

Interpolation and Separation Problems for Linear Temporal Logics

Michael Zakharyashev 

School of Computing and Mathematical Sciences, Birkbeck, University of London, UK

Abstract

The talk gives a survey of recent results on two types of problems for linear temporal logics: one is related to the existence of a Craig interpolant for a given implication in a temporal logic, the other to the existence of a temporal query separating two given sets of temporal data instances.

2012 ACM Subject Classification Theory of computation → Modal and temporal logics

Keywords and phrases Linear temporal logic, Craig interpolation, query-by-example

Digital Object Identifier 10.4230/LIPIcs.TIME.2025.1

Category Invited Talk

1 Extended Abstract

This talk discusses two types of problems that have recently been posed and investigated in the context of linear time temporal logics.

The first problem is related to Craig interpolation. Recall that a logic L has the *Craig interpolation property* (CIP, for short) if, for any two formulas φ and ψ in the language of L , whenever $\varphi \rightarrow \psi$ is valid in L , then there exists a formula χ , constructed from the common non-logical symbols of φ and ψ , such that both $\varphi \rightarrow \chi$ and $\chi \rightarrow \psi$ are valid in L . Such a formula χ is called an *interpolant of φ and ψ* in L . Classical and intuitionistic propositional and first-order logics, standard modal and description logics as well as numerous other logics enjoy the CIP. The vast majority of temporal logics do not, including all logics with the operators $\Diamond$ (some time in the future) and $\Box$ (always in the future) interpreted over any class of connected Kripke frames of unbounded depth, their Priorean extensions with the past-time operators, and full linear temporal logic LTL over any interesting time line.

A different, non-uniform approach to Craig interpolation in logics L without the CIP was suggested in [4] by posing the following *interpolant existence problem* (IEP): given any formulas φ and ψ in the language of L , decide whether there exists an interpolant χ for φ and ψ in L . For L with the CIP, interpolant existence reduces to validity; for L without the CIP, the IEP is typically harder as far as computational complexity is concerned.

The first part of the talk gives a survey of the recent results on the IEP in temporal logics obtained in [5, 6]. In particular, it discusses how a semantic criterion of interpolant existence in terms of descriptive frames can be used to design a CONP-algorithm that is capable of deciding the IEP for any given finitely axiomatisable temporal logic with the operators $\Diamond$ and $\Box$. In this case, the IEP is as complex as the decision problem for the logic. For LTL over finite strict linear orders, the talk discusses how the IEP can be reduced to the following separation problem: given two regular languages $\mathcal{L}_1$ and $\mathcal{L}_2$, decide whether there is a first-order definable (= star-free) language $\mathcal{L}$ separating $\mathcal{L}_1$ and $\mathcal{L}_2$ in the sense that $\mathcal{L}_1 \subseteq \mathcal{L}$ and $\mathcal{L}_2 \cap \mathcal{L} = \emptyset$. A classical result of [7] shows that deciding first-order separability of two regular languages is in 2EXPTIME in the size of the DFAs specifying the given languages. Thus, the IEP for LTL is decidable in 4EXPTIME, being PSPACE-hard. The same bounds hold for LTL over $(\mathbb{N}, <)$. In both cases, the exact complexity remains open.



© Michael Zakharyashev;

licensed under Creative Commons License CC-BY 4.0

32nd International Symposium on Temporal Representation and Reasoning (TIME 2025).

Editors: Thierry Vidal and Przemysław Andrzej Wałęga; Article No. 1; pp. 1:1–1:2

Leibniz International Proceedings in Informatics



LIPICs Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

The second part of the talk is concerned with the scenario where temporal models over finite strict linear orders represent timestamped (qualitative) data. We discuss the reverse engineering (or query-by-example) setting that deals with pairs (E^+, E^-) of finite sets of “positive” and “negative” data examples. An LTL-formula (query) q *separates* (E^+, E^-) if $\mathfrak{M}, 0 \models q$ for all $\mathfrak{M} \in E^+$, and $\mathfrak{M}, 0 \not\models q$ for all $\mathfrak{M} \in E^-$. Based on the results from [1, 2, 3], we give a survey of recent developments in the study of the following problems:

- Given any pair (E^+, E^-) of positive and negative examples and a class $\mathcal{Q}$ of LTL-queries,
1. How hard is it to decide whether (E^+, E^-) is separable by a query in $\mathcal{Q}$?
 2. How hard is it to compute a $\mathcal{Q}$ -separator of (E^+, E^-) , which is (i) shortest/longest or (ii) most specific (logically strongest)/most general (weakest) $\mathcal{Q}$ -separator?
 3. How hard is it to decide whether there is a unique most specific/general $\mathcal{Q}$ -separator?
 4. Given a $\mathcal{Q}$ -separator q of (E^+, E^-) , how hard is it to decide whether q is (i) shortest/longest, (ii) most general/specific, (iii) unique, etc.?
 5. Given any $q \in \mathcal{Q}$, how hard is it to decide whether there is (E^+, E^-) that *uniquely characterises* q in $\mathcal{Q}$ in the sense that $q' \in \mathcal{Q}$ separates (E^+, E^-) iff $q' \equiv q$?
 6. Does there exist a *polynomial-size* unique characterisation of a given $q \in \mathcal{Q}$?

Interesting and practically relevant classes $\mathcal{Q}$ consist of conjunctive LTL formulas. We also outline generalisations of the problems above to combinations of LTL and description logics.

References

- 1 Marie Fortin, Boris Konev, Vladislav Ryzhikov, Yury Savateev, Frank Wolter, and Michael Zakharyashev. Unique characterisability and learnability of temporal instance queries. In Gabriele Kern-Isberner, Gerhard Lakemeyer, and Thomas Meyer, editors, *Proceedings of the 19th International Conference on Principles of Knowledge Representation and Reasoning, KR 2022, Haifa, Israel, July 31 - August 5, 2022*, 2022. URL: <https://proceedings.kr.org/2022/17/>.
- 2 Jean Christoph Jung, Vladislav Ryzhikov, Frank Wolter, and Michael Zakharyashev. Temporalising unique characterisability and learnability of ontology-mediated queries (extended abstract). In Oliver Kutz, Carsten Lutz, and Ana Ozaki, editors, *Proceedings of the 36th International Workshop on Description Logics (DL 2023) co-located with the 20th International Conference on Principles of Knowledge Representation and Reasoning and the 21st International Workshop on Non-Monotonic Reasoning (KR 2023 and NMR 2023)*, Rhodes, Greece, September 2-4, 2023, volume 3515 of *CEUR Workshop Proceedings*. CEUR-WS.org, 2023. URL: <https://ceur-ws.org/Vol-3515/abstract-13.pdf>.
- 3 Jean Christoph Jung, Vladislav Ryzhikov, Frank Wolter, and Michael Zakharyashev. Extremal separation problems for temporal instance queries. In *Proceedings of the Thirty-Third International Joint Conference on Artificial Intelligence, IJCAI 2024, Jeju, South Korea, August 3-9, 2024*, pages 3448–3456. ijcai.org, 2024. URL: <https://www.ijcai.org/proceedings/2024/382>.
- 4 Jean Christoph Jung and Frank Wolter. Living without beth and craig: Definitions and interpolants in the guarded and two-variable fragments. In *36th Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2021, Rome, Italy, June 29 - July 2, 2021*, pages 1–14. IEEE, 2021. doi:10.1109/LICS52264.2021.9470585.
- 5 Agi Kurucz, Frank Wolter, and Michael Zakharyashev. A non-uniform view of Craig interpolation in modal logics with linear frames. *CoRR*, abs/2312.05929, 2023. doi:10.48550/arXiv.2312.05929.
- 6 Agi Kurucz, Frank Wolter, and Michael Zakharyashev. From interpolating formulas to separating languages and back again, 2026. To appear, preprints accessible from <https://cibd.bitbucket.io/taci/>.
- 7 Thomas Place and Marc Zeitoun. Separating regular languages with first-order logic. *Log. Methods Comput. Sci.*, 12(1), 2016. doi:10.2168/LMCS-12(1:5)2016.