0,1]} \frac{T(\theta, x)}{1 + x} . \tag{11}$$

Recall from Equation (1) that we take $\theta = \frac{1}{2}\ln\varphi$. We empirically identified θ as the angle that maximizes the approximation ratio in this analysis. Substituting this value into $T(\theta, x)$ in Equation (10), the RHS of Equation (11) yields the minimization problem

$$\min_{x\in[0,1]} \left[\frac{1}{2(1+x)}\left(1 + \varphi^{-(1-x)} + 2\sqrt{1 - \varphi^{-x}} \cdot \varphi^{-\frac{1}{2}(1-x)}\right)\right],$$

whose value is found to be $\varphi/2$ by the following lemma:

▶ **Lemma 8.** *Given* $\varphi \stackrel{\text{def}}{=} \frac{1+\sqrt{5}}{2}$*, we have that*

$$\min_{x\in[0,1]} \left[\frac{1}{2(1+x)}\left(1+\varphi^{-(1-x)}+2\sqrt{1-\varphi^{-x}}\cdot\varphi^{-\frac{1}{2}(1-x)}\right)\right] = \frac{1+\sqrt{5}}{4} = \frac{\varphi}{2}\,. \tag{12}$$

The proof of Lemma 8 is given in the appendix of [2].

4.2 QMC

We now analyze Algorithm 3 and prove the 0.611-approximation in Theorem 2. We first introduce some helpful notation. Fix a graph $G(V, E, w)$. Let $\tilde{\rho}$ be the pseudo-density matrix outputted by solving the SDP of the level-ℓ quantum moment-SOS hierarchy (described in Section 2.3). We define

$$g_{ij} \stackrel{\text{def}}{=} \mathrm{Tr}[h_{ij}^{QMC}\tilde{\rho}]\,, \qquad s_{ij} \stackrel{\text{def}}{=} \frac{1}{3}\mathrm{Tr}[(X_iX_j + Y_iY_j + Z_iZ_j)\tilde{\rho}]\,.$$

The first value is the energy obtained by $\tilde{\rho}$ on h_{ij}^{QMC}; the second value is the expected value of $\tilde{\rho}$ with respect to the traceless components of the term h_{ij}^{QMC}. It is straightforward to show that $g_{ij} = \frac{1}{2}(1-3s_{ij})$. The output of the SDP gives an upper bound to the optimal energy:

$$\lambda_{max}(H_G) \leq \sum_{(i,j)\in E} w_{ij}\cdot g_{ij} = \sum_{(i,j)\in E} w_{ij}\cdot\frac{1-3s_{ij}}{2}\,. \tag{13}$$

Fix a positive integer k and suppose Lemma 7 is true for $d = \frac{2k}{2k+1}$. Then, consider a *reweighting* $\widetilde{G}(V, E, u)$ of the graph G where $u_{ij} \geq 0$. Let $x \in \mathbb{R}^E$ be a vector with

$$x_{ij} \stackrel{\text{def}}{=} d\,(g_{ij}-1)^+ = d\left(-\frac{1+3s_{ij}}{2}\right)^+.$$

Here, we again use the notation $(\cdot)^+ \stackrel{\text{def}}{=} \max(\,\cdot\,, 0)$ from [14], which ensures

$$\left(-\frac{1+3s_{ij}}{2}\right)^+ \in [0,1]\,, \qquad \forall (i,j)\in E.$$

It is shown (e.g. in [8, Lemma 3.4]) that x is in the *integral* matching polytope of $\widetilde{G}$. This means that x is a convex combination of integral matchings $\{m_\ell\}_{1\leq\ell\leq k}$. Since the weight of any integral matching m_ℓ on $\widetilde{G}$ is at most $\mathrm{M}_{\widetilde{G}}$, the weight of x on $\widetilde{G}$ is also at most $\mathrm{M}_{\widetilde{G}}$. Thus,

$$\sum_{(i,j)\in E} u_{ij}\cdot d\cdot\left(-\frac{1+3s_{ij}}{2}\right)^+ \leq \mathrm{M}_{\widetilde{G}}\,. \tag{14}$$

Equation (14) in this context was first used in [16] and also appears in [14, Lemma 4] and [10, Lemma C.2].

