Amnesiac Flooding: Easy to Break, Hard to Escape

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Abstract

Broadcast is a central problem in distributed computing. Recently, Hussak and Trehan [PODC'19/STACS'20/DC'23] proposed a stateless broadcasting protocol (Amnesiac Flooding), which was surprisingly proven to terminate in asymptotically optimal time (linear in the diameter of the network). However, it remains unclear: (i) Are there other stateless terminating broadcast algorithms with the desirable properties of Amnesiac Flooding, (ii) How robust is Amnesiac Flooding with respect to *faults*?

In this paper we make progress on both of these fronts. Under a reasonable restriction (obliviousness to message content) additional to the fault-free synchronous model, we prove that Amnesiac Flooding is the *only* strictly stateless deterministic protocol that can achieve terminating broadcast. We achieve this by identifying four natural properties of a terminating broadcast protocol that Amnesiac Flooding uniquely satisfies. In contrast, we prove that even minor relaxations of *any* of these four criteria allow the construction of other terminating broadcast protocols.

On the other hand, we prove that Amnesiac Flooding can become non-terminating or non-broadcasting, even if we allow just one node to drop a single message on a single edge in a single round. As a tool for proving this, we focus on the set of all *configurations* of transmissions between nodes in the network, and obtain a *dichotomy* characterizing the configurations, starting from which, Amnesiac Flooding terminates. Additionally, we characterise the structure of sets of Byzantine agents capable of forcing non-termination or non-broadcast of the protocol on arbitrary networks.

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1 Introduction

The dissemination of information to disparate participants is a fundamental problem in both the construction and theory of distributed systems. A common strategy for solving this problem is to "broadcast", i.e. to transmit a piece of information initially held by one agent to all other agents in the system [1, 18, 21, 24, 25]. In fact, broadcast is not merely

a fundamental communication primitive in many models, but also underlies solutions to other fundamental problems such as leader election and wake-up. Given this essential role in the operation of distributed computer systems and the potential volume of broadcasts, an important consideration is simplifying the algorithms and minimizing the overhead required for each broadcast.

Within a synchronous setting, *Amnesiac Flooding* as introduced by Hussak and Trehan in 2019 [13, 14] eliminates the need of the standard flooding algorithm to store historical messages. The algorithm terminates in asymptotically optimal $O(D)$ time (for D the diameter of the network) and is stateless as agents are not required to hold any information between communication rounds. The algorithm in the fault-free synchronous message passing model is defined as follows:

▶ **Definition 1** (Amnesiac flooding algorithm, adapted from [16]). *Let $G = (V, E)$ be an undirected graph, with vertices V and edges E (representing a network where the vertices represent the nodes of the network and edges represent the connections between the nodes). Computation proceeds in synchronous "rounds" where each round consists of nodes receiving messages sent from their neighbours. A receiving node then sends messages to some neighbours which arrive in the next round. No messages are lost in transit. The algorithm is defined by the following rules:*

- **(i)** *All nodes from a subset of sources or initial nodes $I \subseteq V$ send a message M to all of their neighbours in round 1.*
- **(ii)** *In subsequent rounds, every node that received M from a neighbour in the previous round, sends M to all, and only, those nodes from which it did not receive M. Flooding* terminates *when M is no longer sent to any node in the network.*

These rules imply several other desirable properties. Firstly, the algorithm only requires the ability to forward the messages, but does not read the content (or even the header information) of any message to make routing decisions. Secondly, the algorithm only makes use of local information and does not require knowledge of a unique identifier. Thirdly, once the broadcast has begun, the initial broadcaster may immediately forget that they started it.

However, extending Amnesiac Flooding and other stateless flooding algorithms (such as those proposed in [27, 29, 4]) beyond synchronous fault-free scenarios is challenging. This is due to the fragility of these algorithms and their inability to build in complex fault-tolerance due to the absence of state and longer term memory. It has subsequently been shown that no stateless flooding protocol can terminate under moderate asynchrony, unless it is allowed to perpetually modify a super-constant number (i.e. $\omega(1)$) of bits in each message [27]. Yet, given the fundamental role of broadcast in distributed computing, the resilience of these protocols is extremely important even on synchronous networks.

Outside of a partial robustness to crash failures, the fault sensitivity of Amnesiac Flooding under synchrony has not been explored in the literature. This omission is further compounded by the use of Amnesiac Flooding as an underlying subroutine for the construction of other broadcast protocols. In particular, multiple attempts have been made to extend Amnesiac Flooding to new settings (for example routing multiple concurrent broadcasts [4] or flooding networks without guaranteed edge availability [29]), while maintaining its desirable properties. However, none have been entirely successful, typically requiring some state-fulness. It has not in fact been established that *any other* protocol can retain all of Amnesiac Flooding's remarkable properties even in its original setting. These gaps stem fundamentally from the currently limited knowledge of the dynamics of Amnesiac Flooding beyond the fact of its termination and its speed to do so. In particular, all of the existing techniques (e.g. parity

arguments such as in [16] and auxiliary graph constructions such as in [28]) used to obtain termination results for Amnesiac Flooding are unable to consider faulty executions of the protocol and fail to capture the underlying structures driving terminating behaviour.

We address these gaps through the application of novel analysis and by considering the structural properties of Amnesiac Flooding directly. By considering the sequence of message configurations, we are able to identify the structures underlying Amnesiac Flooding's termination and use these to reason about the algorithm in arbitrary configurations. The resulting dichotomy gives a comprehensive and structured understanding of termination in Amnesiac Flooding. For example, we apply this to investigate the sensitivity of Amnesiac Flooding with respect to several forms of fault and find it to be quite fragile. Furthermore, we show that under reasonable assumptions on the properties of a synchronous network, any strictly stateless deterministic terminating broadcast algorithm oblivious to the content of messages, must produce the exact same sequence of message configurations as Amnesiac Flooding on any network from any initiator. We therefore argue that Amnesiac Flooding is unique. However, we show that if any of these restrictions are relaxed, even slightly, distinct terminating broadcast algorithms can be obtained. As a result of this uniqueness and simplicity, we argue that Amnesiac Flooding represents a prototypical broadcast algorithm. This leaves open the natural question: do there exist fundamental stateless algorithms underlying solutions to other canonical distributed network problems? Though memory can be essential or naturally useful in certain scenarios [5, 7, 8, 10, 17, 20], understanding what we can do with statelessness can help us push fundamental boundaries.

1.1 Our Contributions

In this work, we investigate the existence of other protocols possessing the following four desirable properties of Amnesiac Flooding:

1. *Strict Statelessness*: Nodes maintain no information other than their port labellings between rounds. This includes whether or not they were in the initiator set.
2. *Obliviousness*: Routing decisions may not depend on the contents of received messages.
3. *Determinism*: All decisions made by a node must be deterministic.
4. *Unit Bandwidth*: Each node may send at most one message per edge per round.

Our main technical results regarding the existence of alternative protocols to Amnesiac Flooding are given in the next two theorems (reworded in Section 4).

▶ **Theorem 2** (Uniqueness of Amnesiac Flooding). *Any terminating broadcast algorithm possessing all of* Strict Statelessness*,* Obliviousness*,* Determinism *and* Unit Bandwidth *behaves identically to Amnesiac Flooding on all graphs under all valid labellings for all source nodes.*

Note that the last theorem allows, but does not require, that nodes have access to unique identifiers labelling themselves and their ports. However, we enforce the condition that these identifiers, should they exist, may be drawn adversarially from some super set of $[n + \kappa]$ where n is the number of nodes on the networks and $\kappa = R(9, 8)$ where $R(9, 8)$ is a Ramsey Number (≤ 2662)[22]. It is important to stress here that Theorem 2 holds even if the space of unique identifiers is only greater than n by an additive constant. In contrast to the last theorem, the next one does not assume that agents have access to unique identifiers.

▶ **Theorem 3** (Existence of relaxed Algorithms). *There exist terminating broadcast algorithms which behave distinctly from Amnesiac Flooding on infinitely many networks possessing any three of:* Strict Statelessness*,* Obliviousness*,* Determinism *and* Unit Bandwidth*.*

We derive four of these relaxed algorithms which all build upon Amnesiac Flooding:

1. Neighbourhood-2 Flooding: *Strict Statelessness* is relaxed and agents are given knowledge of their neighbours' neighbours. This allows for distinct behaviour on networks of radius one as some agents are aware of the entire network topology.
2. Random Flooding: *Determinism* is relaxed and agents are given access to a random coin. Agents randomly choose in each round whether to use Amnesiac Flooding or to forward messages to all of their neighbours.
3. 1-Bit Flooding (Message Dependent): *Obliviousness* is relaxed and the source is allowed to include one bit of read-only control information in the messages. The source records in the message whether they are a leaf vertex, and if so agents perform Amnesiac Flooding upon receiving the message. Otherwise, agents perform a modified version where leaf vertices return messages.
4. 1-Bit Flooding (High Bandwidth): *Unit Bandwidth* is relaxed and agents are allowed to send either one or two messages over an edge. This permits the same algorithm as the previous case by encoding the control information in the number of messages sent.

We note that despite being a non-deterministic algorithm Random Flooding achieves broadcast with certainty and terminates almost surely in finite time.

