

Revisiting True Concurrency Bisimilarities: On the Role of Backward Ready Multisets and Why They Are Not Enough for HPB and HHPB

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Abstract

Bisimilarities over stable configuration structures can be divided into three families. In the first one – including interleaving, step, pomset, and forward-reverse bisimilarities – no isomorphism is required between the events matched during the bisimulation game. In the second one – including weak history-preserving, weak history-preserving pomset, and weak hereditary history-preserving bisimilarities – a labeling- and causality-preserving isomorphism is required between matched events, which is specific to each pair of configurations related by the bisimulation relation and hence can vary for a matched event from pair to pair. In the third one – including history-preserving and hereditary history-preserving bisimilarities – a single isomorphism is built incrementally, which is therefore fixed for all matched events. We revisit true concurrency bisimilarities by introducing variants that additionally check that the backward ready multisets of related configurations coincide. While the distinguishing power of the bisimilarities of the second and third families does not change, the power of the revised bisimilarities of the first family is equal to that of the bisimilarities of the second family. The latter bisimilarities can thus be characterized by replacing variable isomorphisms with simply counting incoming transitions. In contrast, backward ready multisets are not enough to characterize the third family in the simultaneous presence of autoconcurrency and non-local conflicts. We show that a further check for the existence of diamond and half-diamond substructures is necessary in that case to achieve the same distinguishing power as incremental isomorphisms.

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1 Introduction

Bisimilarity is a central notion in concurrency theory. It expresses the capability of two processes of mimicking each other's behavior stepwise, yielding a behavioral equivalence that is extremely sensitive to branching points. Over labeled transition systems [20], i.e., state-transition graphs whose transitions are labeled with actions, its classical definition [22, 21] complies with the *interleaving view* of concurrency based on the *expansion law* [18]. This means that the parallel composition of two processes reduces to a nondeterministic choice among all possible sequencings of the actions individually executable by the two processes.

Several variants have been subsequently introduced, which are *truly concurrent* in the sense that they do not meet the expansion law. Their spectrum, depicted in Figure 1, has been investigated in [14, 12, 25], where their definitions have been formulated over stable configuration structures [15]. These are variants of event structures [30] akin to labeled transition systems in which causality, concurrency, and conflicts can be retrieved a posteriori.



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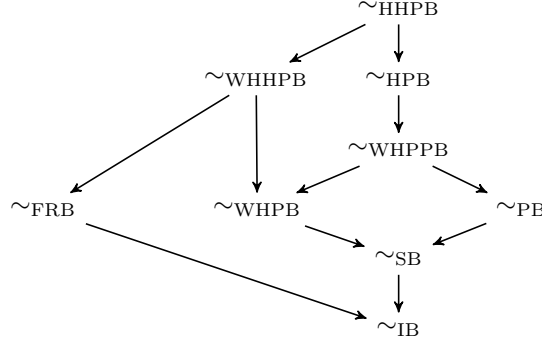
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■ **Figure 1** Main bisimilarities over stable configuration structures examined in [14, 12, 25].

We can distinguish among three families of bisimilarities in the true concurrency spectrum (whose member definitions will be recalled in Section 2):

1. The bisimilarities of the first family do not require any isomorphism between the events matched so far during the bisimulation game. The *no-iso family* includes:
 - *Interleaving bisimilarity* $\sim_{IB}$ [22, 21], which considers transitions labeled with individual actions.
 - *Step bisimilarity* $\sim_{SB}$ [27], which considers transitions labeled with multisets of actions corresponding to concurrent events.
 - *Pomset bisimilarity* $\sim_{PB}$ [6], which considers transitions labeled with partially ordered multisets (pomsets for short) of actions corresponding to events that are causally related or concurrent.
 - *Forward-reverse bisimilarity* $\sim_{FRB}$ [23], which examines not only outgoing transitions (forward direction) but also incoming ones (backward direction) labeled with actions.
2. The bisimilarities of the second family require a labeling- and causality-preserving isomorphism between the events matched so far, which is specific to the pair of configurations at hand and hence can vary for a matched event from pair to pair. The *var-iso family* includes:
 - *Weak history-preserving bisimilarity* $\sim_{WHPB}$ [11], which considers transitions labeled with individual actions.
 - *Weak history-preserving pomset bisimilarity* $\sim_{WHPPB}$ [14], which considers transitions labeled with pomsets.
 - *Weak hereditary history-preserving bisimilarity* $\sim_{WHHPB}$ [25], which examines incoming transitions too.
3. The bisimilarities of the third family incrementally build a single labeling- and causality-preserving isomorphism between the events matched so far, with their underlying bisimulations being ternary relations and their distinguishing power not depending on the kind (i.e., action/step/pomset) of considered transitions [14]. The *incr-iso family* includes:
 - *History-preserving bisimilarity* $\sim_{HPB}$ [28], which examines only outgoing transitions.
 - *Hereditary history-preserving bisimilarity* $\sim_{HHPB}$ [3], which examines incoming transitions as well.

The two bisimilarities of the last family are especially important because they are the coarsest and the finest equivalences, respectively, that are preserved under action refinement and are capable of respecting causality, branching, and their interplay while abstracting from choices between identical alternatives [14]. Moreover, $\sim_{HHPB}$ can be obtained as a special case of a categorical definition of bisimilarity over concurrency models [19].

While $\sim_{\text{HPB}}$ is known to coincide with causal bisimilarity [8, 9], characterizing $\sim_{\text{HHPB}}$ is more complicated. To be precise, in the absence of *autoconcurrency*, i.e., identically labeled concurrent events, $\sim_{\text{HHPB}}$ (which is based on *ternary* bisimulation relations) coincides with $\sim_{\text{FRB}}$ (which is based on *binary* bisimulation relations). This was established in [3] for configuration graphs of prime event structures and later in [24] by requiring the absence of repeated, identically labeled events along forward computations.

In [25] it was then shown that, on stable configuration structures, it suffices to impose the absence of equidepth autoconcurrency, i.e., the absence of identically labeled concurrent events occurring at the same depth within a configuration (where the depth of an event is defined as the length of the longest causal chain of events up to and including the considered event). Another characterization was provided in [1] by relying on an extension of stable configuration structures that somehow encodes the past behavior by uniquely identifying events. No restriction is needed in this case, but the characterizing bisimilarity is based again on ternary bisimulation relations, witnessing once more that the forward and backward bisimulation games alone are not enough to capture $\sim_{\text{HHPB}}$.