We now find a lower bound for the energy obtained by the matching state $\rho_{PMATCH}(\theta)$ in PMATCH. We consider the energy on a case-by-case basis in the following lemma:

▶ **Lemma 9.** *Consider the state ρ_{PMATCH} and matching $\widetilde{M}$ from Algorithm 2. For $x \in \{0,1,2\}$, let $\widetilde{S}_x$ be the subset of edges not in $\widetilde{M}$ (e.g. $\widetilde{S}_x \subseteq E \setminus \widetilde{M}$) where each edge in $\widetilde{S}_x$ has x matched endpoints in $\widetilde{M}$. Then the energy of ρ_{PMATCH} on an edge (i,j) is exactly*

$$\mathrm{Tr}\left[h_{ij}^{QMC}\rho_{PMATCH}\right] = \begin{cases} \frac{1}{2}\left(1+\sin\theta\right)\left(1-t_{ij}\right), & (i,j)\in\widetilde{M}, \\ \frac{1}{2}(1-t_{ij}), & (i,j)\in\widetilde{S}_0, \\ \frac{1}{2}\left(1-t_{ij}\cos\theta\right), & (i,j)\in\widetilde{S}_1, \\ \frac{1}{2}\left(1-t_{ij}\cos^2\theta\right), & (i,j)\in\widetilde{S}_2. \end{cases}$$

where, t_{ij} is defined in Algorithm 2.

Proof. The case when $(i,j)\in\widetilde{M}$ follows directly from how we choose ρ_{PMATCH} in Algorithm 2, Step 4. For all other edges, recall that t_{ij} is the inner product of the Bloch vectors of qubits i and j with respect to the product state ρ_{PROD}. The reduced density matrix of ρ_{PMATCH} on qubits i and j is a product state $\sigma_i\otimes\sigma_j$. By [12, Lemma 10], the energy of $\sigma_i\otimes\sigma_j$ on h_{ij}^{QMC} is

$$\frac{1}{2}\left(1-\vec{B}(\sigma_i)\cdot\vec{B}(\sigma_j)\right).$$

We handle each remaining case separately:

1. $(i,j)\in\widetilde{S}_0$: Here, the product state is exactly $\rho_i\otimes\rho_j$, and so $\vec{B}(\sigma_i)\cdot\vec{B}(\sigma_j) = t_{ij}$ by definition.[1]
2. $(i,j)\in\widetilde{S}_1$: Without loss of generality, i belongs to a matched edge in $\widetilde{M}$ and j is not in a matched edge. Lemma 5 implies that $\vec{B}(\sigma_i)$ is rescaled to $\vec{B}(\sigma_i)\cos\theta$, and so $\vec{B}(\sigma_i)\cdot\vec{B}(\sigma_j) = t_{ij}\cos\theta$.
3. $(i,j)\in\widetilde{S}_2$: i and j belong to two different matched edges. Therefore, it has *both* Bloch vectors rescaled, i.e. $\vec{B}(\sigma_i)\cdot\vec{B}(\sigma_j) = \left(\vec{B}(\rho_i)\cdot\vec{B}(\rho_j)\right)\cos^2\theta = t_{ij}\cos^2\theta$. ◀

Lemma 9 allows us to compute the energy obtained by the state $\rho_{PMATCH}(\theta)$:

$$\begin{aligned}
&\mathrm{PMATCH}(\theta) \stackrel{\text{def}}{=} \sum_{(i,j)\in E} w_{ij}\cdot\mathrm{Tr}[h_{ij}^{QMC}\rho_{PMATCH}] \\
&= \sum_{(i,j)\in\widetilde{M}} \frac{w_{ij}}{2}\left(1+\sin\theta\right)\left(1-t_{ij}\right) + \sum_{(i,j)\in\widetilde{S}_0}\frac{w_{ij}}{2}\left(1-t_{ij}\right) \\
&\qquad + \sum_{(i,j)\in\widetilde{S}_1}\frac{w_{ij}}{2}\left(1-t_{ij}\cos\theta\right) + \sum_{(i,j)\in\widetilde{S}_2}\frac{w_{ij}}{2}\left(1-t_{ij}\cos^2\theta\right) \\
&= \sum_{(i,j)\in E}\frac{w_{ij}}{2}\left(1-t_{ij}\cos^2\theta\right) + \sum_{(i,j)\in\widetilde{M}}\frac{w_{ij}}{2}\left(\left(1+\sin\theta\right)\left(1-t_{ij}\right)-\left(1-t_{ij}\cos^2\theta\right)\right) \\
&\qquad + \sum_{(i,j)\in\widetilde{S}_0}\frac{w_{ij}t_{ij}}{2}\left(\cos^2\theta-1\right) + \sum_{(i,j)\in\widetilde{S}_1}\frac{w_{ij}t_{ij}}{2}\left(\cos^2\theta-\cos\theta\right) \\
&= \sum_{(i,j)\in E}\frac{w_{ij}}{2}\left(1-t_{ij}\cos^2\theta\right) + \sum_{(i,j)\in\widetilde{M}}\frac{w_{ij}}{2}\left(\sin\theta\left(1-t_{ij}\left(1+\sin\theta\right)\right)\right) \\
&\qquad - \sum_{(i,j)\in\widetilde{S}_0}\frac{w_{ij}t_{ij}}{2}\sin^2\theta + \sum_{(i,j)\in\widetilde{S}_1}\frac{w_{ij}t_{ij}}{2}\left(\cos^2\theta-\cos\theta\right). \qquad (15)
\end{aligned}$$

[1] Although $\widetilde{M}$ is a maximum matching, its edge set E' is a subset of E, and so $\widetilde{S}_0$ may be non-empty.

Note that $\widetilde{M}$ is a maximum-weight *integral* matching with respect to the rescaled weights defined in Equation (4). Thus, we can invoke Equation (14) to replace the sum over $\widetilde{M}$ in Equation (15) with a sum over E. Since the rescaled weights are positive for all edges in $\widetilde{M}$, we have

$$\begin{aligned}\mathrm{PMATCH}(\theta) \geq \sum_{(i,j)\in E} \frac{w_{ij}}{2}\left(1 - t_{ij}\cos^2\theta + d\big(\sin\theta\,(1 - t_{ij}\,(1+\sin\theta))\big)\left(-\frac{1+3s_{ij}}{2}\right)^+\right) \\ - \sum_{(i,j)\in\widetilde{S}_0} \frac{w_{ij}t_{ij}}{2}\sin^2\theta + \sum_{(i,j)\in\widetilde{S}_1} \frac{w_{ij}t_{ij}}{2}\left(\cos^2\theta - \cos\theta\right). \end{aligned} \tag{16}$$

Note that t_{ij} is a random variable because PROD is a randomized algorithm. The analysis of PROD in [7] is based on [4], which shows that $\mathbb{E}[t_{ij}] = F(s_{ij})$. A definition of F in the context of QMC appears in [17, Equation 3]. For our purposes, it is enough to note that F is an odd function because it has the form

$$F(s) = c \cdot s \cdot G(s^2)\,, \tag{17}$$

where $c > 0$ is a constant and $G \geq 0$ is a hypergeometric function. Since Equation (16) is linear in t_{ij}, we use linearity of expectation to conclude

$$\begin{aligned}&\mathbb{E}[\mathrm{PMATCH}(\theta)] \geq \\ &\sum_{(i,j)\in E} \frac{w_{ij}}{2}\left(1 - F(s_{ij})\cos^2\theta + d\sin\theta\,(1 - F(s_{ij})\,(1+\sin\theta))\left(-\frac{1+3s_{ij}}{2}\right)^+\right) \\ &\quad - \sum_{(i,j)\in\widetilde{S}_0} \frac{w_{ij}}{2}F(s_{ij})\sin^2\theta + \sum_{(i,j)\in\widetilde{S}_1} \frac{w_{ij}}{2}\Big(\big(\cos^2\theta - \cos\theta\big)\;F(s_{ij})\Big). \end{aligned} \tag{18}$$

The following lemma lets us analyze the performance of Algorithm 3, which takes the better of two algorithms, MATCH and PMATCH.