We also perform a comprehensive investigation of the fault sensitivity of Amnesiac Flooding in a synchronous setting. Through the use of a method of invariants, we obtain stronger characterizations of termination than were previously known, for both Amnesiac Flooding, and a subsequently proposed Stateless Flooding protocol [27]. This allows us to provide precise characterizations of the behaviour of Amnesiac Flooding under the loss of single messages, uni-directional link failure, and time bounded Byzantine failures. The above invariants may be of independent interest, beyond fault sensitivity, as they provide strong intuition for how asynchrony interferes with the termination of both Amnesiac Flooding and the Stateless Flooding proposed in [27]. The main technical result concerning fault sensitivity is a *dichotomy* characterizing the configurations, starting from which, Amnesiac Flooding terminates. As the rigorous statement of the result requires some additional notation and terminology, we will state it only informally here. We show in Theorem 15 that, whether Amnesiac Flooding terminates when begun from a configuration or not, is determined exclusively by the parity of messages distributed around cycles and so-called faux-even cycles (FEC) (essentially pairs of disjoint odd cycles connected by a path, see section 5 for a full definition) within the graph. It follows from this characterisation that Amnesiac Flooding terminates from a configuration if and only if it the configuration can be reached from some sequence of multi-casts. Theorem 15 also implies the following three theorems which demonstrate the fragility of Amnesiac Flooding under three increasingly strong forms of fault. We give our fault model explicitly in Section 5.2.

▶ **Theorem 4** (Single Message Failure). *If single-source Amnesiac Flooding experiences a* single message drop failure *for the message* (u, v) *then it fails to terminate if and only if either or both of the following hold:*

- *uv is not a bridge*
- *uv lies on a path between vertex disjoint odd cycles*

Moreover, it fails to broadcast if and only if this is the first message sent along uv, uv is a bridge, and the side of the cut containing u does not contain an odd cycle.

We also consider the possibility of a link/edge failing in one direction (or if we consider an undirected edge as two directed links in opposite directions, only one of the directed links fails).

▶ **Theorem 5** (Uni-directional link failure). *For any graph $G = (V, E)$ and any initiator set $I \subsetneq V$ there exists an edge $e \in E$ such that a uni-directional link failure at e will cause Amnesiac Flooding to either fail to broadcast or fail to terminate when initiated from I on G. Furthermore, for any non-empty set of uni-directional link failures there exists $v \in V$ such that, when Amnesiac Flooding is initiated at v, it will either fail to broadcast or fail to terminate.*

We, finally, consider a set of Byzantine nodes, who know the original message, are free to collude among themselves and may decide to forward this message in any arbitrary pattern to their neighbours. However, for a discussion of termination to be meaningful, we require that the nodes have Byzantine behaviour for only a finite number of rounds.

▶ **Theorem 6** (Byzantine Failure). *If Amnesiac Flooding on $G = (V, E)$ initiated from $I \subsetneq V$ experiences a* weak Byzantine failure *at $J \subseteq V \setminus I$, then the adversary can force:*

- *Failure to broadcast if and only if J contains a cut vertex set.*
- *Non-termination if and only if at least one member of J lies on either a cycle or a path between odd-cycles.*

Two natural corollaries of the Theorem 4 we wish to highlight here are: (i) on any network from any initial node there exists a single message, the dropping of which, will produce either non-termination or non-broadcast and (ii) dropping any message on a bipartite network will cause either non-termination or non-broadcast. The latter requires the additional observation that each edge is traversed by a message precisely once in a bipartite graph under Amnesiac Flooding. Similarly, Theorem 6 implies that any Byzantine set containing a non-leaf node on any network can force either non-termination or non-broadcast for Amnesiac Flooding with any initial set.

1.2 Organisation of the paper

The initial part of the paper presents the required technical and motivational background, statements of our results and a technical outline of the more interesting proofs. Due to space limitations detailed proofs are deferred to the appendices and the full version [2]. Related work is presented in Section 2. Section 3 presents the model and notation required for the following technical sections. *Uniqueness* of the algorithm is discussed in Section 4. In subsection 4.1 we discuss the conditions under which the algorithm is unique and in subsection 4.2 we relax the conditions individually to derive additional algorithms. The proof of the uniqueness result is given in Appendix A. We present our work on the *termination dichotomy* for Amnesiac Flooding and its applications to *fault sensitivity* in Section 5 with the proof of the dichotomy given in Appendix B. We end with our conclusions and pointers to future work in Section 6.

2 Related Work

The literature surrounding both the broadcast problem and fault sensitivity is vast, and a summary of even their intersection is well beyond the scope of this work. Instead, we shall focus on work specifically concerning stateless, or nearly stateless broadcast.

The termination of Amnesiac Flooding and its derivatives has been the focus of several works. In combination they provide the following result that Amnesiac Flooding terminates under any sequence of multi-casts, i.e.

▶ **Theorem 7** (Termination of Amnesiac Flooding (adapted from [15, 16, 28])). *For $G = (V, E)$ a graph and $I_1, ..., I_k \subseteq V$ a sequence of sets of initiator nodes, Amnesiac Flooding on G terminates when initiated from I_1 in round 1, then I_2 in round 2 and so forth.*

Two independent proofs of the algorithm's termination have been presented, using either parity arguments over message return times [16] or axillary graph constructions [28]. The latter technique has further been used to establish tight diameter independent bounds on the termination of multicast using Amnesiac Flooding, complementing the eccentricity based bounds of [16]. The techniques we develop in this work, however, are more closely aligned with those of [15], as we exploit a similar notion to their "even flooding cycles" in our message path argument. In contrast to that work we focus on arbitrary configurations of messages, rather than just those resulting purely from a correct broadcast which allows us to obtain a stronger characterisation. Combining our techniques with the dual "reverse" flood introduced by [15], we are able to show the complement of Theorem 7, that only those configurations reachable from a sequence of multi-casts lead to termination.

There have, further, been multiple variants of Amnesiac Flooding introduced. It was observed by [27] in a result reminiscent of the BASIC protocol proposed by [12], that sending a second wave of messages from a subset of the initial nodes reduces the worst case $2D + 1$ termination time to the optimal $D + 1$ in all but a specific subset of bipartite graphs. We note that our fault sensitivity results extend naturally to this algorithm as well, as the same invariants apply to this setting. Beyond this, there have been several approaches to deal with the flooding of multiple messages simultaneously. In [15], the original authors of [13] show that under certain conditions termination can be retained, even when conflicting floods occur. Since then, two partially stateless algorithms have been proposed, both making use of message buffers and a small amount of local memory [29, 4]. We will not be directly concerned with these approaches, however, as we assume a single concurrent broadcast throughout. However, the mechanism employed in [29] should be highlighted as it rather cleverly exploits the underlying parity properties we identify as driving termination. Furthermore, as reduction to Amnesiac Flooding is used as a technique for proofs in many of these works, the comprehensive understanding of its termination we present here could prove a powerful tool for future work in these areas.

While the robustness of Amnesiac flooding and its variants have been previously studied, this has been focused on two forms of fault. The first is the disappearance and reappearance of nodes and links. The termination of Amnesiac Flooding is robust to disappearance and vulnerable to reappearance [16]. We will observe that this is a necessary consequence of the invariants driving termination and their relation to cyclicity. In particular, the disappearance of nodes and links cannot form new cycles violating the invariant, whereas their reappearance can. A pseudo-stateless extension to Amnesiac Flooding has been proposed to circumvent this [29], implicitly exploiting the parity conditions of [16]. The second are faults that violate synchrony. Under a strong form of asynchrony, truly stateless and terminating broadcast is impossible [27]. However, the landscape under a weaker form of asynchrony (namely, the case of fixed delays on communication links) is more fine-grained. Although termination results have been obtained for cycles, as well as the case of single delayed edges in bipartite graphs [16], there is no clear understanding of the impact of fixed channel delays. While we do not directly address this, we believe that techniques mirroring our invariant

characterizations may prove fruitful in this area. To our knowledge, this work is the first to consider both the uniqueness of Amnesiac Flooding, as well as its fault sensitivity beyond node/link unavailability in a synchronous setting.

Beyond Amnesiac Flooding and its extensions, the role of memory in information dissemination is well studied in a variety of contexts. Frequently, stateful methods obtain faster termination time, such as in the phone-call model where the ability to remember one's communication partners and prevent re-communication dramatically improves termination time and message efficiency [10, 5, 8]. Similarly, for bit dissemination in the passive communication model the addition of only $\log \log n$ bits of memory is sufficient to break the near linear time convergence lower bound of [7] and achieve polylogarithmic time [17]. Even more strongly, a recent work [20] has shown that in the context of synchronous anonymous dynamic networks, stabilizing broadcast from an idle start is impossible with $O(1)$ memory and even with $o(\log n)$ memory if termination detection is required. Despite this, low memory and even stateless broadcasts remain desirable [12]. The possibility of solving other canonical distributed computing problems beyond broadcast, in a stateless manner, remains intriguing. In this direction, various low memory stateful models have been proposed to handle more complex distributed problems e.g. the compact local streaming (CLS) model in [6] with deterministic solutions (routing, self-healing fault-tolerance etc.) and randomised solutions for distributed colouring in a similar model [11].

Stateless broadcast schemes have been studied in a variety of contexts. To give an example, they are used in mobile ad hoc networks, which, due to the lower power and rapid movement of devices, see diminishing returns from maintaining information about the network [19]. However, given a lack of synchronisation as well as the wish to avoid so-called broadcast storms [26], these techniques typically rely on either some form of global knowledge (such as the direction or distance to the initiator) or the ability to sample network properties by eavesdropping on communications over time [3, 23]. It should be noted however, that in contrast to many models, such as anonymous dynamic networks, radio networks and many mobile ad hoc networks, the typical framework for studying stateless flooding ("true statelessness" as defined by [27] restricting the model of [9]) permits the knowledge and distinguishing of neighbours in both broadcasting and receiving.