A different approach has been recently proposed in [4]. Instead of focusing on the unique identification of identically labeled events, the idea is to count them. A variant $\sim_{\text{FRB:brm}}$ of $\sim_{\text{FRB}}$ has been introduced there, which additionally checks whether pairs of related configurations have the same *backward ready multiset*, intended as the multiset of actions labeling incoming transitions.

Let us consider for example a process that can perform a in parallel with a – which we can represent as $a \parallel_{\emptyset} a$ by using a CSP-like parallel composition [7] – and its interleaving expansion – which we can represent as $a . a + a . a$ by using CCS-like action prefix and nondeterministic choice [21]. The two processes are told apart by $\sim_{\text{HHPB}}$ because the two a -labeled events in the former process are concurrent while the two a -labeled events along either branch of the latter process are causally related, hence no causality-preserving isomorphism can be established among these events. The two processes are distinguished by $\sim_{\text{FRB:brm}}$ too. Indeed, the process reached from $a \parallel_{\emptyset} a$ after performing both a -actions – which we can represent as $a^{\dagger} \parallel_{\emptyset} a^{\dagger}$ according to the notation introduced in [5, 4] with $\dagger$ decorating executed actions – has $\{a, a\}$ as backward ready multiset, whereas the two processes reached from $a . a + a . a$ after performing both a -actions along either branch – which we can respectively represent as $a^{\dagger} . a^{\dagger} + a . a$ and $a . a + a^{\dagger} . a^{\dagger}$ – have $\{a\}$ as backward ready multiset.

From the example above it is clear that the backward ready multiset of a configuration is a measure of the degree of concurrency of the events in the configuration itself. Moreover, comparing the backward ready multisets of two configurations is much simpler than building a possibly incremental, labeling- and causality-preserving isomorphism between the events matched so far. Therefore, it is interesting to investigate whether backward ready multisets can concur to provide an alternative characterization of isomorphisms for all bisimilarities in the var-iso and incr-iso families. This is the objective of the present paper.

Our first contribution (Section 3) is twofold. Let us call *brm-variant* of a bisimilarity the variant that is obtained by additionally requiring that any two related configurations possess the same backward ready multiset. On the one hand, we show that the brm-variants of var-iso and incr-iso bisimilarities coincide with the corresponding original bisimilarities, meaning that for those two families the backward ready multiset equality check does not increase the distinguishing power. On the other hand, we show that the brm-variants of no-iso bisimilarities are strictly finer than the corresponding original bisimilarities. Interestingly, $\sim_{\text{IB:brm}}$ and $\sim_{\text{SB:brm}}$ coincide with $\sim_{\text{WHPB}}$ (and $\sim_{\text{WHPB:brm}}$), while $\sim_{\text{PB:brm}}$ coincides with $\sim_{\text{WHPPB}}$ (and $\sim_{\text{WHPPB:brm}}$) and $\sim_{\text{FRB:brm}}$ coincides with $\sim_{\text{WHHPB}}$ (and $\sim_{\text{WHHPB:brm}}$).

In other words, the var-iso bisimilarities are characterized by the brm-variants of the no-iso bisimilarities, i.e., variable isomorphisms can be replaced by simply counting incoming transitions (see the forthcoming Figure 2).

Our second contribution (Section 4) is the development of an extension of Hennessy-Milner logic [18] that includes backward ready multisets [4], a pomset-based forward modality [13, 17, 29, 6], and an action-based backward modality [10]. We show that its fragments characterize all bisimilarities of the no-iso and var-iso families, including their brm-variants.

Our third contribution (Section 5) is an investigation of why backward ready multisets are not enough to capture incr-iso bisimilarities in the simultaneous presence of autoconcurrency and *non-local conflicts*, i.e., conflicting events occurring in distinct processes that are composed in parallel. What we observe from a number of significant examples – most of which are taken from [25] – is that, when trying to relate two configurations, in addition to the bisimulation game – which is the baseline – and backward ready multisets – which measure concurrent events by counting *incoming* transitions – we should also consider the existence of diamond-shaped configuration substructures – which indicate the presence of concurrent events by looking at *outgoing* transitions – along with half-diamond-shaped configuration substructures – which reveal the presence of *conflicting* events. We show that the resulting diamond-based variants of $\sim_{\text{IB:brm}}$ and $\sim_{\text{FRB:brm}}$, denoted by $\sim_{\text{IB:brm:diam}}$ and $\sim_{\text{FRB:brm:diam}}$, respectively coincide with $\sim_{\text{HPB}}$ and $\sim_{\text{HHPB}}$ (see the forthcoming Figure 2).

Section 6 concludes the paper by outlining some directions for future work.

2 Background Definitions and Results

In this section we recall the stable configuration structure model (Section 2.1) and the three families of bisimilarities defined over it that have been studied in [14, 12, 25] (Sections 2.2, 2.3, 2.4). The spectrum of the considered bisimilarities has already been shown in Figure 1. Every arrow stands for the strict inclusion of the source bisimilarity in the target one, while the absence of a chain of arrows between two equivalences means that those two equivalences are incomparable.

We will use the following process algebraic notation to express examples: a stands for an action (i.e., the label of an event), $a.$ $_$ is the action prefix operator (the a -labeled event is a cause of the subsequent ones), $_ + _$ is the alternative composition operator (events in different branches are in conflict with each other), and $_ \|_L _$ is the parallel composition operator (events in different branches are concurrent, i.e., independent of each other, unless they have to synchronize because their labels are identical and belong to L). When an action is decorated with $\dagger$, it means that it has already been executed.

2.1 Stable Configuration Structures

Configuration structures [15] are variants of event structures [30] expressing their underlying labeled transition system view. A configuration, which represents a state, is the set of non-conflicting events that occurred so far. Every configuration is finite and left-closed with respect to causality, i.e., it contains all the events causing the ones in the configuration itself. Therefore, given two arbitrary events in a configuration, they are either causally related, i.e., can occur only in a specific order, or concurrent, i.e., can occur in any order.

In the following definitions taken from [14], we assume a countable set $\mathcal{A}$ of action labels and we denote by $\mathcal{P}_{\text{fin}}(\mathcal{E})$ the set of finite subsets of set $\mathcal{E}$.

► **Definition 1.** A configuration structure is a triple $C = (\mathcal{E}, \mathcal{C}, \ell)$ where:

- $\mathcal{E}$ is a set of events.
- $\mathcal{C} \subseteq \mathcal{P}_{\text{fin}}(\mathcal{E})$ is a set of configurations.
- $\ell : \bigcup_{X \in \mathcal{C}} X \rightarrow \mathcal{A}$ is a labeling function for events occurring in configurations. ┐

As shown in [14], the configurations of a prime event structure [30] – based on causality and conflict relations – form a configuration structure, while those of a stable event structure [30] – based on consistency and enabling notions – form a configuration structure that is stable in the following sense.

