Computational and Combinatorial Results on Conflict-Free Choosability

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Abstract

The *conflict-free closed neighborhood (CFCN*) chromatic number* of a graph $G = (V, E)$ is the smallest positive integer k for which there exists a coloring of a subset of vertices using k colors such that, for every vertex in V, there exists a color that appears exactly once in its closed neighborhood. The *conflict-free open neighborhood (CFON*) chromatic number* is defined analogously. In this paper, we study "list variants" of the above-mentioned coloring parameters. The *conflict-free closed neighborhood (CFCN*) choice number* of a graph $G = (V, E)$ is the smallest positive integer k such that for every assignment of lists of size k to its vertices, there exists a coloring of a subset of vertices, say V', in which (i) every vertex in V' receives a color from its list, and (ii) for every vertex in V there exists some color that appears exactly once in its closed neighborhood. The *conflict-free open neighborhood (CFON*) choice number* is defined analogously.

Dębski and Przybyło [Journal of Graph Theory, 2022] showed that for any graph G with maximum degree Δ, the CFCN* chromatic number of its line graph is $O(\ln \Delta)$. This result was later extended to claw-free graphs by Bhyravarapu et al. [Journal of Graph Theory, 2023], who proved that every $K_{1,k}$-free graph G admits a CFCN* coloring using $O(k \ln \Delta)$ colors. In this paper, we generalize this result to the list setting and show that every $K_{1,k}$-free graph G has a CFCN* choice number of $O(k \ln \Delta)$. Further, we answer some questions concerning the hardness of computing CFCN*/CFON* choice numbers posed by Gupta and Mathew [SOFSEM, 2026]; in particular, we show that it is NP-hard to determine whether the CFCN*/CFON* choice number a graph is equal to k, for $k = 1, 2$.

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1 Introduction

We begin by defining the notion of conflict-free (CF) choice number of hypergraphs and the two variants of this parameter on graphs, namely conflict-free open neighborhood (CFON) choice number and conflict-free closed neighborhood (CFCN) choice number. Let $\mathcal{H} = (V, \mathcal{E})$ be a hypergraph.

▶ **Definition 1** (k-assignment). *Let $\mathcal{L} = \{L_v : v \in V\}$, where each L_v is a list of colors. We say that $\mathcal{L}$ is a* k-assignment *for $\mathcal{H}$ if $|L_v| = k$ for every $v \in V$.*

▶ **Definition 2** (k-CF*-choosable). *Let $\mathcal{L} = \{L_v : v \in V\}$ be a list assignment. We say that $\mathcal{H}$ admits an* $\mathcal{L}$-CF*-coloring *if there exists a coloring $f : V' \to \bigcup_{v \in V'} L_v$, for some $V' \subseteq V$, such that $f(v) \in L_v$ for all $v \in V'$, and every hyperedge $E \in \mathcal{E}$ contains a vertex whose color is unique within E. The hypergraph $\mathcal{H}$ is called* k-CF*-choosable *if it admits an $\mathcal{L}$-CF*-coloring for every k-assignment $\mathcal{L}$.*

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▶ **Definition 3** (CF* choice number). *The minimum positive integer k such that $\mathcal{H}$ is k-CF*-choosable is called the* CF*-choice number *of $\mathcal{H}$. It is denoted by $ch^*_{CF}(\mathcal{H})$.*

Let G be a finite, simple, undirected graph. Let $V(G)$ denote its vertex set and $E(G)$ denote its edge set. For a vertex $v \in V(G)$, the *open neighborhood* of v in G is the set of all vertices adjacent to v, and is denoted by $N_G(v)$. The *closed neighborhood* of v is the set $\{v\} \cup N_G(v)$, and is denoted by $N_G[v]$.

▶ **Definition 4** (CFON* choice number). *Given a list assignment $\mathcal{L} = \{L_v \ : \ v \in V(G)\}$ for the graph G, we say G is $\mathcal{L}$-*CFON*-colorable *if there exists a coloring $f : V' \rightarrow \bigcup_{v\in V'} L_v$, where $V' \subseteq V(G)$, such that $f(v) \in L_v \ \forall v \in V'$, and every vertex v sees a unique color in its open neighborhood under f. We say that G is k-CFON*-choosable if for every k-assignment $\mathcal{L}$, G is $\mathcal{L}$-CFON*-colorable. The minimum k for which G is k-CFON*-choosable is called the* CFON* choice number *of G. This is denoted by $ch^*_{ON}(G)$.*

Analogously, one can define the CFCN* choice number of a graph G which will be denoted by $ch^*_{CN}(G)$.

Let us define a special list assignment $\mathcal{L}^{[k]} = \{L_v : v \in V\}$, where $L_v = \{1, 2, \ldots, k\}$, for every $v \in V$. That is, under this special list assignment, every vertex is assigned the same list which is $\{1, 2, \ldots, k\}$. We say $\mathcal{H}$ is *k-CF*-colorable* if $\mathcal{H}$ admits an $\mathcal{L}^{[k]}$-CF*-coloring. The minimum k for which $\mathcal{H}$ is k-CF*-colorable is called the *CF*-chromatic number* of $\mathcal{H}$, denoted by $\chi^*_{CF}(\mathcal{H})$. We say G is *k-CFON*-colorable* if G is *$\mathcal{L}^{[k]}$-CFON*-colorable*. The minimum k for which G is k-CFON*-colorable is called the *CFON* chromatic number* of G. This is denoted by $\chi^*_{ON}(G)$. Analgously, one can define *CFCN* chromatic number* of G which is denoted by $\chi^*_{CN}(G)$.

▶ Remark 5. In the definitions above, we did not insist on coloring all the vertices of the hypergraph (or graph) under consideration. We colored only a subset V' of the set of vertices thus leading to a partial coloring. The star symbol as a superscript in the parameters $ch^*_{CF}(\mathcal{H})$, $ch^*_{ON}(G)$, $ch^*_{CN}(G)$, $\chi^*_{CF}(\mathcal{H})$, $\chi^*_{ON}(G)$, and $\chi^*_{CN}(G)$ indicate the underlying partial nature of coloring. That is, we do not insist on coloring all the vertices. However, if we insist on coloring all the vertices, then the corresponding parameters are denoted $ch_{CF}(\mathcal{H})$, $ch_{ON}(G)$, $ch_{CN}(G)$, $\chi_{CF}(\mathcal{H})$, $\chi_{ON}(G)$, and $\chi_{CN}(G)$, respectively.

