

The Mixed Birth-Death/death-Birth Moran Process

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Abstract

We study evolutionary dynamics on graphs in which each step consists of one birth and one death, referred to generally as Moran processes. In standard simplified models, there are two types of individuals: residents, who have a fitness of 1, and mutants, who have a fitness of r . Two standard update rules are used in the literature. In *Birth-death* (*Bd*), a vertex is chosen to reproduce proportional to fitness, and one of its neighbors is selected uniformly at random to die and be replaced by the offspring. In *death-Birth* (*dB*), a vertex is chosen uniformly to die, and then one of its neighbors is chosen, proportional to fitness, to place an offspring into the vacancy. Two crucial quantities are: the unconditional absorption time, which is the expected time until only residents or only mutants remain, and the fixation probability of the mutant, which is the probability that at some time the mutants occupy the whole graph. Birth-death and death-Birth rules can yield significantly different outcomes for these quantities on the same graph, rendering conclusions dependent on the update rule.

We formalize and study a unified model, the λ -mixed Moran process, in which each step is independently a *Bd* step with probability $\lambda \in [0, 1]$ and a *dB* step otherwise. We analyze this mixed process and establish a few results that form a starting point for its further study. All of our results are for undirected, connected graphs. As an interesting special case, we show at $\lambda = 1/2$ for any graph that the fixation probability when $r = 1$ with a single mutant initially on the graph is exactly $1/n$, and also at $\lambda = 1/2$ that the absorption time for any r is $O_r(n^4)$ (that is, with an r -dependent constant). We also show results for graphs that are “almost regular,” in a manner defined in the paper. We use this to show that for suitable random graphs from $G \sim G(n, p)$ and fixed $r > 1$, with high probability over the choice of graph, the absorption time is $O_r(n^4)$, the fixation probability is $O_r(n^{-2})$, and we can approximate the fixation probability in polynomial time.

Another special case is when the graph has only two possible values for the degree $\{d_1, d_2\}$ with $d_1 \leq d_2$. For those graphs, we give exact formulas for fixation probabilities under $r = 1$ and any λ , and establish $O_r(n^4 \alpha^4)$ absorption time regardless of λ , where $\alpha = d_2/d_1$. We also provide explicit formulas for the star and cycle under any r or λ .

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