► **Definition 2.** A configuration structure $C = (\mathcal{E}, \mathcal{C}, \ell)$ is said to be stable iff it is:

- Rooted: $\emptyset \in \mathcal{C}$.
- Connected: $\forall X \in \mathcal{C} \setminus \{\emptyset\}. \exists e \in X. X \setminus \{e\} \in \mathcal{C}$.
- Closed under bounded unions/intersections: $\forall X, Y, Z \in \mathcal{C}. X \cup Y \subseteq Z \implies X \cup Y, X \cap Y \in \mathcal{C}$. ┐

The notions of causality and concurrency for events can be defined locally to configurations, while the notion of conflict has to be defined globally in a stable configuration structure.

► **Definition 3.** Let $C = (\mathcal{E}, \mathcal{C}, \ell)$ be a stable configuration structure and $X \in \mathcal{C}$:

- The causality relation over $X \in \mathcal{C}$ is defined by letting $e_1 \leq_X e_2$, for two events $e_1, e_2 \in X$, iff $e_2 \in Y$ implies $e_1 \in Y$ for all $Y \in \mathcal{C}$ such that $Y \subseteq X$. We write $e_1 <_X e_2$ when $e_1 \leq_X e_2$ and $e_1 \neq e_2$.
- Two distinct events $e_1, e_2 \in X$ are concurrent in X , written $e_1 \text{ co}_X e_2$, iff $e_1 \not\leq_X e_2$ and $e_2 \not\leq_X e_1$.
- Two distinct events $e_1, e_2 \in \mathcal{E}$ are conflicting, written $e_1 \# e_2$, iff there exists no $Z \in \mathcal{C}$ such that $e_1, e_2 \in Z$. ┐

A notion of transition between configurations can be naturally introduced.

► **Definition 4.** Let $C = (\mathcal{E}, \mathcal{C}, \ell)$ be a stable configuration structure, $a \in \mathcal{A}$, and $X, X' \in \mathcal{C}$. We say that there exists an a -labeled transition from X to X' , written $X \xrightarrow{a}_C X'$, iff $X \subseteq X'$, $X' \setminus X = \{e\}$, and $\ell(e) = a$. ┐

2.2 Bisimilarities with No Event Isomorphisms

The bisimilarities of the first family do not require any isomorphism between the events matched so far during the bisimulation game. We start by recalling the standard bisimilarity of [22, 21], in which a single action at a time is considered. It can be termed interleaving because it complies with the view according to which the parallel composition of two processes is equivalent to a nondeterministic choice among all possible sequencings of the actions executable by the two processes themselves.

► **Definition 5.** We say that two stable configuration structures $C_i = (\mathcal{E}_i, \mathcal{C}_i, \ell_i)$, $i \in \{1, 2\}$, are interleaving bisimilar, written $C_1 \sim_{\text{IB}} C_2$, iff there exists an interleaving bisimulation between them, i.e., a relation $\mathcal{B} \subseteq \mathcal{C}_1 \times \mathcal{C}_2$ such that $(\emptyset, \emptyset) \in \mathcal{B}$ and, whenever $(X_1, X_2) \in \mathcal{B}$, then:

- For each $X_1 \xrightarrow{a}_{C_1} X'_1$ there exists $X_2 \xrightarrow{a}_{C_2} X'_2$ such that $(X'_1, X'_2) \in \mathcal{B}$, and vice versa. ┐

Secondly, we have step bisimilarity [27], which considers transitions labeled with multisets of actions corresponding to concurrent events. Formally $X \xrightarrow{A}_C X'$ for $A : \mathcal{A} \rightarrow \mathbb{N}$ iff $X \subseteq X'$, $X' \setminus X = E$ with $e_1 \text{ co}_{X'} e_2$ for all $e_1, e_2 \in E$, and $\ell(E) \triangleq \{\ell(e) \mid e \in E\} = A$, with $\{\}$ and $\}$ being multiset delimiters.

► **Definition 6.** We say that two stable configuration structures $C_i = (\mathcal{E}_i, \mathcal{C}_i, \ell_i)$, $i \in \{1, 2\}$, are step bisimilar, written $C_1 \sim_{\text{SB}} C_2$, iff there exists a step bisimulation between them, i.e., a relation $\mathcal{B} \subseteq \mathcal{C}_1 \times \mathcal{C}_2$ such that $(\emptyset, \emptyset) \in \mathcal{B}$ and, whenever $(X_1, X_2) \in \mathcal{B}$, then:

- For each $X_1 \xrightarrow{A}_{C_1} X'_1$ there exists $X_2 \xrightarrow{A}_{C_2} X'_2$ such that $(X'_1, X'_2) \in \mathcal{B}$, and vice versa. $\lrcorner$

$\sim_{\text{SB}}$ is strictly finer than $\sim_{\text{IB}}$: for example, $a \parallel_\emptyset b$ and $a.b + b.a$ are $\sim_{\text{IB}}$ -equivalent but not $\sim_{\text{SB}}$ -equivalent, because a and b in either branch of the second process are not concurrent and hence the $\{a, b\}$ -transition of the first process cannot be matched by the second one.

Thirdly, we have pomset bisimilarity [6], which considers transitions labeled with partially ordered multisets of actions corresponding to events that are causally related or concurrent. Formally $X \xrightarrow{U}_C X'$ iff $X \subseteq X'$ and $X' \setminus X = H$ with $U = [(H, <_{X'} \cap (H \times H), \ell \cap (H \times \mathcal{A}))]_{\cong}$, where $\cong$ is a labeling- and causality-preserving isomorphism over events and U is the $\cong$ -class of equivalence for event set H .

► **Definition 7.** We say that two stable configuration structures $C_i = (\mathcal{E}_i, \mathcal{C}_i, \ell_i)$, $i \in \{1, 2\}$, are pomset bisimilar, written $C_1 \sim_{\text{PB}} C_2$, iff there exists a pomset bisimulation between them, i.e., a relation $\mathcal{B} \subseteq \mathcal{C}_1 \times \mathcal{C}_2$ such that $(\emptyset, \emptyset) \in \mathcal{B}$ and, whenever $(X_1, X_2) \in \mathcal{B}$, then:

- For each $X_1 \xrightarrow{U}_{C_1} X'_1$ there exists $X_2 \xrightarrow{U}_{C_2} X'_2$ such that $(X'_1, X'_2) \in \mathcal{B}$, and vice versa. $\lrcorner$

$\sim_{\text{PB}}$ is strictly finer than $\sim_{\text{SB}}$: for instance, $a \parallel_\emptyset b$ and $(a \parallel_\emptyset b) + a.b$ are identified by $\sim_{\text{SB}}$ but told apart by $\sim_{\text{PB}}$, because the transition of the second process labeled with a pomset containing the two causally-related actions a and b to the right of $+$ cannot be matched by the first process as its actions a and b are concurrent and hence originate a different pomset.