▶ Remark 6. Given a graph G, let (i) $\mathcal{H}_{(G)} = (V, \mathcal{E})$, where $V = V(G)$ and $\mathcal{E} = \{N_G(v) \ : \ v \in V(G)\}$, and (ii) $\mathcal{H}_{[G]} = (V', \mathcal{E}')$, where $V' = V(G)$ and $\mathcal{E}' = \{N_G[v] \ : \ v \in V(G)\}$. A CFON* (resp. CFCN*) coloring of G is a CF* coloring of the hypergraph $\mathcal{H}_{(G)}$ (resp., $\mathcal{H}_{[G]}$). The converse is also true.

Let $\mathcal{H}$ (resp. G) be a hypergraph (resp., graph) on n vertices. The following propositions connect the various conflict-free parameters with each other. Proposition 7 establishes the relationship between partial and full conflict-free coloring. Using this we can extend all the bounds established in this paper to the full coloring setting.

▶ **Proposition 7.** [13] (i) $\chi_{CF}(\mathcal{H}) \leq \chi^*_{CF}(\mathcal{H}) + 1$, (ii) $ch_{CF}(\mathcal{H}) = O(ch^*_{CF}(\mathcal{H}) + \ln n)$.

The following proposition connects the conflict-free notions in the classical and list settings.

▶ **Proposition 8.** [7] $\chi^*_{\mathrm{CF}}(\mathcal{H}) \leq ch^*_{\mathrm{CF}}(\mathcal{H}) \leq \chi^*_{\mathrm{CF}}(\mathcal{H}) \cdot \ln n + 1$.

Proposition 9 below relates the CFON* notion with the CFCN* notion.

▶ **Proposition 9.** (i) [19] $\chi^*_{CN}(G) \leq 2\,\chi^*_{ON}(G)$, (ii) [13] $ch^*_{CN}(G) \leq 2\,ch^*_{ON}(G) \cdot \ln n + 1$.

Motivated by its applications in the frequency-assignment problem in wireless networks, conflict-free coloring was first studied by Even et al. [11] in 2003. Conflict-free coloring was later studied in several different settings (see [2, 4–6, 8, 9, 12, 14, 16, 19, 20]). For a survey on conflict-free coloring and its applications, see Smorodinsky [21]. Conflict-free coloring has found applications in various areas, including frequency assignment in cellular networks, energy-efficient sensor networks, vertex ranking with applications in VLSI design and operations research, and the pliable index coding problem in coding theory [15]. The classical conflict-free coloring framework assumes that all vertices can choose colors from a common global set, an assumption that is often too restrictive in practical settings. This motivates the study of the list variant, in which each vertex is assigned a list of permissible colors, providing a more realistic and general model of constrained coloring scenarios. For detailed applications of conflict-free choosability, see [13].

1.1 Background and Our Contribution

Conflict-free choosability was first studied by Cheilaris, Smorodinsky, and Sulovskỳ [7] in the context of certain geometric hypergraphs. Gupta and Mathew [13] showed that, for a graph G with maximum degree Δ, the CFCN* choice number of G is $O(\ln^2 \Delta)$ thus generalizing a similar result of Bhyravarapu et al. [4] for the CFCN* chromatic number of G. Recently, Wu and Zhang [22] showed that for every planar graph G, $ch_{ON}(G) \leq 12$.

Dębski and Przybyło [8] showed that, for any graph G with maximum degree Δ, $\chi^*_{CN}(L(G)) = O(\ln \Delta)$ where $L(G)$ denotes the line graph of G. Line graphs are known to be a subclass of claw-free (or $K_{1,3}$-free) graphs. It was asked as an open question in [8] to extend the above result to claw-free graphs. Bhyravarapu et al. [3] solved this open question by showing that if G is $K_{1,k}$-free with maximum degree Δ, then $\chi^*_{CN}(G) = O(k \ln \Delta)$. In this paper, we generalize this result to a list coloring setting. We prove that if G is a $K_{1,k}$-free graph with maximum degree Δ, then $ch^*_{CN}(G) = O(k \ln \Delta)$.

In Section 3, we investigate the hardness of decision problems related to CFON/CFCN choosability and their partial variants which are defined below.

k-CFON*-CHOOSABILITY PROBLEM (k-CFON*-CH PROBLEM)
Input: A graph G and a positive integer k.
Question: Is $ch^*_{ON}(G) \leq k$?

k-CFON-CHOOSABILITY PROBLEM (k-CFON-CH PROBLEM)
Input: A graph G and a positive integer k.
Question: Is $ch_{ON}(G) \leq k$?

Analogously, we define the k-CFCN*-CHOOSABILITY PROBLEM (k-CFCN*-CH PROBLEM) and the k-CFCN-CHOOSABILITY PROBLEM (k-CFCN-CH PROBLEM).

Gupta and Mathew [13] proved that the following decision problems are Π_2^P-complete: (i) the k-CFON-CHOOSABILITY problem, (ii) the k-CFON*-CHOOSABILITY problem, and (iii) the $(2, k)$-CFCN-CHOOSABILITY problem. These hardness results hold for bipartite graphs for any fixed $k \geq 3$, for planar triangle-free graphs when $k = 3$, and for planar graphs when $k = 4$. They also posed the following problems as open questions. The computational complexity of (i) k-CFON*-CHOOSABILITY for $k = 1, 2$, (ii) k-CFON-CHOOSABILITY for $k = 2$, (iii) k-CFCN*-CHOOSABILITY for all $k \geq 1$, and (iv) k-CFCN-CHOOSABILITY for all $k \geq 2$ remains unresolved.

In this paper, we make progress in some of the open questions posed by Gupta and Mathew [13]. In particular, we show that the following are NP-hard: (i) k-CFON*-CHOOSABILITY for $k = 1, 2$, and (ii) k-CFCN*-CHOOSABILITY for $k = 1, 2$.

1.2 Preliminaries

1.2.1 Definitions and Notations

We study simple, finite, and undirected graphs throughout this paper. When discussing open neighborhood coloring, we assume the graph under consideration has no isolated vertices. The *degree* of a vertex $v \in V$ in a given hypergraph $\mathcal{H} = (V, \mathcal{E})$, denoted by $d_{\mathcal{H}}(v)$, is the number of hyperedges that v is present in. The *maximum degree* of $\mathcal{H}$ is defined as $\max\{d_{\mathcal{H}}(v) : v \in V\}$. Let $K_{1,k}$ denote the complete bipartite graph with one vertex in one part and k vertices in other part. A graph is called claw-free if it does not contain an induced subgraph isomorphic to the claw graph $K_{1,3}$.