3 Model and Notation

Throughout this work we consider only finite, connected graphs on at least two nodes. We denote the set $\{1, ..., x\}$ by $[x]$ and $R(r, s)$ the Ramsey number such that any graph on $R(r, s)$ vertices contains either a clique on r vertices or an independent set on s vertices. In this work, we make use of a generic synchronous message passing model (as described in definition 1) with several additional assumptions based on the truly stateless model of [27]. Our agents are the nodes of a network and are able to communicate via messages of arbitrary size sent over the edges of this network. Computation occurs in synchronous rounds, consisting of three phases: (i) nodes receive messages sent in the previous round, (ii) nodes perform computation and (iii) nodes send messages to be delivered in the next round. Unless stated otherwise, agents do not suffer faults and no messages are lost. In addition to these standard assumptions, we enforce that the model is stateless, i.e. nodes cannot maintain any additional information between rounds (such as routing information, previous participation in the flood or even a clock value), cannot hold onto messages and can only forward, not modify the messages.

Unless stated otherwise we do not assume that nodes have access to unique identifiers, however they have locally labelled ports that are distinguishable and totally orderable for both receiving and sending messages. When we do work with identifiers these identifiers are assumed to be unique, drawn from $[|V| + \kappa]$ for $\kappa > 0$ a constant and assigned adversarially. We refer to such an assignment of IDs to a network as a labelling of the network and we identify the port label of a link leading to a node with the ID of that node. We further assume that individual nodes have access to arbitrarily powerful computation on information they do have.

We are principally interested in the problem of *broadcast*, although we will occasionally consider the related *multicast* problem i.e. there are multiple initiators who may potentially wake up in different rounds with the same message to be broadcast. For a graph $G = (V, E)$ and an *initiator* set $I \subseteq V$ we say that a node is informed if it has ever received a message from a previously informed node (where initiators are assumed to begin informed). An algorithm *correctly* solves broadcast (resp. multicast) on G if for all singleton (resp. non-empty) initiator sets there exists a finite number of rounds after which all nodes will be informed. Unless specified otherwise, we assume that initiator nodes remain aware of their membership for only a single round. We say that an algorithm *terminates* on $G = (V, E)$ if, for all valid initiator sets, there exists a finite round after which no further messages are sent (i.e. the communication network quiesces).
Formally, for the message M, we denote a *configuration* of Amnesiac Flooding as follows:

▶ **Definition 8.** *A configuration of Amnesiac Flooding on graph $G = (V, E)$ is a collection of messages/edges $S \subseteq \{(u, v) | uv \in E\}$ where $(u, v) \in S$ implies that in the current round u sent a message to v.*

Further, for H a subgraph of G, we denote by S_H, S restricted to H. Below, we define the operator $A_{I,G} : 2^{V^2} \to 2^{V^2}$ to implement one round of Amnesiac Flooding on the given configuration where agents in I receive the message from outside the network:

▶ **Definition 9.** *The operator $A_{I,G} : 2^{V^2} \to 2^{V^2}$ is defined as follows: The set I of nodes initiate the broadcast. The message $(u, v) \in A_{I,G}(S)$ if $uv \in E$, $(v, u) \notin S$ and either $\exists w \in V : (w, u) \in S$ or $u \in I$.*

We will drop the subscript when G is obvious from context and $I = \emptyset$. Further, we adopt the standard convention of using $A^k_{I,G}$ to mean $A_{I,G}$ applied k times. Theorem 7 in this notation can be expressed as follows:

▶ **Theorem 10** (Theorem 7 restated)**.** *For any graph $G = (V, E)$, and any finite sequence $I_1, ..., I_k \subseteq V$, there exists $m \in \mathbb{N}$ such that*

$$A^m_{\emptyset,G}(A_{I_k,G}(...A_{I_1,G}(\emptyset))) = \emptyset.$$

4 Uniqueness

In this section, we investigate broadcast protocols similar to Amnesiac Flooding and establish four desirable properties that Amnesiac Flooding uniquely satisfies in combination. On the other hand, we show that this result is sharp and that by relaxing any of these conditions one can obtain similar terminating broadcast protocols.

4.1 Uniqueness

Our first major result concerns the uniqueness of Amnesiac Flooding. Given the algorithms surprising properties, a natural question is whether other broadcast algorithms exist maintaining these properties. Specifically, does there exist a terminating protocol for broadcast which obeys all of the following for all graphs and valid port labellings:

1. *Strict Statelessness*: Nodes maintain no information other than their port labellings between rounds. This includes whether or not they were in the initiator set.
2. *Obliviousness*: Routing decisions may not depend on the contents of received messages.
3. *Determinism*: All decisions made by a node must be deterministic.
4. *Unit Bandwidth*: Each node may send at most one message per edge per round.

The answer is negative. We will actually prove the slightly stronger case, that this holds even if agents are provided with unique identifiers, are aware of the identifiers of their neighbours and that these identifiers have bounded size. Intuitively, the *Strict Statelessness* condition forces any broadcast protocol to make its forwarding decisions based only on the messages it receives in a given round. The combination of *Obliviousness* and *Unit Bandwidth* forces any protocol meeting the conditions to view messages as atomic. Finally, *Determinism* forces the protocol to make identical decisions every time it receives the same set of messages. Formally, any broadcast protocol meeting the four conditions must be expressible in the following form:

▶ **Definition 11.** *A* protocol $P = (b, f)$ *is a pair of functions, an* initial function b *and a* forwarding function f*, where* $b : \mathbb{N} \times 2^{\mathbb{N}} \to 2^{\mathbb{N}}$ *and* $f : \mathbb{N} \times 2^{\mathbb{N}} \times 2^{\mathbb{N}} \to 2^{\mathbb{N}}$*. The protocol is implemented as follows. On the first round the initiator node* s *with neighbourhood* $N(s)$ *sends messages to every node with a label in* $b(s, N(s))$*. On future rounds, each node* u *sends messages to every node with a label in* $f(u, N(u), S)$ *where* S *is the set of labels of nodes* u *received messages from in the previous round. Further, we require that for any* $B \subseteq A \subset \mathbb{N}$ $b(u, A), f(u, A, B) \subseteq A$ *and that* $f(u, A, \emptyset) = \emptyset$*, enforcing that agents can only communicate over edges of the graph and can only forward messages they have actually received respectively.*

In this setting, achieving broadcast is equivalent to every node receiving a message at least once and terminating in finite time corresponds to there existing a finite round after which no messages are sent. For example, Amnesiac Flooding is defined by the following functions:

▶ **Definition 12** (Amnesiac Flooding Redefinition)**.** *Amnesiac Flooding is defined by* $P_{AF} = (b_{AF}, f_{AF})$ *where, for all* $T \subseteq S \subset \mathbb{N}$*,* $b(u, S) = S$ *and* $f(u, S, T) = S \setminus T$ *if* $T \neq \emptyset$ *and* $\emptyset$ *otherwise.*

In order, to argue that Amnesiac Flooding is unique we require a notion of what it means for two broadcast algorithms to be distinct.

▶ **Definition 13.** *Let* $G = (V, E)$ *be a graph and* $L \subseteq \mathbb{N}$ *a set of labels. We say that the pair* (G, L) distinguishes *the protocols* P *and* Q *if there exists a labelling of* G *using only labels from* L *such that for some initial vertex* s*,* P *and* Q *send messages over different sets of edges in the same round when implementing broadcast on* G *initiated from* s*. If there exists some pair* (G, L) *which distinguishes* P *and* Q*, we describe* P *and* Q *as* distinct*. Otherwise, we consider them the same protocol.*

For a protocol $P = (b, f)$, a set $S \subseteq \mathbb{N}$ and a number $k \in \mathbb{N}$ we describe P as AF up to degree k on S if there is no graph G of maximum degree k such that (G, S) distinguishes P from P_{AF}. From here on we will assume that all unique labels are drawn from $[n + \kappa]$

where κ is a sufficiently large constant and n is the number of nodes in the graph. As this only eliminates possible pairs distinguishing protocols from Amnesiac Flooding, this only strengthens the result. We obtain the following result, discussion and sketching the proof of which makes up the remainder of this section, provided $\kappa \geq R(9, 8)$.

▶ **Theorem 14.** *(Restatement of Theorem 2) Let $P = (b, f)$ be a correct and terminating broadcast protocol defined according to definition 11, then P is not distinct from Amnesiac Flooding.*

Proof sketch for Theorem 14. The basic argument is to show that any correct and terminating broadcast protocol meeting the criteria is identical to Amnesiac Flooding. Our core technique is to construct a set of network topologies such that any policy distinct from Amnesiac Flooding fails on at least one of them. However, the behaviour of these alternative protocols can be quite complex, as can the relationship between the constraints enforced by different networks under different labellings. In order to circumvent this, we find very simple networks, and labellings there of, where any algorithm must behave like Amnesiac Flooding and then modify them to obtain new instances, with the property that only a small number of vertices may ever behave distinctly from Amnesiac Flooding. In these more manageable cases, we are able to then show that any distinct behaviour leads to an incorrect algorithm. More precisely, for any given protocol we derive a directed graph (separate from the network topology) describing its behaviour and demonstrate via a forbidden subgraph argument that any set of IDs of size $R(x, 8)$ must contain a subset T of size at least x such that P is AF up to degree 1 on T. We take $\kappa = R(9, 8)$ and show that there must then exist $U \subseteq T$ containing at least 6 identifiers such that P is AF up to degree 2 on U. By constructing a set of small sub-cubic graphs, we are able to extend this to degree 3.
These form the base case of a pair of inductive arguments. First, we construct a progression of sub-cubic graphs which enforce that if P is AF on $[m]$ up to degree 3 it must be AF on $[m + 1]$ up to degree 3. We then construct a family of graphs which have a single node of high-degree, while all other nodes have a maximum degree of 3 and so must behave as though running Amnesiac Flooding. These graphs permit a second inductive argument showing that this unique high degree node must also behave as if running Amnesiac Flooding. In combination, these two constructions enforce that P behaves like Amnesiac Flooding in all possible cases. The proof is given in appendix A. ◀

4.2 Relaxing the constraints

Despite the uniqueness established in the previous subsection, we are able to derive four relaxed algorithms distinct from Amnesiac Flooding each obeying only three of the four conditions.