Finally, we have forward-reverse bisimilarity [23], which considers transitions labeled with individual actions like $\sim_{\text{IB}}$ but examines incoming transitions (backward direction of computation) in addition to outgoing ones (forward direction of computation).

► **Definition 8.** We say that two stable configuration structures $C_i = (\mathcal{E}_i, \mathcal{C}_i, \ell_i)$, $i \in \{1, 2\}$, are forward-reverse bisimilar, written $C_1 \sim_{\text{FRB}} C_2$, iff there exists a forward-reverse bisimulation between them, i.e., a relation $\mathcal{B} \subseteq \mathcal{C}_1 \times \mathcal{C}_2$ such that $(\emptyset, \emptyset) \in \mathcal{B}$ and, whenever $(X_1, X_2) \in \mathcal{B}$, then:

- For each $X_1 \xrightarrow{a}_{C_1} X'_1$ there exists $X_2 \xrightarrow{a}_{C_2} X'_2$ such that $(X'_1, X'_2) \in \mathcal{B}$, and vice versa.
- For each $X'_1 \xrightarrow{a}_{C_1} X_1$ there exists $X'_2 \xrightarrow{a}_{C_2} X_2$ such that $(X'_1, X'_2) \in \mathcal{B}$, and vice versa. $\lrcorner$

$\sim_{\text{FRB}}$ is strictly finer than $\sim_{\text{IB}}$: as an example, $a \parallel_\emptyset b$ and $a.b + b.a$ are $\sim_{\text{IB}}$ -equivalent but not $\sim_{\text{FRB}}$ -equivalent, because $a^\dagger \parallel_\emptyset b^\dagger$ has two differently labeled incoming transitions whereas $a^\dagger.b^\dagger + b.a$ and $a.b + b^\dagger.a^\dagger$ have only one. $\sim_{\text{FRB}}$ is incomparable with $\sim_{\text{SB}}$ and $\sim_{\text{PB}}$. On the one hand, $a.(b+c) + (a \parallel_\emptyset b)$ and $a.(b+c) + (a \parallel_\emptyset b) + a.b$ are identified by $\sim_{\text{PB}}$ and hence $\sim_{\text{SB}}$ but told apart by $\sim_{\text{FRB}}$, because $a.(b+c) + (a \parallel_\emptyset b) + a^\dagger.b$ can only be matched by $a.(b+c) + (a^\dagger \parallel_\emptyset b)$ – as $a^\dagger.(b+c) + (a \parallel_\emptyset b)$ enables both b and c – but then $a.(b+c) + (a \parallel_\emptyset b) + a^\dagger.b^\dagger$ has a single incoming transition whilst $a.(b+c) + (a^\dagger \parallel_\emptyset b^\dagger)$ has two differently labeled ones. On the other hand, $a \parallel_\emptyset a$ and $a.a$ are equated by $\sim_{\text{FRB}}$ but distinguished by $\sim_{\text{SB}}$ and hence $\sim_{\text{PB}}$, because the two occurrences of a in the second process are not concurrent and hence the $\{a, a\}$ -transition of the first process cannot be matched by the second one.

2.3 Bisimilarities with Variable Event Isomorphisms

The bisimilarities of the second family build suitable isomorphisms between the events matched so far during the bisimulation game. The isomorphisms are *specific* to the pairs of configurations at hand, hence can *vary* for a matched event from pair to pair. The resulting bisimilarities are called *weak history-preserving* in [14]. Unlike [21], the word weak is not to be intended as abstracting from unobservable actions, but as preserving history in a weak sense.

► **Definition 9.** Given two stable configuration structures $C_i = (\mathcal{E}_i, C_i, \ell_i)$ and two configurations $X_i \in C_i$, $i \in \{1, 2\}$, a labeling- and causality-preserving isomorphism between X_1 and X_2 is induced by a bijection $f : X_1 \rightarrow X_2$ such that:

- $\ell_1(e) = \ell_2(f(e))$ for all $e \in X_1$ (labeling preservation).
- $e \leq_{X_1} e' \iff f(e) \leq_{X_2} f(e')$ for all $e, e' \in X_1$ (causality preservation). ┐

We start by recalling the one of these bisimilarities that considers transitions labeled with individual actions [11].

► **Definition 10.** We say that two stable configuration structures $C_i = (\mathcal{E}_i, C_i, \ell_i)$, $i \in \{1, 2\}$, are weak history-preserving bisimilar, written $C_1 \sim_{\text{WHPB}} C_2$, iff there exists a weak history-preserving bisimulation between them, i.e., a relation $\mathcal{B} \subseteq C_1 \times C_2$ such that $(\emptyset, \emptyset) \in \mathcal{B}$ and, whenever $(X_1, X_2) \in \mathcal{B}$, then:

- There exists a labeling- and causality-preserving bijection $f : X_1 \rightarrow X_2$.
- For each $X_1 \xrightarrow{a}_{C_1} X'_1$ there exists $X_2 \xrightarrow{a}_{C_2} X'_2$ such that $(X'_1, X'_2) \in \mathcal{B}$, and vice versa. ┐