1.2.2 Probabilistic Inequalities and Known Results

The following results on conflict-free coloring are used in the proof of Theorem 15.

▶ **Theorem 10.** [7] *For any hypergraph $\mathcal{H}$ with maximum degree at most Δ, $ch_{CF}(\mathcal{H}) \leq \Delta+1$.*

▶ **Theorem 11.** [13] *For any graph G with maximum degree Δ, $ch^*_{\mathrm{CN}}(G) = O(\ln^2 \Delta)$.*

We state the Local Lemma [10] and a variant of the Talagrand's inequality due to [17] below. These are used in the proof of Lemma 14.

▶ **Lemma 12** (The Local Lemma, [10])**.** *Let $A_1, \ldots, A_n$ be events in an arbitrary probability space. Suppose that each event A_i is mutually independent of a set of all the other events A_j but at most d, and that $Pr[A_i] \leq p$ for all $i \in [n]$. If $4pd \leq 1$, then $Pr[\cap_{i=1}^{n} \overline{A_i}] > 0$.*

▶ **Theorem 13** (Talagrand's Inequality, [17])**.** *Let X be a non-negative random variable determined by the independent trials $T_1, \ldots, T_n$. Suppose that for every set of possible outcomes of the trials, we have:*

(i) *changing the outcome of any one trial can affect X by at most a; and*
(ii) *for each $s > 0$, if $X \geq s$ then there is a set of at most bs trials whose outcomes certify that $X \geq s$.*

Then for any $t \geq 0$, we have

$$Pr[|X - E[X]| > t + 20a\sqrt{bE[X]} + 64a^2 b] \leq 4e^{-\frac{t^2}{8a^2 b(E[X]+t)}}.$$

2 Combinatorial Results for Conflict-free Choosability

The following lemma gives a coloring of the vertices of a "nearly uniform" hypergraph in which every hyperedge E sees at least $\frac{|E|}{8}$ distinctly colored vertices. This lemma plays a crucial role in proving Theorem 15.

▶ **Lemma 14.** *Let $\mathcal{H} = (V, \mathcal{E})$ be a hypergraph satisfying the conditions below:*

(i) *Every hyperedge intersects with at most Γ other hyperedges, and*
(ii) *For every hyperedge $E \in \mathcal{E}$, $\alpha \leq |E| \leq \beta$, where $\alpha = \max(2^{12}, \lceil 136 \ln(16\Gamma) \rceil)$.*

Let $\mathcal{L} = \{L_v : v \in V(\mathcal{H})\}$ be a 32β-assignment for $\mathcal{H}$. Then there is an $\mathcal{L}$-coloring of $\mathcal{H}$ such that every hyperedge E sees at least $\frac{|E|}{8}$ unique colors.

Proof. For each vertex in V, assign a color that is chosen independently, uniformly at random from its list of size 32β. For any hyperedge $E \in \mathcal{E}$, let X_E be a random variable that denotes the number of vertices in E whose color is not unique in E. Thus, for any vertex v in E, we have

$$Pr[\text{color of } v \text{ is not unique in } E] \leq 1 - \left(1 - \frac{1}{32\beta}\right)^{|E|-1} \leq 1 - \left(1 - \frac{|E|-1}{32\beta}\right) \leq \frac{|E|}{32\beta}.$$

Then, by linearity of expectation,

$$E[X_E] \leq \frac{|E|^2}{32\beta} \leq \frac{|E|}{32}.$$

We claim that the random variable X_E satisfies the assumptions of Theorem 13 (Talagrand's Inequality) with $n = |E|$, $a = 2$, and $b = 2$. The value of X_E is determined by $|E|$ independent trials. Changing the outcome of any trial can affect X_E by at most 2. For any $s > 0$, if it is given that $X_E \geq s$, then there is a set of at most $2s$ trials whose outcomes would ensure that X_E is at least s, regardless of the outcomes of the remaining trials. This proves our claim. Let $A = \frac{|E|}{2} + 20a\sqrt{bE[X]} + 64a^2b = \frac{|E|}{2} + 40\sqrt{2E[X]} + 512$. Applying Theorem 13 with $t = |E|/2$, we get

$$\begin{aligned}
Pr[|X_E - E[X_E]| > A] &\leq 4e^{-\frac{|E|^2/4}{64(E[X_E]+|E|/2)}} \\
&\leq 4e^{-\frac{|E|^2}{256(17|E|/32)}} \quad (\text{since } E[X_E] \leq \frac{|E|}{32}) \\
&\leq 4e^{-\frac{|E|}{136}} \\
&\leq 4e^{-\ln(16\Gamma)} \quad (\text{since } |E| \geq \alpha \geq \lceil 136\ln(16\Gamma)\rceil) \\
&= \frac{1}{4\Gamma}.
\end{aligned}$$

Since $E[X_E] \leq \frac{|E|}{32}$, we have

$$E[X_E] + A \leq \frac{17|E|}{32} + 40\sqrt{\frac{2|E|}{32}} + 512 = \frac{17|E|}{32} + 10\sqrt{|E|} + 512 < \frac{7|E|}{8},$$

where the last inequality follows from the fact that $|E| \geq \alpha \geq 2^{12}$. Thus,

$$\begin{aligned}
Pr\left[X_E \geq \frac{7|E|}{8}\right] &= Pr\left[X_E - \frac{7|E|}{8} \geq 0\right] \\
&\leq Pr[X_E - (E[X_E] + A) > 0] \\
&= Pr[(X_E - E[X_E]) > A] \\
&\leq Pr[|X_E - E[X_E]| > A] \\
&\leq \frac{1}{4\Gamma}.
\end{aligned}$$

Let A_E denote the bad event that $X_E \geq \frac{7|E|}{8}$. From the above calculations, we know that $Pr[A_E] \leq \frac{1}{4\Gamma}$. We can apply the Local Lemma (Lemma 12) on the events A_E, for all hyperedges $E \in \mathcal{E}$. Since each hyperedge intersects with at most Γ other hyperedges, and $4 \cdot \frac{1}{4\Gamma} \cdot \Gamma \leq 1$, we get $Pr[\cap_{E\in\mathcal{E}}(\overline{A}_E)] > 0$. Thus, $\mathcal{H}$ admits an $\mathcal{L}$-coloring for any 32β-assignment $\mathcal{L}$ such that every hyperedge E in $\mathcal{H}$ sees at least $\frac{|E|}{8}$ unique colors. ◀

We show an upper bound for the CFCN* choice number for $K_{1,k}$-free graphs.