▶ **Lemma 10** (Reducing worst-case bounds to a single edge). *Suppose we have a collection of k approximation algorithms $\{A_\ell\}_{\ell\in[k]}$ and an upper bound on the maximum energy $\lambda_{max}(H_G) \leq \sum_{e\in E} w_e b_e$, where $b_e \in B = (0, b_{max}]$ for all $e \in E$, and b_{max} is a constant. Furthermore, suppose that the energy that algorithm A_ℓ earns on edge e is a function of b_e, which we denote $a_\ell(b_e)$, such that A_ℓ earns $\sum_{e\in E} w_e\, a_\ell(b_e)$. Then, by running each of $A_{\ell\in[k]}$ and taking the output with the maximum energy, we can obtain an approximation ratio*

$$\alpha \geq \max_{\mu_\ell}\ \min_{b} \sum_{\ell} \mu_\ell \frac{a_\ell(b)}{b},$$

where the maximum is taken over valid probability distributions

$$\{\ \mu \mid \sum_{\ell\in[k]} \mu_\ell = 1,\ 0 \leq \mu_\ell \leq 1\ \forall \ell \in [k]\ \}\,,$$

and the minimum is taken over $b \in B$.

Proof. Given a specific graph G, outputting the best of A_ℓ is at least as good as outputting the result of A_ℓ with probability μ_ℓ, for all $\ell \in [k]$ and for any possible μ. Now, given a fixed distribution μ, the approximation ratio is at least

$$\begin{aligned}
\alpha &\geq \sum_\ell \mu_\ell \frac{\sum_{e\in E} w_e\, a_\ell(b_e)}{\sum_{e\in E} w_e b_e} \\
&= \sum_\ell \mu_\ell \sum_{e\in E} \left(\frac{w_e b_e}{\sum_{e\in E} w_e b_e}\right) \frac{a_\ell(b_e)}{b_e} \\
&\geq \min_{e\in E} \sum_\ell \mu_\ell \frac{a_\ell(b_e)}{b_e} \\
&\geq \min_{b\in B} \sum_\ell \mu_\ell \frac{a_\ell(b)}{b}\,.
\end{aligned}$$

The third line follows because

$$\left\{\frac{w_e b_e}{\sum_{e\in E} w_e b_e}\right\}_{e\in E}$$

is a distribution over E. Since the statement is true for all possible distributions μ, we take the maximum over μ to finish the lemma. ◀

We apply Lemma 10 with the SDP upper bound in Equation (13) and our two algorithms MATCH and PMATCH as described in Algorithm 3. For PMATCH, we consider the minimum of three different functions, depending on if an edge is in $\widetilde{S}_0$, in $\widetilde{S}_1$, or otherwise. For MATCH, we derived in Section 2.3 that the energy obtained by MATCH on G is $\frac{3\mathrm{M}+W}{2}$. Since M is a maximum-weight matching with respect to G, we may invoke Equation (14) to express

$$\mathrm{M} \geq \sum_{(i,j)\in E} w_{ij} \cdot d \cdot \left(-\frac{1+3s_{ij}}{2}\right)^+,$$

implying that MATCH achieves energy at least

$$\frac{3\mathrm{M}+W}{2} \geq \sum_{(i,j)\in E} \frac{w_{ij}}{2}\left(1+3d\left(-\frac{1+3s_{ij}}{2}\right)^+\right).$$