▶ **Theorem 3** (Existence of relaxed Algorithms)**.** *There exist terminating broadcast algorithms which behave distinctly from Amnesiac Flooding on infinitely many networks possessing any three of:* Strict Statelessness*,* Obliviousness*,* Determinism *and* Unit Bandwidth*.*

The algorithms we obtain all build upon Amnesiac Flooding, and we believe illuminate the role of each of the four conditions in the uniqueness result by showing what they prevent. The algorithms for *Obliviousness* and *Unit Bandwidth* are presented together, as they differ only in how control information is encoded.

- *Strict Statelessness*: Several relaxations of this already exist, such as Stateless Flooding (the initiator retains information for one round) or even classical non-Amnesiac Flooding (nodes are able to retain 1-bit for one round). We present Neighbourhood-2 Flooding. Nodes know the ID of their neighbours' neighbours. The protocol behaves distinctly on star graphs, as the hub can determine the entire network topology.

- *Obliviousness* and *Unit Bandwidth*: 1-BIT FLOODING. Nodes are allowed to send a single bit of read-only control information (in the message header or encoded in the number of messages sent) communicating whether the initiator is a leaf vertex. If it is, nodes implement Amnesiac Flooding, otherwise they use a different mechanism called PARROT FLOODING (leaves bounce the message back) which always terminates when begun from a non-leaf vertex.
- *Determinism*: RANDOM-FLOODING. Nodes have access to one bit of randomness per round. Each round every node randomly chooses to implement Amnesiac Flooding or to forward to all neighbours. RANDOM-FLOODING is correct with certainty and terminates almost surely in finite time.

Since each of these constitutes only a minor relaxation of the restrictions, we argue that the uniqueness of Amnesiac Flooding is in some sense sharp. We present the protocols fully and demonstrate their correctness and termination for each of these cases independently in the full version [2].

5 Termination Dichotomy

With the uniqueness of Amnesiac Flooding established, a greater understanding of its properties is warranted. In this section, we study the configuration space of Amnesiac Flooding and obtain an exact characterisation of terminating configurations. We then apply this to investigate the algorithm's fault sensitivity.

5.1 Obtaining a termination dichotomy

In order to consider the fault sensitivity of Amnesiac Flooding, we need to be able to determine its behaviour outside of correct broadcasts. Unfortunately, neither of the existing termination proofs naturally extend to the case of arbitrary message configurations. Fortunately, we can derive an invariant property of message configurations when restricted to subgraphs that exactly captures non-termination, which we will call "balance" (see Definition 20).

▶ **Theorem 15.** *Let S be a configuration on $G = (V, E)$ then there exists $k \geq 0$ such that $A^k_G(S) = \emptyset$ if and only if S is balanced on G.*

In fact we obtain that not only do balanced configurations terminate, they terminate quickly.

▶ **Corollary 16.** *Let S be a balanced configuration on $G = (V, E)$ then there exists $k \leq 2|E|$ such that $A^k_G(S) = \emptyset$.*

Intuitively, for the protocol not to terminate, we require that a message is passed around forever and since it is impossible for a message to be passed back from a leaf node the message must traverse either a cycle or system of interconnected cycles. As we will demonstrate in the rest of the section we need only consider systems of at most two cycles. Specifically, we introduce an invariant property determined by parity constraints on the number of messages travelling in each direction and their spacing around: odd-cycles, even cycles and what we will dub, faux-even cycles.

▶ **Definition 17.** *A* faux-even cycle *(FEC) is a graph comprised of either two node disjoint odd cycles connected by a path or two odd cycles sharing only a single node. We denote by $FEC_{x,y,z}$ the FEC with one cycle of length $2x+1$, one of length $2z+1$ and a path containing y edges between them. We emphasize that if $y = 0$ the two cycles share a common node and if $y = 1$ the two cycles are connected by a single edge.*

FECs get their name from behaving like even cycles with respect to the operator A. In order to capture this we can perform a transformation to convert them into an equivalent even cycle.

▶ **Definition 18.** *Let $F = (V, E)$ be $FEC_{x,y,z}$. Then the* even cycle representation *of F denoted F_2 is the graph constructed by splitting the end points of the interconnecting path in two, and duplicating the path to produce an even cycle. Formally, if the two cycles are of the form $a_0...a_{2x}a_0$ and $c_0...c_{2z}c_0$, with a_0 and c_0 connected by the path $b_1...b_{y-1}$, we construct the following large even cycle from four paths: $a_0...a_{2x}a_{-1}d_1...d_{y-1}c_{-1}c_{2z}...c_0b_{y-1}...b_1a_0$. Here a_{-1} is a copy of a_0, c_{-1} is a copy of c_0, and the path $d_1...d_{y-1}$ is a copy of the path $b_1...b_{y-1}$ (See figure 1). Note that if $y = 0$, $a_0 = c_0$ and so we do not include any nodes from b or d. Similarly, if $y = 1$, a_0 and c_0 are connected by a single edge, as are a_{-1} and c_{-1}.*

There then exists a corresponding message configuration over the even cycle representation. Essentially (other than a few technical exceptions), this new configuration is the same as the old configuration but with two copies of each message on the path, one on each corresponding edge of the even cycle representation. Formally,

▶ **Definition 19.** *Let $F = (V, E)$ be $FEC_{x,y,z}$ and S a configuration of Amnesiac Flooding on F, the* even cycle representation *of S on F denoted $S_{2,F}$ is determined as follows. For each $m \in S$*

- *If $m = (a_{2x}, a_0)$ (resp. (a_0, a_{2x})) we add (a_{2x}, a_{-1}) (resp. (a_{-1}, a_{2x})) to $S_{2,F}$.*
- *If $m = (b_i, b_j)$ for some $i, j \in \{1, ..., y-1\}$, we add both (b_i, b_j) and (d_i, d_j) to $S_{2,F}$.*
- *If $m = (c_{2z}, c_0)$ (resp. (c_0, c_{2z})) we add (c_{2z}, c_{-1}) (resp. (c_{-1}, c_{2z})) to $S_{2,F}$.*
- *If $m = (a_0, b_1)$ (resp. (b_1, a_0)) we add both (a_0, b_1) and (a_{-1}, d_1) (resp. (b_1, a_0) and (d_1, a_{-1})) to $S_{2,F}$.*
- *If $m = (c_0, b_{y-1})$ (resp. (b_{y-1}, c_0)) we add both (c_0, b_{y-1}) and (c_{-1}, d_{y-1}) (resp. (b_{y-1}, c_0) and (d_{y-1}, c_{-1})) to $S_{2,F}$.*
- *If $m = (a_0, c_0)$ (resp. (c_0, a_0)) we add both (a_0, c_0) and (a_{-1}, c_{-1}) (resp. (c_0, a_0) and (c_{-1}, a_{-1})) to $S_{2,F}$.*
- *Otherwise we add m to $S_{2,F}$.*

With this established we can now define the notion of balance.

▶ **Definition 20.** *A configuration S is* balanced *on $G = (V, E)$ if for all subgraphs H of G one of the following holds:*

- *H is not a cycle or FEC.*
- *H is an odd cycle and S_H contains an equal number of messages travelling clockwise and anti-clockwise on H.*
- *H is an even cycle and for any given message m in S_H, there is an equal number of messages travelling clockwise and anti-clockwise on H such that their heads are an even distance from m's.*
- *H is an FEC and $S_{2,H}$ is balanced on H_2 (i.e. obeys the previous condition).*

With these definitions established, we can present the intuition behind the proof of Theorem 15.