$\sim_{\text{WHPB}}$ is strictly finer than $\sim_{\text{SB}}$: for example, $a \parallel_\emptyset b$ and $(a \parallel_\emptyset b) + a.b$ are identified by $\sim_{\text{SB}}$ but told apart by $\sim_{\text{WHPB}}$, because a and b to the right of $+$ in the second process are causally related and hence cannot be mapped to the concurrent a and b in the first process. $\sim_{\text{WHPB}}$ is incomparable with $\sim_{\text{PB}}$ and $\sim_{\text{FRB}}$. On the one hand, $a.(b+c) + (a \parallel_\emptyset b)$ and $a.(b+c) + (a \parallel_\emptyset b) + a.b$ are identified by $\sim_{\text{PB}}$ but told apart by $\sim_{\text{WHPB}}$, while $a \parallel_\emptyset a$ and $a.a$ are equated by $\sim_{\text{FRB}}$ but distinguished by $\sim_{\text{WHPB}}$. On the other hand, the two complex processes in the forthcoming Figure 3 (where parallel transitions have the same action label) are identified by $\sim_{\text{WHPB}}$ but told apart by $\sim_{\text{PB}}$, while $(a \parallel_\emptyset (b+c)) + (a \parallel_\emptyset b) + ((a+c) \parallel_\emptyset b)$ and $(a \parallel_\emptyset (b+c)) + ((a+c) \parallel_\emptyset b)$ are equated by $\sim_{\text{WHPB}}$ but distinguished by $\sim_{\text{FRB}}$, because $(a \parallel_\emptyset (b+c)) + (a^\dagger \parallel_\emptyset b^\dagger) + ((a+c) \parallel_\emptyset b)$ has two incoming transitions departing from configurations having no alternative outgoing transitions whilst one of the two incoming transitions of $(a^\dagger \parallel_\emptyset (b^\dagger+c)) + ((a+c) \parallel_\emptyset b)$ and $(a \parallel_\emptyset (b+c)) + ((a^\dagger+c) \parallel_\emptyset b^\dagger)$ departs from a configuration having an outgoing c -transition too.

Then we have the one that considers transitions labeled with pomsets [14].

► **Definition 11.** We say that two stable configuration structures $C_i = (\mathcal{E}_i, C_i, \ell_i)$, $i \in \{1, 2\}$, are weak history-preserving pomset bisimilar, written $C_1 \sim_{\text{WHPPB}} C_2$, iff there exists a weak history-preserving pomset bisimulation between them, i.e., a relation $\mathcal{B} \subseteq C_1 \times C_2$ such that $(\emptyset, \emptyset) \in \mathcal{B}$ and, whenever $(X_1, X_2) \in \mathcal{B}$, then:

- There exists a labeling- and causality-preserving bijection $f : X_1 \rightarrow X_2$.
- For each $X_1 \xrightarrow{U}_{C_1} X'_1$ there exists $X_2 \xrightarrow{U}_{C_2} X'_2$ such that $(X'_1, X'_2) \in \mathcal{B}$, and vice versa. ┐

$\sim_{\text{WHPPB}}$ is strictly finer than $\sim_{\text{WHPB}}$ and $\sim_{\text{PB}}$: for instance, the two complex processes in the forthcoming Figure 3 are identified by $\sim_{\text{WHPB}}$ but told apart by $\sim_{\text{WHPPB}}$, while $a.(b+c) + (a \parallel_\emptyset b)$ and $a.(b+c) + (a \parallel_\emptyset b) + a.b$ are equated by $\sim_{\text{PB}}$ but distinguished by $\sim_{\text{WHPPB}}$, because the rightmost a in the second process can only be matched by the rightmost a in the first process but then the rightmost causally-related a and b in the second process cannot be mapped to the rightmost concurrent a and b in the first process. $\sim_{\text{WHPPB}}$ is incomparable with $\sim_{\text{FRB}}$: as an example, $a \parallel_\emptyset a$ and $a.a$ are identified by $\sim_{\text{FRB}}$ but told apart by $\sim_{\text{WHPPB}}$, whereas $(a \parallel_\emptyset (b+c)) + (a \parallel_\emptyset b) + ((a+c) \parallel_\emptyset b)$ and $(a \parallel_\emptyset (b+c)) + ((a+c) \parallel_\emptyset b)$ are equated by $\sim_{\text{WHPPB}}$ but distinguished by $\sim_{\text{FRB}}$.

Finally, we have the one that considers transitions labeled with individual actions like $\sim_{\text{WHPB}}$ but examines incoming transitions in addition to outgoing ones [25].

► **Definition 12.** We say that two stable configuration structures $C_i = (\mathcal{E}_i, \mathcal{C}_i, \ell_i)$, $i \in \{1, 2\}$, are weak hereditary history-preserving bisimilar, written $C_1 \sim_{\text{WHHPB}} C_2$, iff there exists a weak hereditary history-preserving bisimulation between them, i.e., a relation $\mathcal{B} \subseteq C_1 \times C_2$ such that $(\emptyset, \emptyset) \in \mathcal{B}$ and, whenever $(X_1, X_2) \in \mathcal{B}$, then:

- There exists a labeling- and causality-preserving bijection $f : X_1 \rightarrow X_2$.
- For each $X_1 \xrightarrow{a}_{C_1} X'_1$ there exists $X_2 \xrightarrow{a}_{C_2} X'_2$ such that $(X'_1, X'_2) \in \mathcal{B}$, and vice versa.
- For each $X'_1 \xrightarrow{a}_{C_1} X_1$ there exists $X'_2 \xrightarrow{a}_{C_2} X_2$ such that $(X'_1, X'_2) \in \mathcal{B}$, and vice versa. $\lrcorner$

$\sim_{\text{WHHPB}}$ is strictly finer than $\sim_{\text{WHPB}}$ and $\sim_{\text{FRB}}$: for example, $(a \parallel_\emptyset (b + c)) + (a \parallel_\emptyset b) + ((a + c) \parallel_\emptyset b)$ and $(a \parallel_\emptyset (b + c)) + ((a + c) \parallel_\emptyset b)$ are identified by $\sim_{\text{WHPB}}$ but told apart by $\sim_{\text{WHHPB}}$, while $a \parallel_\emptyset a$ and $a . a$ are equated by $\sim_{\text{FRB}}$ but distinguished by $\sim_{\text{WHHPB}}$. $\sim_{\text{WHHPB}}$ is incomparable with $\sim_{\text{WHPPB}}$: for instance, the two complex processes in the forthcoming Figure 3 are identified by $\sim_{\text{WHHPB}}$ but told apart by $\sim_{\text{WHPPB}}$, whilst $(a \parallel_\emptyset (b + c)) + (a \parallel_\emptyset b) + ((a + c) \parallel_\emptyset b)$ and $(a \parallel_\emptyset (b + c)) + ((a + c) \parallel_\emptyset b)$ are equated by $\sim_{\text{WHPPB}}$ but distinguished by $\sim_{\text{WHHPB}}$.

2.4 Bisimilarities with Incremental Event Isomorphisms

The bisimilarities of the third family additionally require that a *single* labeling- and causality-preserving isomorphism between the events matched so far be built *incrementally*. As a consequence, the underlying bisimulations become *ternary* relations because they must include the bijection inducing the isomorphism. These bisimilarities are termed *history-preserving* in a strong sense. In the following, $f \upharpoonright X$ denotes the restriction of function f to set X .

