Discrepancy Beyond Additive Functions with Applications to Fair Division

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Abstract

We consider a setting where we have a ground set $\mathcal{M}$ together with real-valued set functions $f_1, \ldots, f_n$, and the goal is to partition $\mathcal{M}$ into two sets S_1, S_2 such that $|f_i(S_1) - f_i(S_2)|$ is small for every i. Many results in discrepancy theory can be stated in this form with the functions f_i being additive. In this work, we initiate the study of the unstructured case where f_i is *not* assumed to be additive. We show that even without the additivity assumption, the upper bound remains at most $O(\sqrt{n \log n})$.

Our result has implications on the fair allocation of indivisible goods. In particular, we show that a consensus halving up to $O(\sqrt{n \log n})$ goods always exists for n agents with monotone utilities. Previously, only an $O(n)$ bound was known for this setting.

2012 ACM Subject Classification Mathematics of computing → Combinatorics; Applied computing → Economics

Keywords and phrases Discrepancy Theory, Fair Division

Digital Object Identifier 10.4230/LIPIcs.ITCS.2026.77

Category Extended Abstract

Related Version *Full Version*: https://arxiv.org/abs/2509.09252

Funding *Raghu Meka*: Supported by the NSF Award CCF-2217033 (EnCORE: Institute for Emerging CORE Methods in Data Science).
Warut Suksompong: Supported by the Singapore Ministry of Education under grant number MOE-T2EP20221-0001 and by an NUS Start-up Grant.