




Advancements in Online Edge Coloring Algorithms

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Abstract

We study online edge coloring, where edges of an n -vertex graph arrive sequentially and must be colored irrevocably so that adjacent edges receive different colors. The goal is to use as few colors as possible as a function of the maximum degree Δ .

This talk surveys recent progress that achieves near-optimal guarantees by leveraging martingale concentration arguments. Specifically, we show that near-optimal colorings (using $(1 + o(1))\Delta$ colors) exhibit *sharp threshold* phenomena that match long-standing lower bounds, resolving and strengthening a conjecture of Bar-Noy, Motwani, and Naor [1].

First, while the conjecture posited the existence of a *randomized* algorithm achieving a $(1 + o(1))\Delta$ -edge-coloring for maximum degree $\Delta = \omega(\log n)$, we present a *deterministic* online algorithm that achieves this guarantee in the same regime. This result matches the known impossibility result for deterministic algorithms when $\Delta = O(\log n)$, establishing a sharp threshold.

Second, improving the conditions under which near-optimal coloring is known to be possible with randomness, we present a randomized online algorithm achieving a $(1 + o(1))\Delta$ -edge-coloring already for graphs with maximum degree $\Delta = \omega(\sqrt{\log n})$. This establishes a sharp threshold for randomized algorithms, matching the lower bound in [1] for the $\Delta = O(\sqrt{\log n})$ regime.

This is joint work with Joakim Blikstad, Radu Vintan, and David Wajc [2, 3].

2012 ACM Subject Classification Theory of computation \rightarrow Online algorithms

Keywords and phrases Edge coloring, Martingale, Online algorithms

Digital Object Identifier 10.4230/LIPIcs.STACS.2026.3

Category Invited Talk

Funding Supported by the Swiss State Secretariat for Education, Research and Innovation (SERI) under contract number MB22.00054.

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43rd International Symposium on Theoretical Aspects of Computer Science (STACS 2026).

Editors: Meena Mahajan, Florin Manea, Annabelle McIver, and Nguyễn Kim Thăng

Article No. 3; pp. 3:1–3:1



Leibniz International Proceedings in Informatics

LIPICs Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