Sketch of the proof of Theorem 15. We first establish that balance, and therefore imbalance, is conserved by Amnesiac flooding and, as the empty configuration is balanced, Amnesiac Flooding cannot terminate from any imbalanced configuration. For Amnesiac Flooding not to terminate it requires that some message travels around the communication graph and returns to the same edge, in the same direction. We show that if a configuration is balanced, the trajectory of any message can spend only a bounded number of consecutive steps on any

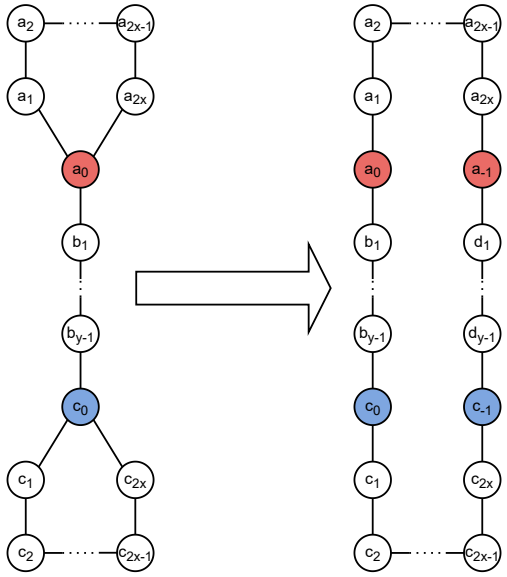


Figure 1 Left: An $FEC_{x,y,z}$. Right: The corresponding even cycle representation. Please note that this depiction only holds for $y \geq 2$. For $y = 1$: a_0 and c_0 are connected directly by an edge in both sub figures (as are a_{-1} and c_{-1}). For $y = 0$: $a_0 = c_0$ and $a_{-1} = c_{-1}$.

given cycle or FEC. However, we can also show that any message's trajectory which crosses the same edge twice in the same direction, must have spent a large number of consecutive steps on some cycle or FEC, and therefore could not have begun in a balanced configuration. Thus, Amnesiac Flooding started from any balanced configuration must terminate. The proof is given in appendix B. ◀

5.2 Applying the termination dichotomy

In this section, we apply the dichotomy to obtain a number of results.

5.2.1 Extended Dichotomy

While Theorem 15 provides a full dichotomy over the configuration space of Amnesiac Flooding and is easier to reason about than previous results, the definition is somewhat unwieldy. In this subsection, we demonstrate the effectiveness of the dichotomy and unify it with the existing results [15, 16, 28]. It has previously been observed that running Amnesiac Flooding backwards obtains another instance of multi-cast Amnesiac Flooding [15]. Formally,

▶ **Definition 21.** *Let $G = (V, E)$ be a graph and S be a configuration of messages on G. Then $\bar{S} = \{(v, u) | (u, v) \in S\}$.*

The following lemma is similar to the argument made in Corollary 4.6. of [15].

▶ **Lemma 22.** *Let $G = (V, E)$ be a graph, S a configuration of messages on G and $T = \{u \in V | \forall v \in N(u) : (u, v) \in S\}$ the set of source vertices. Then $A_{T,G}\left(\overline{A_{\emptyset,G}(\bar{S})}\right) = S$*

Which gives the following immediately via induction,

▶ **Lemma 23.** *Let $G = (V, E)$ be a graph and S a configuration of messages on G. Then for any $k \in \mathbb{N}$ there exists a sequence $I_1, ..., I_k \subseteq V$ such that $A_{I_1,G}\left(...\left(A_{I_k,G}\left(\overline{A^k_{\emptyset,G}\left(\bar{S}\right)}\right)\right)...\right) = S$.*

Intuitively, this means that given any configuration S of Amnesiac Flooding, we can run it backwards through time to some earlier configuration S'. Further we obtain a sequence of vertex sets $I_1, ..., I_k$ that were sinks in the time-reversed process and therefore sources in the forwards process. We can therefore reconstruct S beginning from S' via some sequence of fresh multi-casts from $I_1, ..., I_k$. We will use this fact to obtain all configurations from which Amnesiac Flooding terminates (i.e. balanced configurations) from the empty configuration. The following lemma is immediate from the definition of balance (definition 20), as reversing the direction of all messages in a configuration does not affect its balance.

▶ **Lemma 24.** *$\bar{S}$ is balanced on G if and only if S is balanced on G*

Putting it all together, we obtain the following extension of the dichotomy result, as well as the complement to Theorem 7.

▶ **Theorem 25.** *Let $G = (V, E)$ be a graph and S a configuration of G, the following are all equivalent:*

1. $\exists k \in \mathbb{N} : A^k_{\emptyset,G}(S) = \emptyset$
2. $\exists k \in \mathbb{N}, I_1, ..., I_k \subseteq V : A_{I_k,G}\left(... \left(A_{I_1,G}(\emptyset)\right) ...\right) = S$
3. *S is balanced on G*

Proof. The equivalence of (1) and (3) follow immediately from Theorem 15. Further, we have that (2) implies (1) from Theorem 7. Now assume S is balanced, then by lemma 24, so is $\bar{S}$. Thus by Theorem 15 there exists a finite k such that after k rounds Amnesiac flooding started from $\bar{S}$ must terminate, i.e. $A^k_{\emptyset,G}(\bar{S}) = \emptyset$. Therefore, by lemma 23 we have a sequence $I_1, ..., I_k \subseteq V$ such that $S = A_{I_1,G}\left(...\left(A_{I_k,G}\left(\overline{A^k_{\emptyset,G}\left(\bar{S}\right)}\right)\right)...\right) = A_{I_1,G}\left(... \left(A_{I_k,G}(\emptyset)\right) ...\right)$. Thus, we have (3) implies (2) and the result follows. ◀

5.2.2 Fault Sensitivity

In this work we consider three key forms of fault of increasing severity: message dropping, uni-directional link failure and weak-Byzantine failures. Intuitively, these correspond to a set of messages failing to send in a specific round, a link failing in one direction creating a directed edge and a set of nodes becoming transiently controlled by an adversary.

More precisely, let $\mathbf{S} = (S_i)_{i\in\mathbb{N}}$ be the sequence of actual message configurations on our network. We say that $\mathbf{S}$ is *fault free* for $G = (V, E)$ **if** $S_{i+1} = A_G(S_i)$ for all $i \in \mathbb{N}$. Otherwise, we say it experienced a *fault*. In this case we say $\mathbf{S}$ has suffered from,

- Message dropping, if there exists $T \subseteq V^2$ and $k \geq 1$ such that $S_{k+1} = A(S_k) \setminus T$ and for all $i \neq k$, $S_{i+1} = A(S_i)$. This corresponds to all messages in T being dropped on round k.
- Uni-directional link failure, if there exists $X \subseteq V^2$ such that for all $i \geq 1$, $S_{i+1} = A(S_i) \setminus X$. This corresponds to all oriented links in X failing.
- Weak-Byzantine failure, if there exists $Y \subseteq V$ such that for some k at least twice the diameter, for all $i < k$, $S_{i+1} \setminus \{(u, v)|u \in Y\} = A(S_i) \setminus \{(u, v)|u \in Y\}$. This corresponds to a possible failure where an adversary determines the forwarding decisions of the nodes in Y until round k.

Note that we refer to the Byzantine failures as weak, since they are transient and only interfere with the forwarding of the message, not its content. It is obvious to see that in a stateless setting there is no way to deal with a Byzantine fault that changes the message

as there is no method to verify which message is authentic. Intuitively, in our setting, Weak-Byzantine agents may choose to send messages to an arbitrary set of neighbours in each round and they are all controlled by a single coordinated adversary. We say that a Weak-Byzantine adversary with control of a given set of nodes can force some behaviour if there exists any weak byzantine failure on that set of nodes producing the forced behaviour. We can now express our fault sensitivity results, the proofs of which we defer to the full version [2], and begin with an extreme case of single message dropping.

▶ **Theorem 4** (Single Message Failure). *If single-source Amnesiac Flooding experiences a* single message drop failure *for the message* (u, v) *then it fails to terminate if and only if either or both of the following hold:*

- *uv is not a bridge*
- *uv lies on a path between vertex disjoint odd cycles*

Moreover, it fails to broadcast if and only if this is the first message sent along uv, uv is a bridge, and the side of the cut containing u does not contain an odd cycle.

Thus, Amnesiac Flooding is extremely fault-sensitive with respect to message dropping. Secondly, considering uni-directional link failures we obtain the following.

▶ **Theorem 5** (Uni-directional link failure). *For any graph $G = (V, E)$ and any initiator set $I \subsetneq V$ there exists an edge $e \in E$ such that a uni-directional link failure at e will cause Amnesiac Flooding to either fail to broadcast or fail to terminate when initiated from I on G. Furthermore, for any non-empty set of uni-directional link failures there exists $v \in V$ such that, when Amnesiac Flooding is initiated at v, it will either fail to broadcast or fail to terminate.*

Finally for the weak-Byzantine case.

▶ **Theorem 6** (Byzantine Failure). *If Amnesiac Flooding on $G = (V, E)$ initiated from $I \subsetneq V$ experiences a* weak Byzantine failure *at $J \subseteq V \setminus I$, then the adversary can force:*

- *Failure to broadcast if and only if J contains a cut vertex set.*
- *Non-termination if and only if at least one member of J lies on either a cycle or a path between odd-cycles.*

6 Conclusions and Future Work

In this paper, we prove a *uniqueness* result: Under standard synchronous message passing assumptions, any strictly stateless deterministic algorithm oblivious to the message content which solves terminating broadcast is indistinguishable from *Amnesiac Flooding*. We therefore argue due to both its uniqueness and simplicity, that Amnesiac Flooding is a fundamental or prototypical broadcast algorithm. We formalise the four properties required for this uniqueness to hold, and show that by relaxing each individually one can obtain other correct and terminating broadcast algorithms, of which we present several. These present the following natural questions: To what extent does Amnesiac Flooding represent a "minimal" broadcast algorithm? Are there identifiable families of algorithms solving terminating broadcast with a subset of these restrictions? Are any of these independent of (i.e. not derivatives of) Amnesiac Flooding? Lastly, we are not aware of any similar uniqueness results in algorithms literature, restricting the number of successful algorithms to just one (or even to small finite numbers) - can this result stimulate a study into enumerating distinct algorithms for solutions to interesting problems? Note that our model of true statelessness rules out trivial extensions to algorithms such as delaying algorithm start or holding messages for a certain count. Are there reasonable ways to define distinctiveness i.e. discard "trivial" extensions to algorithms in less restrictive models than ours?