▶ **Theorem 15.** *Let G be a $K_{1,k}$-free graph with maximum degree Δ. Then $ch^*_{CN}(G) = O(k \ln \Delta)$.*

Proof. Suppose $k > \frac{\ln \Delta}{8}$. Then by Theorem 11, $ch^*_{CN}(G) = O(k \ln \Delta)$. For the rest of the proof, we shall assume that $k \leq \frac{\ln \Delta}{8}$. Construct a maximal independent set A of G. We remove all the vertices that belong to A from G to obtain a new graph G'.

▶ **Observation 16.** *Since A is a maximal independent set in G, every vertex in G' has some neighbor in A. Further, since G is $K_{1,k}$-free, every vertex in G' has at most $k-1$ neighbors in A.*

We consider a greedy, proper coloring of G' using s colors, where $1 \leq s \leq \Delta + 1$, as described below. Vertices are colored one by one, and each vertex is given the smallest available color that is not used by any of its colored neighbors so far. Let $S_1, S_2, \ldots, S_s$ be the color classes given by this coloring.

▶ **Observation 17.** *Let $2 \leq i \leq s$. Every vertex in the color class S_i has at least one neighbor in each color class S_j, for every $j < i$.*

We prove the observation by contradiction. Suppose $v \in S_i$ has no neighbor in S_j, for some $j < i$. Then the greedy algorithm would have given v a color of value at most j.

▶ **Observation 18.** *Since G is $K_{1,k}$-free, every vertex in G has at most $k-1$ neighbors in each S_i, where $1 \leq i \leq s$.*

We partition the color classes obtained from the above proper coloring of G' into two parts, B and C. Let $B := S_1 \cup S_2 \cup \cdots \cup S_b$, where $b = \min\{s, \max\{2^{12}, 272 \ln(4\Delta)\}\}$. Thus, $b \leq s$. Let $C := S_{b+1} \cup \cdots \cup S_s$. Suppose $b = s$. Then, $C = \emptyset$ which makes the proof easier as we won't need to find uniquely colored neighbors for the vertices in C. Further, when $C = \emptyset$, our proof will go through as we do not color any vertex in C in our proof and therefore no vertex will have its uniquely colored neighbor in C, For the rest of the proof, we assume that $b < s$ and therefore (i) $b = \max\{2^{12}, 272 \ln(4\Delta)\}$, and (ii) $C \neq \emptyset$. We make the following observation which follows from Observations 17 and 18.

▶ **Observation 19.** *Every vertex in C has at least b neighbors in B. Moreover, for every vertex $v \in V(G)$, the number of neighbors of v that belongs to B is at most $(k-1)b$.*

Let $\mathcal{L} = \{L_v \ : \ v \in V(G)\}$ be an r-assignment for G given by the adversary, where $r = 2^{18} k \ln \Delta$. We obtain the desired $\mathcal{L}$-CFCN*-coloring of G by list CF* coloring two hypergraphs $\mathcal{H}_1$ and $\mathcal{H}_2$ that are defined below.

- Let $\mathcal{H}_1 = (V_1, \mathcal{E}_1)$, where $V_1 = A$ and $\mathcal{E}_1 = \{N_G[v] \cap A : v \in (A \cup B)\}$. By Observation 19, the maximum degree of $\mathcal{H}_1$ is at most $(k-1)b + 1$. Thus, by Theorem 10, we have $ch^*_{CF}(\mathcal{H}_1) \leq (k-1)b+1$. Thus $\mathcal{H}_1$ is $\mathcal{L}$-CF*-colorable. Let f_1 denote this coloring. Under f_1, every vertex in $A \cup B$ sees a unique color in its closed neighborhood.

Next, by list CF* coloring a hypergraph $\mathcal{H}_2$ (defined below) having vertex set B, we intend to take care of every vertex in C. This however can lead to two problems that are described below:

(i) For a vertex $a \in A$, let c_a denote the unique color seen by a under the coloring f_1. We should ensure that none of the neighbors of a in B receive the color c_a. For this, we update the lists of every vertex $u \in B$ as $\mathcal{L}'_u = \mathcal{L}_u \setminus X$, where $X = \{c_a : a \in N_G(u) \cap A\}$. By Observation 16, we have $|X| \leq (k-1)$.

(ii) For a vertex $w \in B$, let c_w denote the unique color seen by w under the coloring f_1. We should ensure that none of the closed neighbors of w in B receive the color c_w. For this, we update the lists of every vertex $u \in B$ as $\mathcal{L}''_u = \mathcal{L}'_u \setminus Y$, where $Y = \{c_w : w \in N_G(u) \cap B\}$. By Observation 19, we have $|Y| \leq (k-1)b$. Thus, for every vertex $v \in B$, we remove at most $(k-1) + (k-1)b = (b+1)(k-1)$ colors from $v's$ list in total.

- Thus $\mathcal{L}'' = \{L''_v : v \in V(B)\}$ is an r'-assignment for every vertex v in B, where $r' = r - (b+1)(k-1) \geq 2^{17}k \ln \Delta$. Let $\mathcal{H}_2 = (V_2, \mathcal{E}_2)$, where $V_2 = B$ and $\mathcal{E}_2 = \{N_{G'}(v) \cap B : v \in C\}$. Fix a vertex $v \in C$. Notice that v shares a common neighbor with at most Δ^2 other vertices in C. Thus, every hyperedge in $\mathcal{H}_2$ overlaps with at most Γ other hyperedges, where $\Gamma \leq \Delta^2$.
  By Observation 19, any vertex $v \in C$ has at least b and at most $(k-1)b$ neighbors in B. So for each $E \in \mathcal{E}_2$, we have $b \leq |E| \leq (k-1)b$. Further, $32(k-1)b = \max\{2^{17}(k-1), 8704(k-1)\ln(4\Delta)\} < 2^{17}k \ln \Delta \leq r'$. By applying Lemma 14 to $\mathcal{H}_2$, with $\alpha = b$, $\beta = (k-1)b$, and $\Gamma \leq \Delta^2$, we get an $\mathcal{L}''$-coloring of $\mathcal{H}_2$ such that every hyperedge E sees at least $\frac{|E|}{8}$ unique color. From the definition of $\mathcal{H}_2$, it follows that every vertex v in C is getting to see at least $\frac{b}{8}$ uniquely colored vertices among its neighbors in B. Since $b \geq 272\ln(4\Delta) \geq 2176k$ (since $k \leq \frac{\ln \Delta}{8}$), v sees at least $272k$ uniquely colored vertices among its neighbors in B. Some of these colors may be used by a neighbor of v in A. Since v has at most $k-1$ neighbors in A, v still sees at least $272k - (k-1)$ uniquely colored vertices in its neighborhood.