We have lower-bounded the energy that each algorithm earns on an edge (i,j) as a function of s_{ij}. Note that the SDP upper bound is trivial (i.e. at most 0) when $s_{ij} \geq \frac{1}{3}$, so we may search over $s \in [-1, 1/3)$. Altogether, Lemma 10 implies that Algorithm 3 obtains approximation ratio at least

$$\begin{aligned}
\alpha \geq \max_{\mu\in[0,1],\,\theta\in[0,\pi/2]} \min\Big\{ &\min_{s\in[-1,1/3)} \ \mu\, E_{PM}(\theta) + (1-\mu)E_M, \\
&\min_{s\in[-1,1/3)} \ \mu\, E'_{PM}(\theta) + (1-\mu)E_M, \\
&\min_{s\in[-1,1/3)} \ \mu\, E''_{PM}(\theta) + (1-\mu)E_M \Big\},
\end{aligned} \tag{19}$$

$$E_{PM}(\theta) \stackrel{\text{def}}{=} \frac{1 - F(s)\cos^2\theta + d\sin\theta\,(1 - F(s)\,(1+\sin\theta)) \cdot \left(-\frac{1+3s}{2}\right)^+}{1-3s}, \tag{20}$$

$$E'_{PM}(\theta) \stackrel{\text{def}}{=} \frac{1 - F(s)\cos\theta + d\sin\theta\,(1 - F(s)\,(1+\sin\theta)) \cdot \left(-\frac{1+3s}{2}\right)^+}{1-3s}, \tag{21}$$

$$E''_{PM}(\theta) \stackrel{\text{def}}{=} \frac{1 - F(s) + d\sin\theta\,(1 - F(s)\,(1+\sin\theta)) \cdot \left(-\frac{1+3s}{2}\right)^+}{1-3s}, \tag{22}$$

$$E_M \stackrel{\text{def}}{=} \frac{1 + 3\,d\,(-\frac{1+3s}{2})^+}{1-3s}. \tag{23}$$

In the above, μ is demoted to a scalar (which fully describes a probability distribution over two variables). Equation (23) is the approximation ratio obtained by MATCH on an edge with $s = s_{ij}$. The expected approximation ratio obtained by PMATCH is Equation (22) on an edge in $\widetilde{S}_0$, Equation (21) on an edge in $\widetilde{S}_1$, and is Equation (20) on all other edges.

We further simplify Equation (19). Recall that F is an odd function (Equation (17)). So, for all $\theta \in [0, \pi/2]$ we have $E''_{PM}(\theta) \le E'_{PM}(\theta) \le E_{PM}(\theta)$ when $s > 0$ and $E_{PM}(\theta) \le E'_{PM}(\theta) \le E''_{PM}(\theta)$ when $s \le 0$. So we may rewrite Equation (19) as

$$\alpha \ge \max_{\mu\in[0,1],\,\theta\in[0,\pi/2]} \min\left\{ \min_{s\in[-1,0)} \mu\,E_{PM}(\theta) + (1-\mu)E_M,\ \min_{s\in[0,1/3)} \mu\,E''_{PM}(\theta) + (1-\mu)E_M \right\}. \tag{24}$$

One may then search over the three free parameters μ, θ, and s using an enumeration of the feasible ranges. For a more efficient approach, it is also possible to solve a linear program where the number of variables and constraints is based on an enumeration of the feasible ranges for s and θ. We obtain $\alpha > 0.611$ for $d = 14/15$ at $\theta = 1.286$ and $\mu = 0.861$. By Lemma 7, our algorithm thus achieves a 0.611-approximation in expectation if $\ell = 13$. If the factor d in Lemma 7 is improved to 1, we would obtain $\alpha > 0.614$ at $\theta = 1.288$ and $\mu = 0.881$.

▶ Remark 11. We may simplify Equation (19) by assuming $s \in [-1, 0)$. This is because if $s \in [0, 1/3)$ then $E_M \ge 1$ and $E''_{PM}(\theta) \ge 1$ for all $\theta \in [0, \pi/2]$. This follows from F being odd, and in addition $|F(s)| \le |s|$ for all $s \in [-1, 1]$.

5 Discussion

In this work, we introduce two novel techniques for constructing algorithms for the Quantum MaxCut and EPR Hamiltonians on arbitrary graphs. The main techniques are a) *partially* entangling edges in a matching towards an entangled state while maintaining the direction of the marginal Bloch vectors as b) choosing matching with respect to *reweighted* graphs. We show that these techniques immediately lead to state-of-the-art approximation ratios for both problems. We believe that these techniques may be combined with other methods to obtain further improvements, and encourage further studies in this direction.

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