The first of these bisimilarities considers transitions labeled with individual actions [28]. Its distinguishing power would not change in the case that step or pomset transitions were considered [14].

► **Definition 13.** We say that two stable configuration structures $C_i = (\mathcal{E}_i, \mathcal{C}_i, \ell_i)$, $i \in \{1, 2\}$, are history-preserving bisimilar, written $C_1 \sim_{\text{HPB}} C_2$, iff there exists a history-preserving bisimulation between them, i.e., a relation $\mathcal{B} \subseteq C_1 \times C_2 \times \mathcal{P}(\mathcal{E}_1 \times \mathcal{E}_2)$ such that $(\emptyset, \emptyset, \emptyset) \in \mathcal{B}$ and, whenever $(X_1, X_2, f) \in \mathcal{B}$, then:

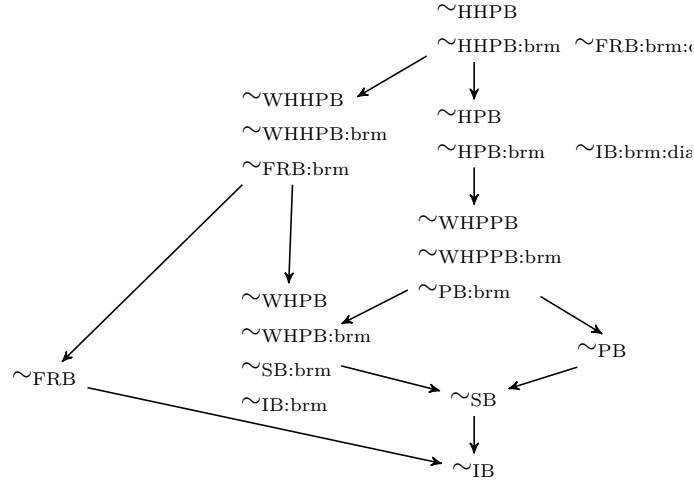
- f is a labeling- and causality-preserving bijection from X_1 to X_2 .
- For each $X_1 \xrightarrow{a}_{C_1} X'_1$ there exist $X_2 \xrightarrow{a}_{C_2} X'_2$ and $f' \subseteq \mathcal{E}_1 \times \mathcal{E}_2$ such that $(X'_1, X'_2, f') \in \mathcal{B}$ and $f' \upharpoonright X_1 = f$, and vice versa. $\lrcorner$

$\sim_{\text{HPB}}$ is strictly finer than $\sim_{\text{WHPPB}}$: as an example, the two complex processes in the forthcoming Figure 5 are identified by $\sim_{\text{WHPPB}}$ but told apart by $\sim_{\text{HPB}}$. $\sim_{\text{HPB}}$ is incomparable with $\sim_{\text{WHHPB}}$: for instance, the two complex processes in the forthcoming Figure 3 are identified by $\sim_{\text{WHHPB}}$ but told apart by $\sim_{\text{HPB}}$, whilst $(a \parallel_\emptyset (b + c)) + (a \parallel_\emptyset b) + ((a + c) \parallel_\emptyset b)$ and $(a \parallel_\emptyset (b + c)) + ((a + c) \parallel_\emptyset b)$ are equated by $\sim_{\text{HPB}}$ but distinguished by $\sim_{\text{WHHPB}}$.

The second one examines incoming transitions in addition to outgoing ones [3]. Like the first one, its distinguishing power would not change in the case that step or pomset transitions were considered [14].

► **Definition 14.** We say that two stable configuration structures $C_i = (\mathcal{E}_i, \mathcal{C}_i, \ell_i)$, $i \in \{1, 2\}$, are hereditary history-preserving bisimilar, written $C_1 \sim_{\text{HHPB}} C_2$, iff there exists a hereditary history-preserving bisimulation between them, i.e., a relation $\mathcal{B} \subseteq C_1 \times C_2 \times \mathcal{P}(\mathcal{E}_1 \times \mathcal{E}_2)$ such that $(\emptyset, \emptyset, \emptyset) \in \mathcal{B}$ and, whenever $(X_1, X_2, f) \in \mathcal{B}$, then:

- f is a labeling- and causality-preserving bijection from X_1 to X_2 .
- For each $X_1 \xrightarrow{a}_{C_1} X'_1$ there exist $X_2 \xrightarrow{a}_{C_2} X'_2$ and $f' \subseteq \mathcal{E}_1 \times \mathcal{E}_2$ such that $(X'_1, X'_2, f') \in \mathcal{B}$ and $f' \upharpoonright X_1 = f$, and vice versa.
- For each $X'_1 \xrightarrow{a}_{C_1} X_1$ there exist $X'_2 \xrightarrow{a}_{C_2} X_2$ and $f' \subseteq \mathcal{E}_1 \times \mathcal{E}_2$ such that $(X'_1, X'_2, f') \in \mathcal{B}$ and $f \upharpoonright X'_1 = f'$, and vice versa. $\lrcorner$



■ **Figure 2** Spectrum of the main bisimilarities examined in [14, 12, 25] and their brm-variants.

$\sim_{\text{HHPB}}$ is strictly finer than $\sim_{\text{HPB}}$ and $\sim_{\text{WHPB}}$: for example, $(a \parallel_{\emptyset} (b + c)) + (a \parallel_{\emptyset} b) + ((a + c) \parallel_{\emptyset} b)$ and $(a \parallel_{\emptyset} (b + c)) + ((a + c) \parallel_{\emptyset} b)$ are identified by $\sim_{\text{HPB}}$ but told apart by $\sim_{\text{HHPB}}$, while the two complex processes in the forthcoming Figure 3 are equated by $\sim_{\text{WHPB}}$ but distinguished by $\sim_{\text{HHPB}}$.

3 Extending Bisimilarities with Backward Ready Multiset Equality

Forward-reverse bisimilarity was extended in [4] by additionally checking that the backward ready multisets of related configurations are equal. The purpose was to approach the distinguishing power of hereditary history-preserving bisimilarity also in the presence of autoconcurrency, i.e., identically labeled concurrent events. We recall from Section 1 that the two $\sim_{\text{HHPB}}$ -inequivalent processes $a \parallel_{\emptyset} a$ and $a . a$ are identified by $\sim_{\text{FRB}}$ but told apart by $\sim_{\text{FRB:brm}}$, because $a^{\dagger} \parallel_{\emptyset} a^{\dagger}$ has two incoming a -labeled transitions while $a^{\dagger} . a^{\dagger}$ has only one.