The above coloring ensures that every vertex sees a uniquely colored vertex in its closed neighborhood. In addition, each vertex in the above coloring process receives a color at most once. Some vertices, that are not part of any hypergarph, remain uncolored. ◀

3 Computational Results for Conflict-free Choosability

This section studies the computational complexity of the CFON*/CFCN* choosability problem.

Mulzer and Rote [18] showed that Positive Planar 1-in-3-SAT is NP-hard. We define a few necessary notions before introducing Positive Planar 1-in-3-SAT. Given a 3-CNF Boolean formula ϕ on n variables, say $X = \{x_1, \ldots, x_n\}$, and m clauses, say $C = \{c_1, \ldots, c_m\}$, its *associated graph* G_ϕ is a bipartite graph on $n+m$ vertices with bipartition $\{X, C\}$ where a vertex x_i is connected by an edge to a vertex c_j if and only if x_i or $\overline{x_i}$ is present in the clause c_j. Throughout this section, when we refer to x_i (or c_i), it can either be the variable x_i (resp., the clause c_i) in ϕ or the corresponding vertex x_i (resp., c_i) in G_ϕ. This will either be explicitely mentioned, or it will be clear from the context. A Boolean formula ϕ is said to be *planar* if its associated graph G_ϕ is planar. A Boolean formula ϕ is called *positive* if every literal in ϕ is positive.

Positive Planar 1-in-3-SAT Problem[see [18]]
Input: A positive planar 3-CNF Boolean formula ϕ on variables $X = \{x_1, \ldots, x_n\}$ and clauses $C = \{c_1, \ldots, c_m\}$ of the form

$$\phi = c_1 \wedge c_2 \wedge \cdots \wedge c_m,$$

where each c_i is of the form $(x_i \vee x_j \vee x_k)$.
Question: Does there exist a truth assignment to the variables such that each clause sees exactly one True variable?

The only connected graph with a CFON-choice-number equal to 1 is a path on 2 vertices. Therefore, deciding whether a graph is 1-CFON choosable can be done in polynomial time. Similarly, the only graph with a CFCN-choice-number equal to 1 is the edgeless graph, i.e., a graph consisting solely of isolated vertices. Therefore, deciding whether a graph is 1-CFCN choosable can be done in polynomial time.

Abel et al. [1] proved that it is NP-complete to decide whether (i) a planar bipartite graph is 1-CFON*-colorable, and (ii) a planar bipartite graph is 1-CFCN*-colorable. Below we prove a proposition that equates the notion of 1-CFON* (resp. 1-CFCN*) colorability with 1-CFON* (resp. 1-CFCN*) choosability.

▶ **Proposition 20.**

(i) A graph G is 1-CFON*-colorable if and only if G is 1-CFON*-choosable.
(ii) A graph G is 1-CFCN*-colorable if and only if G is 1-CFCN*-choosable.

Proof. We prove (i), the proof of (ii) is analogous. First, suppose that G is 1-CFON*-colorable. We show that G is 1-CFON*-choosable. Let $S \subseteq V(G)$ be the set of vertices that receive a color in a 1-CFON*-coloring of G. Let $\mathcal{L} = \{L_v : v \in V(G)\}$ be a 1-assignment for G. We color each vertex in S with the only color in its list. This produces a valid $\mathcal{L}$-CFON*-coloring of G.

Conversely, suppose that G is 1-CFON*-choosable. We show that G is 1-CFON*-colorable. Let $\mathcal{L} = \{L_v : v \in V(G)\}$ be a 1-assignment where $L_v = \{R\}$ for every vertex $v \in V(G)$. Let $Q \subseteq V(G)$ be the set of vertices that receive color R under an $\mathcal{L}$CFON*-coloring of G. Note that the coloring obtained is indeed a valid 1-CFON*-coloring of G. ◀

Thus, by Proposition 20 and Theorems 1.3 and 6.5 in Abel et. al. [1], we conclude that

(i) the 1-CFON*-CHOOSABILITY PROBLEM on planar bipartite graphs is NP-complete, and
(ii) the 1-CFCN*-CHOOSABILITY PROBLEM on planar bipartite graphs is NP-complete.

We provide alternate proofs to these problems. We present a significantly simpler construction than the one used by Abel et al. [1].

▶ **Lemma 21.** *1-CFON*-CH PROBLEM is in NP.*

Proof. Given a graph G, a subset S of vertices of G is a *Perfect Induced Matching Dominating Set* (PIMDS) for G if (i) the subgraph of G induced by S is a matching, and (ii) every vertex in G has exactly one neighbor in S. It can be verified that G is 1-CFON*-choosable if and only if G contains a PIMDS, a property which can be verified in polynomial time. ◀

▶ **Theorem 22.** *1-CFON*-CHOOSABILITY PROBLEM is NP-hard on planar bipartite graphs.*

Proof. We give a polynomial-time reduction from the POSITIVE PLANAR 1-IN-3-SAT PROBLEM.

Construction. From an input 3-CNF Boolean formula ϕ on a variable set $X = \{x_1, \ldots, x_n\}$ and a clause set $C = \{c_1, \ldots, c_m\}$ for the POSITIVE PLANAR 1-IN-3-SAT PROBLEM, we construct its associated graph G_ϕ which is known to be planar and bipartite. We modify G_ϕ as described below. For each $i \in \{1, \ldots, n\}$, we attach a distinct path on 2 vertices to x_i (thus forming an induced path on 3 vertices called the *variable gadget of* x_i). The resultant planar bipartite graph is called G'_ϕ. It has $m + 3n$ vertices. Figure 1 shows the graph G'_ϕ, where $\phi = (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee x_5) \wedge (x_1 \vee x_3 \vee x_5) \wedge (x_3 \vee x_4 \vee x_5)$.

▷ Claim 23. The formula ϕ is a yes instance of the POSITIVE PLANAR 1-IN-3-SAT PROBLEM if and only if G'_ϕ is 1-CFON*-choosable.

