

Automata and Algebras for Probability and Nondeterminism

Ana Sokolova  

University of Salzburg, Austria

Abstract

Probabilistic models of computation have been studied for over three decades now, from foundational, logical, coalgebraic, categorical, as well as more practical verification-motivated point of view. In my work, and in this talk, we focus on the foundational, semantics side of probabilistic automata and transition systems, and in particular the relevant monads, and their algebras. The interplay of probability and non-determinism has been particularly challenging from a semantics point of view for some decades, as it does not just yield a monad. There are by now well-studied solutions to this and several monads for probability and non-determinism: some enriching the structure with convexity to obtain a monad, albeit a non-commutative one, others deliberately simplifying it, by imposing commutativity.

It is well known that monads have two faces: a computational one – we think here of the powerset monad modelling non-determinism, or the probability distribution monad modelling probabilistic choices, and a universal-algebraic one – where we think of the algebraic presentation of the monads, like semilattices for the powerset monad and (variants of) convex algebras for (variants of) the probability distribution monad. Combining non-determinism and probability yields other combined monads, among which probably the most studied is the convex-subsets-of-distributions monad. This monad is presented by so-called convex semilattices, algebraic structures that are both a semilattice and a convex algebra, with suitable distributivity connecting the operations.

From a semantics point of view, the algebraic presentations give us a nice way to define (and sometimes compute) language (aka trace) equivalence of the corresponding automata. Moreover, the presentations are useful for axiomatizations of language equivalence. From an algebraic point of view, these algebras are interesting and many questions about them are still open. We will discuss language semantics, its axiomatization, as well as some obtained and open algebraic problems for convex algebras and convex semilattices – and their computational consequences. In particular, we have a full characterization of congruences of (variants of) convex algebras, which for example yields decidability of distribution semantics; other results on congruences, e.g. a result on a congruence being finitely generated as a subalgebra, surprisingly accelerated the proof of completeness of infinite trace semantics, etc. We will review some existing results: all congruences are described and $\text{fp} = \text{fg}$ for convex algebras, termination is a black hole, useful functors are proper on convex algebras, cancellativity of convex semi-lattices; as well as mention ongoing work on: mid-point-cancellativity, subalgebras, and homomorphisms for convex semi-lattices, as well as topological convex semilattices.

This talk is based on previous joint works with Filippo Bonchi, Alexandra Silva, Valeria Vignudelli, and Harald Woracek, as well as on ongoing work and discussions with Matteo Mio, Alex Simpson, and Harald Woracek.

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