We also obtain an understanding of the structural properties of Amnesiac Flooding. In particular, we study its sensitivity to single message drops, uni-directional link failures, and weak byzantine collusion, showing it can easily become non-terminating or non-broadcasting under such conditions. This is perhaps surprising, as statelessness is frequently associated with fault tolerance, such as in the self stabilizing setting. A reasonable interpretation of Theorem 15, however, is that Amnesiac Flooding, while locally stateless, depends heavily on a distributed "meta-state" contained in the configuration of sent messages. This suggests it is unlikely that any minor modification of Amnesiac Flooding will resolve its fragility without depending on an entirely different mechanism for termination. In support of this, we note that of the four alternatives presented in the proof of Theorem 3, only Random-Flooding is meaningfully more robust (and will in fact terminate from any configuration in finite time almost surely). Nevertheless, we contend that further exploration of stateless algorithms such as Amnesiac Flooding, their properties and related models are important for both theory and practice of distributed networks.

References

1 Hagit Attiya and Jennifer Welch. *Distributed Computing: Fundamentals, Simulations and Advanced Topics*. John Wiley & Sons, 2004.

2 Henry Austin, Maximilien Gadouleau, George B. Mertzios, and Amitabh Trehan. Amnesiac flooding: Easy to break, hard to escape, 2025. doi:10.48550/arXiv.2502.06001.

3 Abhik Banerjee, Chuan Heng Foh, Chai Kiat Yeo, and Bu-Sung Lee. Performance improvements for network-wide broadcast with instantaneous network information. *J. Netw. Comput. Appl.*, 35(3):1162–1174, 2012. doi:10.1016/J.JNCA.2012.01.008.

4 Zahra Bayramzadeh, Ajay D. Kshemkalyani, Anisur Rahaman Molla, and Gokarna Sharma. Weak amnesiac flooding of multiple messages. In Karima Echihabi and Roland Meyer, editors, *Networked Systems - 9th International Conference, NETYS 2021, Virtual Event, May 19-21, 2021, Proceedings*, volume 12754 of *Lecture Notes in Computer Science*, pages 88–94. Springer, 2021. doi:10.1007/978-3-030-91014-3_6.

5 Petra Berenbrink, Robert Elsässer, and Thomas Sauerwald. Randomised broadcasting: Memory vs. randomness. *Theor. Comput. Sci.*, 520:27–42, 2014. doi:10.1016/J.TCS.2013.08.011.

6 Armando Castañeda, Jonas Lefèvre, and Amitabh Trehan. Fully compact routing in low memory self-healing trees. In Nandini Mukherjee and Sriram V. Pemmaraju, editors, *ICDCN 2020: 21st International Conference on Distributed Computing and Networking, Kolkata, India, January 4-7, 2020*, pages 21:1–21:10. ACM, 2020. doi:10.1145/3369740.3369786.

7 Niccolò D'Archivio and Robin Vacus. On the Limits of Information Spread by Memory-Less Agents. In Dan Alistarh, editor, *38th International Symposium on Distributed Computing (DISC 2024)*, volume 319 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 18:1–18:21, Dagstuhl, Germany, 2024. Schloss Dagstuhl – Leibniz-Zentrum für Informatik. doi:10.4230/LIPIcs.DISC.2024.18.

8 Benjamin Doerr, Mahmoud Fouz, and Tobias Friedrich. Social networks spread rumors in sublogarithmic time. In *Proceedings of the Forty-Third Annual ACM Symposium on Theory of Computing*, STOC '11, pages 21–30, New York, NY, USA, 2011. Association for Computing Machinery. doi:10.1145/1993636.1993640.

9 Danny Dolev, Michael Erdmann, Neil Lutz, Michael Schapira, and Adva Zair. Stateless computation. In Elad Michael Schiller and Alexander A. Schwarzmann, editors, *Proceedings of the ACM Symposium on Principles of Distributed Computing, PODC 2017, Washington, DC, USA, July 25-27, 2017*, pages 419–421. ACM, 2017. doi:10.1145/3087801.3087854.

10 Robert Elsässer and Thomas Sauerwald. The power of memory in randomized broadcasting. In Shang-Hua Teng, editor, *Proceedings of the Nineteenth Annual ACM-SIAM Symposium*

on Discrete Algorithms, SODA 2008, San Francisco, California, USA, January 20-22, 2008, pages 218–227. SIAM, 2008. URL: http://dl.acm.org/citation.cfm?id=1347082.1347107.

11 Maxime Flin, Mohsen Ghaffari, Magnús M. Halldórsson, Fabian Kuhn, and Alexandre Nolin. Coloring fast with broadcasts. In Kunal Agrawal and Julian Shun, editors, *Proceedings of the 35th ACM Symposium on Parallelism in Algorithms and Architectures, SPAA 2023, Orlando, FL, USA, June 17-19, 2023*, pages 455–465. ACM, 2023. doi:10.1145/3558481.3591095.

12 Ajei S. Gopal, Inder S. Gopal, and Shay Kutten. Fast broadcast in high-speed networks. *IEEE/ACM Trans. Netw.*, 7(2):262–275, 1999. doi:10.1109/90.769773.

13 Walter Hussak and Amitabh Trehan. On termination of a flooding process. In Peter Robinson and Faith Ellen, editors, *Proceedings of the 2019 ACM Symposium on Principles of Distributed Computing, PODC 2019, Toronto, ON, Canada, July 29 - August 2, 2019*, pages 153–155. ACM, 2019. doi:10.1145/3293611.3331586.

14 Walter Hussak and Amitabh Trehan. On the termination of flooding. In Christophe Paul and Markus Bläser, editors, *37th International Symposium on Theoretical Aspects of Computer Science, STACS 2020, March 10-13, 2020, Montpellier, France*, volume 154 of *LIPIcs*, pages 17:1–17:13. Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2020. doi:10.4230/LIPICS.STACS.2020.17.

15 Walter Hussak and Amitabh Trehan. Terminating cases of flooding. *CoRR*, abs/2009.05776, 2020. arXiv:2009.05776.

16 Walter Hussak and Amitabh Trehan. Termination of amnesiac flooding. *Distributed Comput.*, 36(2):193–207, 2023. doi:10.1007/S00446-023-00448-Y.

17 Amos Korman and Robin Vacus. Early adapting to trends: self-stabilizing information spread using passive communication. *Distributed Comput.*, 37(4):335–362, 2024. doi:10.1007/S00446-024-00462-8.

18 Nancy A. Lynch. *Distributed Algorithms*. Morgan Kaufmann, 1996.

19 Victoria Manfredi, Mark Crovella, and Jim Kurose. Understanding stateful vs stateless communication strategies for ad hoc networks. In Parmesh Ramanathan, Thyaga Nandagopal, and Brian Neil Levine, editors, *Proceedings of the 17th Annual International Conference on Mobile Computing and Networking, MOBICOM 2011, Las Vegas, Nevada, USA, September 19-23, 2011*, pages 313–324. ACM, 2011. doi:10.1145/2030613.2030649.

20 Garrett Parzych and Joshua J. Daymude. Memory Lower Bounds and Impossibility Results for Anonymous Dynamic Broadcast. In Dan Alistarh, editor, *38th International Symposium on Distributed Computing (DISC 2024)*, volume 319 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 35:1–35:18, Dagstuhl, Germany, 2024. Schloss Dagstuhl – Leibniz-Zentrum für Informatik. doi:10.4230/LIPIcs.DISC.2024.35.

21 David Peleg. *Distributed Computing: A Locality Sensitive Approach*. SIAM, 2000.

22 Stanisław Radziszowski. Small ramsey numbers. *The electronic journal of combinatorics*, pages DS1–Sep, 2012.

23 Patricia Ruiz and Pascal Bouvry. Survey on broadcast algorithms for mobile ad hoc networks. *ACM Comput. Surv.*, 48(1):8:1–8:35, 2015. doi:10.1145/2786005.

24 Andrew Tanenbaum. *Computer networks*. Pearson Prentice Hall, Boston, 2011.

25 Gerard Tel. *Introduction to distributed algorithms*. Cambridge University Press, New York, NY, USA, 1994.

26 Yu-Chee Tseng, Sze-Yao Ni, Yuh-Shyan Chen, and Jang-Ping Sheu. The broadcast storm problem in a mobile ad hoc network. *Wirel. Networks*, 8(2-3):153–167, 2002. doi:10.1023/A:1013763825347.

27 Volker Turau. Stateless information dissemination algorithms. In Andréa Werneck Richa and Christian Scheideler, editors, *Structural Information and Communication Complexity - 27th International Colloquium, SIROCCO 2020, Paderborn, Germany, June 29 - July 1, 2020, Proceedings*, volume 12156 of *Lecture Notes in Computer Science*, pages 183–199. Springer, 2020. doi:10.1007/978-3-030-54921-3_11.

28 Volker Turau. Amnesiac flooding: Synchronous stateless information dissemination. In Tomás Bures, Riccardo Dondi, Johann Gamper, Giovanna Guerrini, Tomasz Jurdzinski, Claus Pahl, Florian Sikora, and Prudence W. H. Wong, editors, *SOFSEM 2021: Theory and Practice of Computer Science - 47th International Conference on Current Trends in Theory and Practice of Computer Science, SOFSEM 2021, Bolzano-Bozen, Italy, January 25-29, 2021, Proceedings*, volume 12607 of *Lecture Notes in Computer Science*, pages 59–73. Springer, 2021. doi:10.1007/978-3-030-67731-2_5.