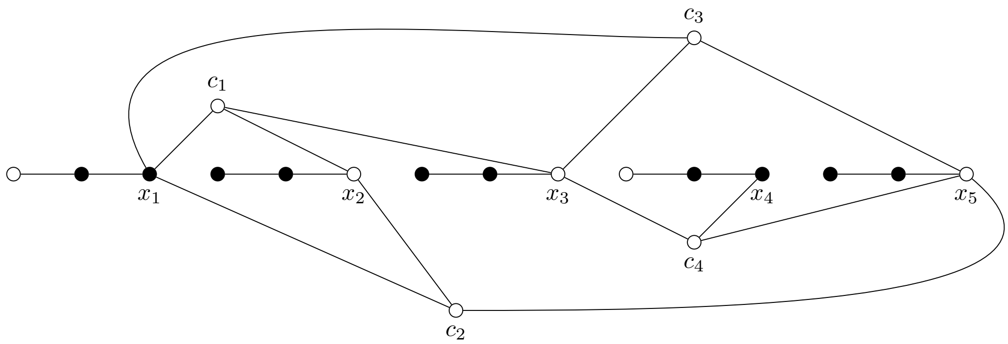

Figure 1 The graph G'_ϕ, where $\phi = (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee x_5) \wedge (x_1 \vee x_3 \vee x_5) \wedge (x_3 \vee x_4 \vee x_5)$. ϕ is satisfiable by setting x_1, x_4 to be true. The black vertices will receive a color from their respective lists in any valid CFON*-coloring of the graph.

Proof. Suppose ϕ is a yes instance, that is, there is an assignment of truth values to its variables where each clause sees exactly one TRUE variable and two FALSE variables. Let $\mathcal{L} = \{L_v \ : \ v \in V(G'_\phi)\}$ be a 1-assignment for G'_ϕ. Let $i \in \{1, \ldots n\}$. If $x_i =$ TRUE, then $\mathcal{L}$-color the vertex x_i and its only neighbor in the variable gadget of x_i. Otherwise, leave x_i uncolored and $\mathcal{L}$-color the other two vertices in the variable gadget of x_i. The reader may verify that, under this coloring, every vertex has exactly one vertex in its open neighborhood that is colored.

Suppose, G'_ϕ is 1-CFON*-choosable. Let $\mathcal{L} = \{L_v \ : \ v \in V(G'_\phi)\}$ be a 1-assignment for G'_ϕ, where $L_v = \{c\}$, for every $v \in V(G'_\phi)$. Consider an $\mathcal{L}$-CFON* coloring f of G'_ϕ. We observe that, under the coloring f, the variable gadget for each variable x_i has exactly two vertices colored; either the middle vertex and the vertex to its right which is x_i or the middle vertex and the vertex to its left. We set $x_i =$ TRUE if the vertex x_i is colored under f. Otherwise, we set $x_i =$ FALSE. Since f is a valid CFON* coloring of G'_ϕ, every clause vertex has exactly one of its neighbors colored. Thus, under the above truth assignment, every clause will see exactly one TRUE variable. ◁

◀

Below, we show that the 1-CFCN*-CH PROBLEM is NP-hard for planar bipartite graphs (by reducing from the POSITIVE PLANAR 1-IN-3-SAT PROBLEM).

▶ **Lemma 24.** *1-CFCN*-CH PROBLEM is in NP.*

Proof. Given a graph G, a *Perfect Independent Dominating Eet (PIDS)* is a set of vertices $S \subseteq V(G)$ such that S is an independent set and every vertex outside S has exactly one neighbor in S. That is, for each $v \in V(G)$, $|N_G(v) \cap S| = 1$. It can be verified that G is 1-CFCN*-choosable if and only if G contains a PIDS, a property which can be verified in polynomial time. ◀

▶ **Theorem 25.** *1-CFCN*-CHOOSABILITY PROBLEM is NP-hard on planar bipartite graphs.*

Proof. We give a polynomial-time reduction from the POSITIVE PLANAR 1-IN-3-SAT PROBLEM.

Construction. From an input 3-CNF Boolean formula ϕ on a variable set $X = \{x_1, \ldots, x_n\}$ and a clause set $C = \{c_1, \ldots, c_m\}$ for the POSITIVE PLANAR 1-3-SAT PROBLEM, we construct its associated graph G_ϕ which is known to be bipartite. We modify G_ϕ as described below. For each $i \in \{1, \ldots, n\}$, we attach a pendant vertex v_i to x_i. The resultant bipartite graph is called G''_ϕ. It has $m + 2n$ vertices. Figure 2 shows the graph G''_ϕ, where $\phi = (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee x_5) \wedge (x_1 \vee x_3 \vee x_5) \wedge (x_3 \vee x_4 \vee x_5)$.

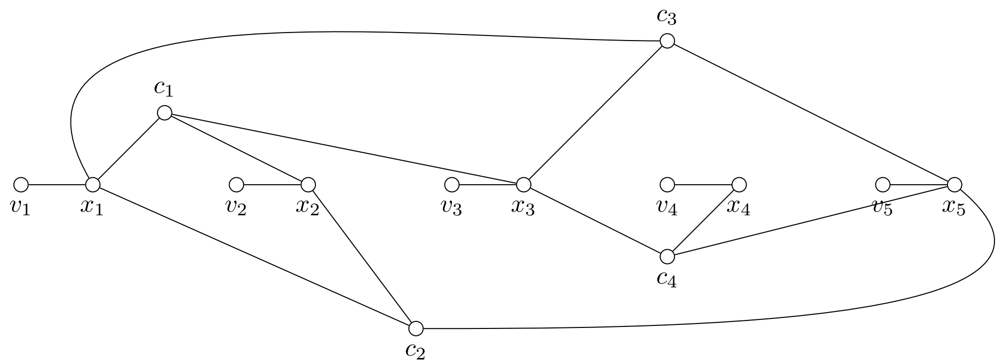

Figure 2 The graph G''_ϕ , where $\phi = (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee x_5) \wedge (x_1 \vee x_3 \vee x_5) \wedge (x_3 \vee x_4 \vee x_5)$. ϕ is satisfiable by setting x_1, x_4 to be true.

▷ Claim 26. The formula ϕ is a yes instance of the Positive Planar 1-in-3-SAT Problem if and only if G''_ϕ is 1-CFCN*-choosable.