► **Definition 15.** Let $C = (\mathcal{E}, \mathcal{C}, \ell)$ be a stable configuration structure and $X \in \mathcal{C}$. The backward ready multiset of X is $\text{brm}(X) = \{ a \in \mathcal{A} \mid X' \xrightarrow{a}_{\mathcal{C}} X \}$. For each bisimilarity $\sim_B$ in Figure 1 we denote by $\sim_{B:\text{brm}}$ its backward ready multiset variant obtained by adding the clause $\text{brm}(X_1) = \text{brm}(X_2)$ for each pair $(X_1, X_2) \in \mathcal{B}$. $\square$

In this section we investigate the discriminating power of any $\sim_{B:\text{brm}}$. We first examine the bisimilarities of the no-iso family (Section 3.1), then those of the var-iso and incr-iso families (Section 3.2). The resulting extended spectrum is shown in Figure 2.

3.1 No-Iso Bisimilarities with Backward Ready Multiset Equality

Adding the backward ready multiset equality check to $\sim_{\text{IB}}$ yields a discriminating power greater than the one of $\sim_{\text{SB}}$. The intuition is that $\text{brm}(X)$ coincides with the label of the incoming step transition of X with the largest label. A counterexample witnessing the strict inclusion of $\sim_{\text{IB:brm}}$ in $\sim_{\text{SB}}$ is given by $a \parallel_{\emptyset} b$ and $a \parallel_{\emptyset} b + a . b$, which are identified by $\sim_{\text{SB}}$ but told apart by $\sim_{\text{IB:brm}}$ because $a^{\dagger} \parallel_{\emptyset} b^{\dagger}$ has two incoming transitions while $a \parallel_{\emptyset} b + a^{\dagger} . b^{\dagger}$ has only one.

► **Theorem 16.** *Let $C_i = (\mathcal{E}_i, \mathcal{C}_i, \ell_i)$, $i \in \{1, 2\}$, be two stable configuration structures. If $C_1 \sim_{\text{IB:brm}} C_2$ then $C_1 \sim_{\text{SB}} C_2$.* $\lrcorner$

Surprisingly, adding the backward ready multiset equality check to $\sim_{\text{SB}}$ yields the same distinguishing power as $\sim_{\text{IB:brm}}$.

► **Theorem 17.** *Let $C_i = (\mathcal{E}_i, \mathcal{C}_i, \ell_i)$, $i \in \{1, 2\}$, be two stable configuration structures. Then $C_1 \sim_{\text{IB:brm}} C_2$ iff $C_1 \sim_{\text{SB:brm}} C_2$.* $\lrcorner$

As far as $\sim_{\text{SB:brm}}$ and $\sim_{\text{PB:brm}}$ are concerned, we notice that they are finer than their original counterparts and that the latter is finer than the former:

- $a \parallel_\emptyset b$ and $a \parallel_\emptyset b + a \cdot b$ are identified by $\sim_{\text{SB}}$ but told apart by $\sim_{\text{SB:brm}}$, because $a^\dagger \parallel_\emptyset b^\dagger$ has two incoming transitions while $a \parallel_\emptyset b + a^\dagger \cdot b^\dagger$ has only one.
- $a \cdot (b + c) + (a \parallel_\emptyset b)$ and $a \cdot (b + c) + (a \parallel_\emptyset b) + a \cdot b$ are equated by $\sim_{\text{PB}}$ but distinguished by $\sim_{\text{PB:brm}}$, because $a \cdot (b + c) + a \parallel_\emptyset b + a^\dagger \cdot b^\dagger$ can only be matched by $a \cdot (b + c) + (a^\dagger \parallel_\emptyset b^\dagger)$ – otherwise c would be enabled after performing a – but the former has a single incoming transition whereas the latter has two.
- The two complex processes in the forthcoming Figure 3 (where parallel transitions have the same action label) are identified by $\sim_{\text{SB:brm}}$ but told apart by $\sim_{\text{PB:brm}}$, because in the former after executing a it is always possible to perform a pomset given by a and b causally related to each other, while in the latter this is not the case due to configuration 6_F .

The main result about no-iso bisimilarities is that the backward ready multiset equality check turns them into var-iso bisimilarities. In the following, given a stable configuration structure $C = (\mathcal{E}, \mathcal{C}, \ell)$ and $X \in \mathcal{C}$, we denote by $\text{brs}_e(X) = \{e \in \mathcal{E} \mid X' \xrightarrow{\ell(e)}_C X\}$ the backward ready set of X expressed in terms of events (which are all distinct); note that $\text{brm}(X) = \{\ell(e) \mid e \in \text{brs}_e(X)\}$. Moreover, we denote by $|M| = \sum_{a \in M} M(a)$ the cardinality of a multiset $M : \mathcal{A} \rightarrow \mathbb{N}$, where $M(a)$ is the multiplicity of a in M ; note that $|\text{brm}(X)| = |\text{brs}_e(X)|$. Given a transition $X \xrightarrow{\ell(e)}_C X'$, the set of immediate causes of e is given by $\text{causes}_{X'}(e) = \text{brs}_e(X) \setminus \text{brs}_e(X')$. The intuition is that, since $\text{brs}_e(X)$ collects the concurrent events that originate the incoming transitions of X , any event in $\text{brs}_e(X) \setminus \text{brs}_e(X')$ must have been a cause of the newly added event e , otherwise it would originate another incoming transition of X' . Some preliminary results are in order – where the possibly occurring bisimulations or bisimilarities are not necessarily no-iso – which highlight the relationships that exist among brs_e cardinality, brs_e inclusion, and causality.