29 Volker Turau. Synchronous concurrent broadcasts for intermittent channels with bounded capacities. In Tomasz Jurdzinski and Stefan Schmid, editors, *Structural Information and Communication Complexity - 28th International Colloquium, SIROCCO 2021, Wrocław, Poland, June 28 - July 1, 2021, Proceedings*, volume 12810 of *Lecture Notes in Computer Science*, pages 296–312. Springer, 2021. doi:10.1007/978-3-030-79527-6_17.

Appendix

In this section we present the proof of our main technical results, i.e. theorems 14 and 15, as well as their constituent lemmas. Due to space constraints, we defer the proofs of all such lemmas as well as the correctness of all other results to the full version. Additionally, we will refer to Amnesiac Flooding as simply AF throughout.

A Proof of Uniqueness (Theorem 14)

We begin with the following observations, which follow from case analysis on paths:

▶ **Observation 26.**

1. *For all $\emptyset \neq S \subset \mathbb{N}$ and $u \in \mathbb{N}$, $b(u, S) \neq \emptyset$. Otherwise no new nodes will be informed after the first round, violating correctness.*
2. *For $u, v, w \in \mathbb{N}$, $f(v, \{u, w\}, \{u\}) \in \{\{w\}, \{u, w\}\}$. Otherwise, if uvw is a subsequence of a path the message started from u will not reach w.*
3. *For $u, v \in [\kappa]$, if $f(u, \{v\}, \{v\}) = v$ then $f(v, \{u\}, \{u\}) = \emptyset$. Otherwise P will not terminate on the two node path labelled with u and v.*
4. *For $u, v, w \in [\kappa]$ if $f(u, \{v\}, \{v\}) = \{v\}$ and $f(w, \{v\}, \{v\}) = \{v\}$ then $f(v, \{u, w\}, \{u\}) = \{u, w\}$ and/or $f(v, \{u, w\}, \{w\}) = \{u, w\}$. Additionally, $f(v, \{u, w\}, \{u, w\}) = \emptyset$. Otherwise the protocol would not terminate on the three node path with labels uvw.*

Via a combinatorial argument followed by case analysis, we show the existence of a small set of labels which when restricted to, P behaves identically to AF for subcubic graphs.

▶ **Lemma 27.** *There exists $T \subseteq [\kappa]$ such that $|T| \geq 6$ and P is AF on T up to degree 3.*

With this established we can extend the result to all labels via an inductive argument. Essentially, we are able to construct a sequence of graph-labelling pairs that grow to include all labels with all possible neighbourhoods. At each state, by our constructions and the induction hypothesis, at most four vertices may behave distinctly from AF. By careful construction, however, any distinct behaviour will lead to an incorrect algorithm.

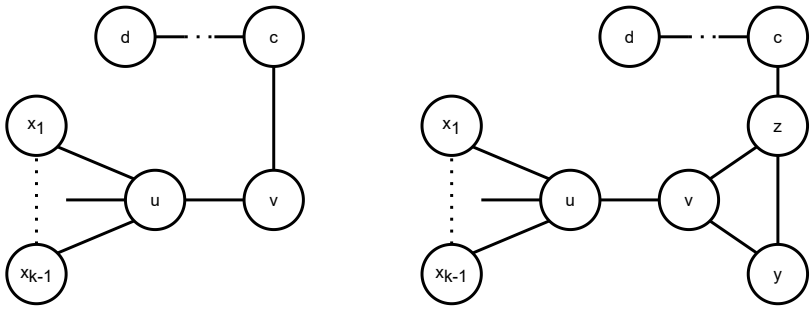
▶ **Lemma 28.** *P is AF on $\mathbb{N}$ up to degree 3.*

The main proof makes use of a very similar argument. However, we now construct graphs with one high degree vertex, while the rest all have degree at most three and so behave indistinguishably from AF.

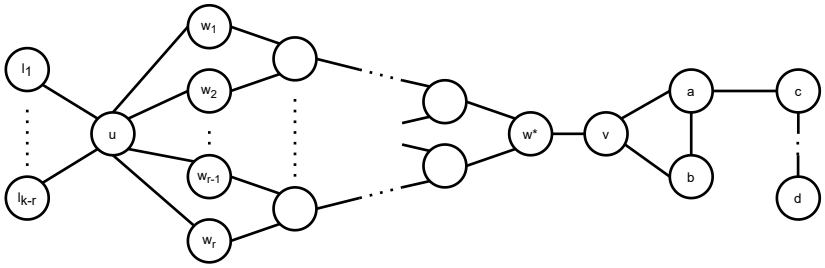
Proof of Theorem 14. By lemma 28 we have that P is AF up to degree 3 on $\mathbb{N}$. It is immediate that for all $u \in \mathbb{N}$, $S \subset \mathbb{N}$ $b(u, S) = S$ as u could be at the centre of a star with one of its leaves replaced by a long path. In this case, u will never receive a message again and so must send to all of its neighbours in the first round.

For an arbitrary label $u \in \mathbb{N}$ and degree $k \in \mathbb{N}$ we can show that u must behave as though it is implementing AF if it is at a node of degree k. Take $S \subset \mathbb{N}$ such that $|S| = k$, we will show that for all non-empty $T \subseteq S$, $f(u, S, T) = S \setminus T$ via induction on the size of T.

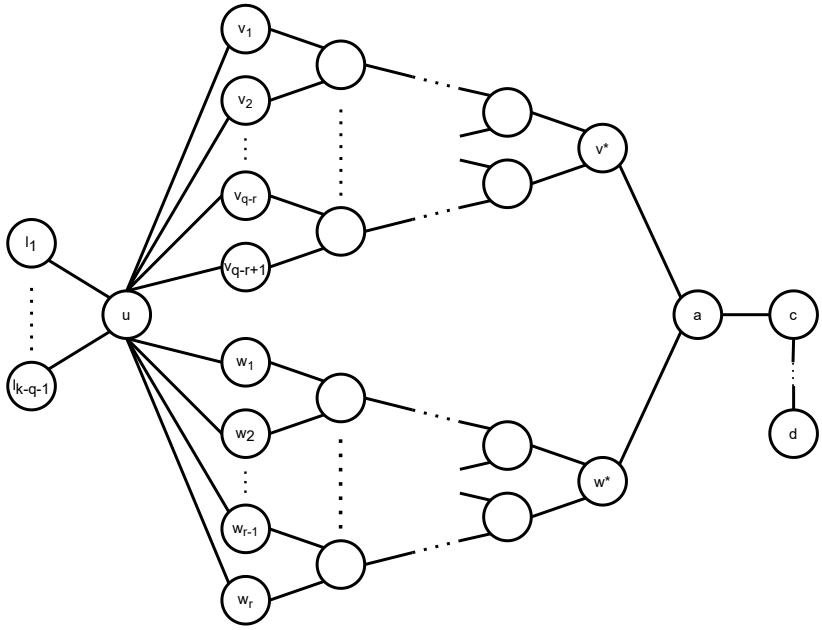
Beginning with our base case. If u receives from a single neighbour we can construct a pair of graphs (special cases of our general construction) that enforce the AF policy. Consider the tree in figure 2a, if u receives a message from v it must send to at least $x_1, ..., x_{k-1}$ as u will receive messages in only a single round. This follows as the rest of tree will use AF policies and since the graph is bipartite each node will be active only once. Thus, the only two options for $f(u, S, \{v\})$ are S or $S \setminus \{v\}$. Now consider the second graph from figure 2a and a broadcast initiated at u. We can see that if $f(u, S, \{v\}) = S$ then the cycle and u will simply pass a message back and forth forever. Thus, $f(u, S, \{v\}) = S \setminus \{v\}$.
We now generalize this construction and perform our induction. Assume that for any non-empty subset T of S of size at most q, $f(u, S, T) = S \setminus T$. For the sake of contradiction assume that this is not true for V where $T \subset V \subseteq S$ with $|V| = q+1$. Thus either $f(u, S, V) \cap V \neq \emptyset$ or $f(u, S, V) \cup V \neq S$.
In the first, case let $W = V \cap f(u, S, V)$, and for some ordering label the elements of W: $w_1, ..., w_r$ and the elements of $V \setminus W$: $v_1, ..., v_{q+1-r}$.
If $W = V$, then we construct a communication graph as in figure 2b and consider a broadcast initiated at u. The messages will travel in only one direction through the binary tree and so when w^* receives a message it will be the only message on the graph. This message will then be passed onto the cycle where it will circulate, before being passed back onto the binary tree in the opposite direction. Again the policy of all nodes other than u is indistinguishable from AF and so when u next receives a message, it receives from all of V and these are the only messages (outside of the path from c to d). We therefore have a repeating sequence as u will forward the message back to every node of W.
If $W \subset V$, then we construct a slightly different communication graph (see figure 2c). This graph instead partitions S into W, $V \setminus W$ and $S \setminus V$, with separate binary trees for W and $V \setminus W$. Here we consider a broadcast initiated at a. When u first receives a message it will receive the message from all identifiers in V and there will be no other messages in the body of the graph. Then by assumption u will send a message to some portion of $\{l_1, ..., l_{k-q-1}\}$ as well as all of W. The leaves will not respond and messages will travel only upwards in the binary tree with W as leaves and w^* as its root. Thus, a will next receive a message only from w^* and will send to v^* and c. Until u receives a message the only messages in the body of the graph will be those travelling down the binary tree with $V \setminus W$ as its leaves and they will all arrive at u simultaneously. Thus, u will then receive only from $V \setminus W$. Since $|V \setminus W| < |V| = q + 1$ by assumption u must make the same decision as AF and so will send messages to all of W and its leaves. This creates a repeating sequence and so we have non-termination. Thus, since $f(u, S, V) \cap V \neq \emptyset$ always allows us to construct a communication graph with a non-terminating broadcast, we must have that $f(u, S, V) \cap V = \emptyset$. Now consider the communication graph from figure 2c again but with W an arbitrary strict subset of V. Since $f(u, S, V)$ does not contain any id from V and none of $\{l_1, ..., l_{k-q-1}\}$ will send a message back to u, u sends messages in only a single round. Therefore, u must send to all of $\{l_1, ..., l_{k-q-1}\}$ otherwise some would not receive the message (and so the protocol would not implement broadcast correctly). Thus, $f(u, S, V) = S \setminus V$ and so we have our contradiction.



