

Homotopy Theory in Complexity of the Graph Homomorphism Problem

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Abstract

I will talk about an emerging application of homotopy theory in computational complexity of combinatorial problems, more precisely *homomorphism problems*. Homomorphism problems appear under many names, including *constraint satisfaction problems* and conjunctive database queries.

A prime example of a homomorphism problem is graph colouring; by a colouring of a graph with k colours, I mean an assignment of colours to vertices under which no edge is monochromatic – this is equivalent to the existence of a homomorphism to the clique with k vertices. Graph 3-colouring is a prototypical example of an NP-complete problem. There are many variations on this problem whose complexity remains widely open. For example, although it is generally believed that colouring a 3-colourable graph with a fixed number of colours is NP-hard, only the hardness of colouring of such a graph with 5 colours is known (and shown only in 2019).

I will give an overview of several related results about variations of graph colouring that share a common theme of using a method based on tools from topological combinatorics and on ideas of Lovász [J. Comb. Theory, Ser. A, 25(3):319–324, 1978].

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References

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