Proof. Suppose ϕ is a yes instance, that is, there is an assignment of truth values to its variables where each clause sees exactly one True variable and two False variables. Let $\mathcal{L} = \{L_v \ : \ v \in V(G''_\phi)\}$ be a 1-assignment for G''_ϕ. Let $i \in \{1, \ldots n\}$. If $x_i =$ True, then $\mathcal{L}$-color the vertex x_i. Otherwise, $\mathcal{L}$-color the vertex v_i. The reader may verify that, under this coloring, every vertex has exactly one vertex in its closed neighborhood that is colored.

Suppose, G''_ϕ is 1-CFCN*-choosable. Let $\mathcal{L} = \{L_v \ : \ v \in V(G''_\phi)\}$ be a 1-assignment for G''_ϕ, where $L_v = \{c\}$, for every $v \in V(G''_\phi)$. Consider an $\mathcal{L}$-CFCN* coloring f of G''_ϕ. We observe that, under the coloring f, the variable gadget for each variable x_i has exactly one vertex colored; either the vertex x_i or the vertex v_i. We set $x_i =$ True if the vertex x_i is colored under f. Otherwise, we set $x_i =$ False. Since f is a valid CFCN* coloring of G''_ϕ, every clause vertex has exactly one of its neighbors colored. Thus, under the above truth assignment, every clause will see exactly one True variable. ◁

◀

Now we show that the 2-CFCN*-choosability problem is NP-hard by reducing from the 1-CFCN*-choosability problem. Below, we present a construction of a graph H_G derived from a given graph G.

Construction. We construct H_G by taking 12 vertex-disjoint copies of G, denoted $G^a_{12}, G^b_{12}, G^a_{13}, G^b_{13}, G^a_{14}, G^b_{14}, G^a_{23}, G^b_{23}, G^a_{24}, G^b_{24}, G^a_{34}, G^b_{34}$, and adding four new vertices v_1, v_2, v_3, v_4. For every $1 \leq i < j \leq 4$, $z \in \{a, b\}$, we add an edge from (i) v_i to every vertex in G^z_{ij} and (ii) v_j to every vertex in G^z_{ij}. These are the only edges of H_G. Thus, $|E(H_G)| = 12 \cdot |E(G)| + |\{v_1, v_2, v_3, v_4\}| \cdot 6 \cdot |V(G)| = 12 \cdot |E(G)| + 24 \cdot |V(G)|$. Figure 3 illustrates the construction of H_G from G.

▶ **Lemma 27.** *A graph G is 1-CFCN*-choosable if and only if the graph H_G is 2-CFCN*-choosable.*

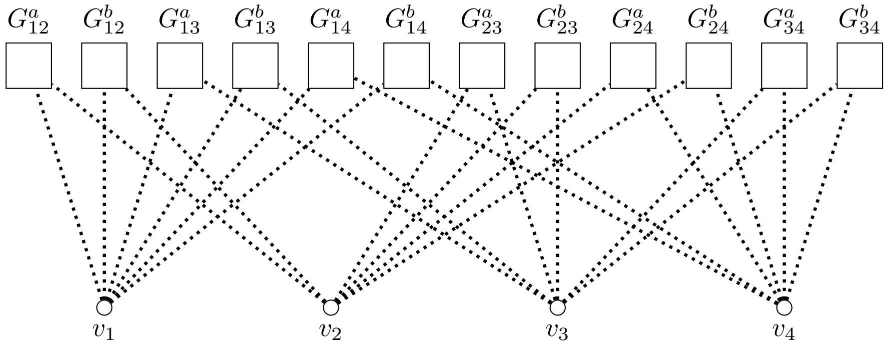

Figure 3 The graph H_G. For $\ell \in [4]$, $1 \leq i < j \leq 4$, and $z \in \{a, b\}$, a dotted edge from v_ℓ to G^z_{ij} indicates that v_ℓ is adjacent to every vertex in G^z_{ij}.

Proof. Suppose G is 1-CFCN*-choosable. We will show that H_G is 2-CFCN*-choosable. Let $\mathcal{L} = \{L_v \ : \ v \in V(H_G)\}$ be a 2-assignment for H_G. We first color the vertices v_1, v_2, and v_3 by assigning each an arbitrary color from its list, ensuring that at most two vertices receive the same color. The vertex v_4 remains uncolored. The vertices v_1, v_2, and v_3 see their own color as a unique color. Let c be the color given to v_1. For a vertex v_4 to see a unique color in its closed neighborhood, we color exactly one vertex w in G^a_{14} with a color other than color c from its list. The color of the vertex w serves as a unique color in the closed neighborhood of v_4. We now have two cases:

(i) If all three vertices v_1, v_2, v_3 receive distinct colors, then it can be verified that every vertex in H_G has a unique color in its closed neighborhood. We do not color the remaining vertices in H_G (only four vertices in H_G are colored i.e., v_1, v_2, v_3, and w).

(ii) If instead two of the vertices among v_1, v_2, v_3 receive the same color, then we proceed as follows. Without loss of generality, assume that v_1 and v_2 receive color c. We then remove the color c from the list of every vertex in G^a_{12} and G^b_{12}. Note that some vertices in G^a_{12} and G^b_{12} may be left with a singleton list after this removal. Since G^a_{12} and G^b_{12} are 1-CFCN*-choosable, G^a_{12} and G^b_{12} admit 1-CFCN*-coloring under the reduced lists. We do not color the remaining vertices in H_G (v_1, v_2, v_3, w and all the vertices in G^a_{12} and G^b_{12} are the only colored vertices in H_G). This yields a valid $\mathcal{L}$-CFCN*-coloring of H_G, where every vertex in G^a_{12} and G^b_{12} finds a unique color within G^a_{12} and G^b_{12}, respectively, and every vertex in the remaining ten copies of G (namely, $G^a_{13}, G^b_{13}, \ldots, G^b_{34}$) has a uniquely colored neighbor in the set $\{v_1, v_2, v_3\}$.