(a) Two graphs used in the proof of theorem 14 to determine identifiers' response to receiving only a single message. Left: A tree that forbids sending to too small a subset of neighbours. Right: A graph that forbids sending a message to all neighbours.



(b) A graph used in the proof of theorem 14. The graph consists of a star centred at u with its leaves partitioned into two sets of size r and $k-r$. The leaves in the set of size r are connected by a binary tree of depth $\lceil \log_2 r \rceil$ with root w^* to a cycle, which in turn is connected to a path.



(c) A graph used in the proof of Theorem 14. The graph consists of a star centred at u with its leaves partitioned into three sets of size r, $q-r+1$ and $k-q-1$. The leaves in the first two sets are then each joined to a single node labelled by $w*$ and $v*$ respectively by binary trees of depth $\lceil \log_2 q \rceil$. The single nodes are connected to a node labelled a which is the start of a path $ac...d$.

Figure 2 Note that in all figures the path $c...d$ contains all identifiers in $[m]$ not used in labelling the body of the graph, where m is the largest id in the whole labelling.

Therefore, if $f(u, S, T) = S \setminus T$ for $S \subset \mathbb{N}$ where $|S| = k$ and all $T \subseteq S$ such that $0 < |T| \leq q$, then $f(u, S, V) = S \setminus V$ for $V \subset S$ such that $|V| = q + 1$. Thus, by induction since we know this to be true for all u and S when $q = 1$ it must hold for all $q \leq |S|$.

This gives our claim, as for every u and k we can apply this argument and show that for any $k \in \mathbb{N}$, P is AF up to degree k. ◀

B Proof of the Dichotomy (Theorem 15)

We begin by showing that (im)balance is preserved by the operation of AF,

▶ **Lemma 29.** *For any set of initiators $I \subseteq V$ and configuration S on $G = (V, E)$, $A_{I,G}(S)$ is balanced if and only if S is balanced.*

This gives the following immediate corollary, as $\emptyset$ is trivially balanced.

▶ **Corollary 30.** *If S is imbalanced on $G = (V, E)$, then for all $k > 0$, $A^k_G(S) \neq \emptyset$.*

This gives us the forward direction of Theorem 15. For the other direction, we need the notion of message paths and their recurrence.

▶ **Definition 31.** *A message $m = (v_0, v_1)$ in configuration $S \subset V^2$ has a* message path *v_0v_1. We define the rest of its paths recursively, i.e. $m = (v_0, v_1)$ has a message path $v_0...v_{k+1}$ from S on G if:*

- *$v_0v_1...v_k$ is a message path of m in S*
- *The message (v_k, v_{k+1}) exists in $A^k_G(S)$*

We say that a message $m = (v_0, v_1)$ is recurrent *on $G = (V, E)$ from S if m has a message path of the form $v_0v_1...v_0v_1$ on G from S.*

We obtain the following property relating message paths and termination immediately.

▶ **Lemma 32.** *Let S be a non-empty configuration on $G = (V, E)$ such that $A^k_G(S) = S$, then S contains a recurrent message on G.*

The following theorem connects the notions of imbalance and recurrence.

▶ **Theorem 33.** *Let S be a configuration on G, S is imbalanced if and only if it contains a recurrent message.*

Since, all non-terminating configurations must either contain a recurrent message or eventually reach a configuration with a recurrent message, we have that all balanced configurations must terminate. Thus, Theorem 15 follows immediately from Theorem 33. The forward direction is itself immediate from Corollary 30. For the reverse we take $G = (V, E)$ to be a communication graph and $S \subseteq V^2$ to be a balanced configuration of messages. For contradiction we assume that S contains a recurrent message m which has a message path W performing exactly one excursion and return to m. We will view $W = w_0w_1...w_0w_1$ as both a walk on G and a word. The key observation we require is that the number of consecutive steps that can be spent on certain subgraphs by a message path is bounded from a balanced configurations.

▶ **Lemma 34.** *W must obey the following rules:*

1. *W cannot return to the node it just came from, i.e. no sequence of the form uvu.*
2. *W cannot take $2x + 2$ consecutive steps around an odd cycle of length $2x + 1$.*
3. *W cannot take $x + 1$ consecutive steps around an even cycle of length $2x$.*

4. *If W takes more than $x+y+z+2$ consecutive steps on an FEC it remains on one cycle.*
5. *If W takes x steps around an even cycle of length $2x$, then there exists W' which takes the opposite path of equal length and is otherwise identical.*

The fifth of these has the following useful interpretation, if the existence of W' implies imbalance then the existence of W implies imbalance. Therefore, we will use this rule as a substitution allowing us to "modify" W to take the alternate path. By application of these rules we can obtain the following further conditions:

▶ **Lemma 35.** *W must contain the following four rules:*

1. *If W contains a factor $u...u$ then that factor contains an odd cycle as a subfactor.*
2. *There can only be one factor of W that forms an odd cycle.*
3. *Every node appears at most twice in W.*
4. *There exists at most one node that is both a member of a consecutive odd cycle and appears twice. Furthermore, such a node must be the start and end of the cycle.*

No matter how we construct W, it will violate one of these rules.

Proof of the reverse direction of Theorem 33. Since W must have the form $w_0w_1...w_0w_1$, it follows from Lemma 35 (3) that there exists a cycle C containing m such that W fully traverses C before returning to m, with possible excursions. Specifically, there exists a sequence of pairs $(w_0, w_1), (w_1, w_2)...(w_k, w_0), (w_0, w_1)$ such that $C = w_0w_1..w_kw_0$ is a cycle of G, each pair appears in W in consecutive order (i.e. $W = w_0w_1...w_1w_2...w_kw_0..w_0w_1$).

▷ Claim 36. C is an odd cycle

Proof. If C is of even length (say $2k$), then there must be an excursion from C otherwise Lemma 34 (3) would be violated. However, by Lemma 35 (1) and (2), there can be at most one such excursion as it must contain a consecutive odd cycle. Thus, the two subsequences on either side must be factors of W. Therefore, since some subsequence of W traverses C fully with one additional step, one of the two factors must take $k+1$ steps around C. This also violates Lemma 34 (3) and so C must be an odd cycle. ◁

▷ Claim 37. W consists of two odd cycles C and $\hat{C}$ connected by a path.

Proof. If C is an odd cycle there must be an excursion from it or Lemma 34 (2) would be violated. By Lemma 35 (1) and (2) the excursion must contain exactly one consecutive odd cycle which we denote by $\hat{C}$. If $\hat{C}$ does not share its starting node with C, W either forms a path between C and $\hat{C}$ or some chain of cycles. We can use Lemma 34 (5) to eliminate all even cycles of this chain, after which any odd cycles in the chain correspond to a fully traversed FEC when paired with $\hat{C}$ and so violate Lemma 34 (4). Thus, W takes a simple path from C to $\hat{C}$ and back, although possibly intersecting C along the way. ◁

▷ Claim 38. C and $\hat{C}$ intersect with each other but not the path between them.

Proof. If the path to $\hat{C}$ does intersect C, since we are taking the same path in both directions any node shared between the path and C appears in W three times. This violates Lemma 35 (3) and so C must be disjoint from the path to $\hat{C}$. Similarly $\hat{C}$ must be disjoint from the path otherwise it would violate the same lemma. If C is disjoint from $\hat{C}$ the pair would form a fully traversed FEC, thereby violating Lemma 34 (4). Thus, C and $\hat{C}$ must intersect. ◁

▷ Claim 39. Claim: C and $\hat{C}$ do not intersect.

Proof. Let $W = uv..wx_1, ..., x_k w..uv$ where $C = u...w...uv$ and the excursion to $\hat{C}$ is given by $wx_1...x_k w$. Now assume that $\hat{C}$ contains a node from C which occurs strictly before w in W. This node is on a consecutive odd cycle and appears twice. There must exist a latest such node in the ordering of C, we denote it y. Since y is on a consecutive odd cycle and appears twice in W it must be the start and end point of $\hat{C}$ by Lemma 35 (4). However, then since $y \neq w$ it must appear three times, violating Lemma 35 (3). The same argument holds taking the earliest node shared by C and $\hat{C}$ strictly after w. Thus, the only node that can be shared by C and $\hat{C}$ is W, implying that W forms two odd cycles sharing a single node. However, this is an FEC which is fully traversed and so violates Lemma 34 (4). ◁

Thus, W cannot exist and so there can be no recurrent message in S ◀