Suppose G is not 1-CFCN*-choosable. We will show that H_G is not 2-CFCN*-choosable. Let $\mathcal{L} = \{L_v \ : \ v \in V(H)\}$ be a 2-assignment for H_G, where $L_v = \{R, B\}$, for every $v \in V(H_G)$. We have the following two cases:

(i) Consider an $\mathcal{L}$-coloring of H_G that colors at least three vertices among v_1, v_2, v_3, v_4. WLOG, assume v_1, v_2, and v_3 are colored. Since $L_v = \{R, B\}$, $\forall v \in V(H_G)$, at least two of the three vertices v_1, v_2, v_3 receive the same color. Without loss of generality, assume that v_1 and v_2 receive color R, under the $\mathcal{L}$-coloring of H_G. Then for G^a_{12} and G^b_{12} to see a unique color, color R should not be assigned to any vertex in G^a_{12} or G^b_{12}. For every vertex in G^a_{12} and G^b_{12}, after the removal of color R from every vertex's list in G^a_{12} and G^b_{12}, it is left with only color B in its list. Since neither G^a_{12} nor G^b_{12} is 1-CFCN*-choosable, by Proposition 20 (ii), neither G^a_{12} nor G^b_{12} is 1-CFCN*-colorable. Therefore G^a_{12} and G^b_{12} are not list CFCN*-colorable under the reduced list. Thus this case does not yield a valid 2-CFCN*-coloring of H_G.

(ii) Consider an $\mathcal{L}$-coloring of H_G where at least two of the vertices among v_1, v_2, v_3, v_4 are uncolored. Without loss of generality, assume that v_1 and v_2 are uncolored under the $\mathcal{L}$-coloring of H_G. Then some of the vertices in G_{12}^a and G_{12}^b must be colored so that every vertex in G_{12}^a and G_{12}^b sees a unique color in its closed neighborhood. For the vertex v_1 (or v_2) to see a unique color in its closed neighborhood one color should be present exactly once among the vertices in G_{12}^a and G_{12}^b. Assume the color R is given to exactly one vertex in G_{12}^a. Then the color R cannot be given to any vertex in G_{12}^b. So after the removal of color R from every vertex list in G_{12}^b, every vertex in G_{12}^b is left with only one color, i. e. the color B. Since G_{12}^b is not 1-CFCN*-choosable, by Proposition 20 (ii), G_{12}^b is not 1-CFCN*-colorable. Therefore, G_{12}^b is not list CFCN*-colorable under the reduced list. Thus this case does not yield a valid 2-CFCN*-coloring of H_G. ◀

Combining Lemma 27 and Theorem 25, we have the following result.

▶ **Theorem 28.** *The* 2*-CFCN***-*CHOOSABILITY PROBLEM *is NP-hard.*

Now we show that the 2-CFON*-CHOOSABILITY problem is NP-hard on bipartite graphs by reducing from the 2-CFCN*-CHOOSABILITY problem.

▶ **Definition 29.** *Given a graph G with $V(G) = \{v_1, v_2, \ldots, v_n\}$, the* extended double cover *of G, denoted D_G, is the bipartite graph with bipartition $\{X, Y\}$, where $X = \{x_1, x_2, \ldots, x_n\}$, $Y = \{y_1, y_2, \ldots, y_n\}$ and $x_i x_j \in E(D_G)$ if and only if $j = i$ or $v_i v_j \in E(G)$.*

We now prove the following lemma.

▶ **Lemma 30.**

(i) *A graph G is k-CFCN*-choosable if and only if its extended double cover D_G is k-CFON*-choosable, where $k \geq 1$.*

(ii) *A graph G is k-CFCN-choosable if and only if its extended double cover D_G is k-CFON-choosable, where $k \geq 1$.*

Proof. We prove (i). The proof of (ii) is analogous. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$. Let D_G be the extended double cover of G as defined above.

Suppose G is k-CFCN*-choosable. We will show that D_G is k-CFON*-choosable. Let $\mathcal{L} = \{L_v : v \in V(D_G)\}$ be a k-assignment for D_G. Let $\mathcal{L}^X = \{L_{v_i}^X : v_i \in V(G)\}$, where $L_{v_i}^X = L_{x_i}$, be a k-assignment for G. Let f^X be any valid $\mathcal{L}^X$-CFCN* coloring of G. In a similar way, let $\mathcal{L}^Y = \{L_{v_i}^Y : v_i \in V(G)\}$, where $L_{v_i}^Y = L_{y_i}$, be a k-assignment for G. Let f^Y be any valid $\mathcal{L}^Y$-CFCN* coloring of G. We now obtain an $\mathcal{L}$-CFON*-coloring f of D_G as described below: $f(x_i) = f^X(v_i)$ and $f(y_i) = f^Y(v_i)$, $\forall i \in [n]$. We claim that f is a CFON*-coloring of D_G. Consider a vertex $x_i \in X$. Suppose v_j is a uniquely colored vertex in the closed neighborhood of v_i in G, under the coloring f^Y. Then, under f, y_j is a uniquely colored neighbor in the open neighborhood of x_i. In a similar way, one can show that every vertex in Y sees a unique color in its open neighborhood under f.

Suppose D_G is k-CFON*-choosable. We will show that G is k-CFCN*-choosable. Let $\mathcal{L} = \{L_v : v \in V(G)\}$ be a k-assignment for G. Let $\mathcal{L}' = \{L'_v : v \in V(D_G)\}$, where $L'_{x_i} = L'_{y_i} = L_{v_i}$, $\forall i \in [n]$, be a k-assignment for D_G. Let f' be an $\mathcal{L}'$-CFON*-coloring of D_G. We now obtain an $\mathcal{L}$-CFCN*-coloring f of G as described below: $f(v_i) = f'(x_i)$, $\forall i \in [n]$. Consider a vertex $v_i \in V(G)$. Let x_j be a uniquely colored vertex in the open neighborhood of y_i under f'. Then v_j is a uniquely colored neighbor in the closed neighborhood of v_i in G, under f. ◀

Combining Lemma 30 and Theorem 28, we have the following result.

▶ **Theorem 31.** *The* 2*-CFON***-choosability problem is NP-hard on bipartite graphs.*

4 Concluding Remarks

In Theorem 15, we show that for a $K_{1,k}$-free graph G, we have $ch^*_{CN}(G) = O(k \ln \Delta)$. It is natural to ask whether a similar bound holds for $ch^*_{ON}(G)$. In Section 3, we show that the k-CFON*/CFCN*-choosability problems, for $k = 1, 2$, are NP-hard. It was shown in [13] that the 3-CFON*-choosability problem is Π_2^P-complete. Is 2-CFON*-choosability problem Π_2^P-complete? Further, the computational complexity of the following problems remain unresolved: (i) 2-CFON-choosability, (ii) k-CFCN*-choosability for $k \geq 3$, and (iii) k-CFCN-choosability for $k \geq 2$.

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