► **Lemma 18.** *Let $C = (\mathcal{E}, \mathcal{C}, \ell)$ be a stable configuration structure and $X \xrightarrow{\ell(e)}_C X'$ for $X, X' \in \mathcal{C}$ and $e \in \mathcal{E}$. Then $|\text{brs}_e(X)| < |\text{brs}_e(X')|$ iff $\text{brs}_e(X) \subsetneq \text{brs}_e(X')$.* $\lrcorner$

► **Lemma 19.** *Let $C_i = (\mathcal{E}_i, \mathcal{C}_i, \ell_i)$, $i \in \{1, 2\}$, be two stable configuration structures and $X_i \xrightarrow{a}_C X'_i$ for $X_i, X'_i \in \mathcal{C}_i$ such that $(X_1, X_2), (X'_1, X'_2) \in \mathcal{B}$ for some brm-bisimulation $\mathcal{B}$. Then $\text{brs}_e(X_1) \subsetneq \text{brs}_e(X'_1)$ iff $\text{brs}_e(X_2) \subsetneq \text{brs}_e(X'_2)$.* $\lrcorner$

► **Lemma 20.** *Let $C = (\mathcal{E}, \mathcal{C}, \ell)$ be a stable configuration structure and $X \xrightarrow{\ell(e)}_C X'$ for $X, X' \in \mathcal{C}$ and $e \in \mathcal{E}$ such that $\text{brs}_e(X) \not\subseteq \text{brs}_e(X')$. Then $e' <_{X'} e$ for all $e' \in \text{causes}_{X'}(e)$.* $\lrcorner$

► **Theorem 21.** *Let $C_i = (\mathcal{E}_i, \mathcal{C}_i, \ell_i)$, $i \in \{1, 2\}$, be two stable configuration structures. If $C_1 \sim_{\text{B:brm}} C_2$ then for each pair of related configurations $X_1 \in \mathcal{C}_1$ and $X_2 \in \mathcal{C}_2$ there exists a labeling- and causality-preserving bijection $f : X_1 \rightarrow X_2$.* $\lrcorner$

► **Corollary 22.** $\sim_{\text{IB:brm}} = \sim_{\text{WHPB}}, \sim_{\text{PB:brm}} = \sim_{\text{WHPPB}}, \sim_{\text{FRB:brm}} = \sim_{\text{WHHPB}}.$ $\lrcorner$

3.2 Iso Bisimilarities with Backward Ready Multiset Equality

Adding the backward ready multiset equality check to var-iso and incr-iso bisimilarities does not increase their distinguishing power. The reason is that the aforementioned check can be derived from the existing isomorphism.

► **Theorem 23.** *Let $C_i = (\mathcal{E}_i, \mathcal{C}_i, \ell_i)$, $i \in \{1, 2\}$, be two stable configuration structures and $B \in \{\text{WHPB}, \text{WHPPB}, \text{WHHPB}, \text{HPB}, \text{HHPB}\}$. Then $C_1 \sim_B C_2$ iff $C_1 \sim_{B:\text{brm}} C_2$. $\square$*

4 Logical Characterizations of No-Iso and Var-Iso Bisimilarities

In this section we introduce an extension of the backward ready multiset logic of [4], which is in turn an extension of Hennessy-Milner logic [18] comprising a backward modality [10], denoted by $\langle a^\dagger \rangle$, and a backward ready multiset proposition, denoted by M . We use its fragments to characterize the bisimilarities of the no-iso and var-iso families, including their brm-variants that allow us to get rid of variable isomorphisms.

What we add is the capability of expressing pomsets, because in this way we can deal with all the three kinds of transitions appearing in the equivalences of Section 2: single action, step, and pomset. This is accomplished by replacing the usual single-action modality $\langle a \rangle$ with a more expressive modality $\langle \alpha \rangle$, where α is a *pomset term* generated by the following syntax in which $a \in \mathcal{A}$, “;” represents causality, and “||” represents concurrency:

$$\alpha ::= a \mid \alpha ; \alpha \mid \alpha \parallel \alpha$$

The pomset $U_\alpha = [(E_\alpha, <_\alpha, \ell_\alpha)]_\cong$ associated with α is defined by induction on the syntactical structure of α as follows where $\uplus$ denotes disjoint union:

$$\begin{aligned} U_a &= [(\{e\}, \emptyset, \{(e, a)\})]_\cong \\ U_{\alpha_1 ; \alpha_2} &= [(E_{\alpha_1} \uplus E_{\alpha_2}, <_{\alpha_1} \cup <_{\alpha_2} \cup (E_{\alpha_1} \times E_{\alpha_2}), \ell_{\alpha_1} \cup \ell_{\alpha_2})]_\cong \\ U_{\alpha_1 \parallel \alpha_2} &= [(E_{\alpha_1} \uplus E_{\alpha_2}, <_{\alpha_1} \cup <_{\alpha_2}, \ell_{\alpha_1} \cup \ell_{\alpha_2})]_\cong \end{aligned}$$

The resulting pomsets are called *series-parallel pomsets* in [13] as they can be constructed by starting from atomic actions and then combining them via sequential and parallel compositions. In [17, 29] it has been proven that a finite pomset is series-parallel iff it is N -free, i.e., it has no subpomsets of the form $N = [(\{e_1, e_2, e_3, e_4\}, \{(e_1, e_3), (e_2, e_4), (e_1, e_4)\}, \{(e_i, a) \mid 1 \leq i \leq 4\})]_\cong$. As shown in [17, 13] and remarked in [6], N -free pomsets can be completely characterized by the syntax of pomset terms. In particular, they constitute the kind of pomsets that label the transitions considered by pomset bisimilarity [6].

The syntax of our modal logic $\mathcal{L}_{\text{brm}, \text{pom}}$ is the following:

$$\phi ::= \text{true} \mid M \mid \neg\phi \mid \phi \wedge \phi \mid \langle \alpha \rangle \phi \mid \langle a^\dagger \rangle \phi$$

with $M : \mathcal{A} \rightarrow \mathbb{N}$, α being a pomset term, and $a \in \mathcal{A}$. Given a stable configuration structure $C = (\mathcal{E}, \mathcal{C}, \ell)$, we let $C \models \phi$ iff $\emptyset \models_C \phi$, where $X \models_C \phi$ for $X \in \mathcal{C}$ is defined by induction on the syntactical structure of $\phi \in \mathcal{L}_{\text{brm}, \text{pom}}$ as follows (when C is clear from the context, we will simply write $\models$):

$$\begin{aligned} X &\models_C \text{true} \\ X &\models_C M && \text{iff } \text{brm}(X) = M \\ X &\models_C \neg\phi' && \text{iff } X \not\models_C \phi' \\ X &\models_C \phi_1 \wedge \phi_2 && \text{iff } X \models_C \phi_1 \text{ and } X \models_C \phi_2 \\ X &\models_C \langle \alpha \rangle \phi' && \text{iff there exists } X \xrightarrow{U_\alpha}_C X' \text{ such that } X' \models_C \phi' \\ X &\models_C \langle a^\dagger \rangle \phi' && \text{iff there exists } X' \xrightarrow{a}_C X \text{ such that } X' \models_C \phi' \end{aligned}$$

■ **Table 1** Fragments of $\mathcal{L}_{\text{brm}, \text{pom}}$ characterizing no-iso and var-iso bisimilarities.

	true	M	$\neg$	$\wedge$	$\langle a \rangle$	$\langle \alpha_S \rangle$	$\langle \alpha \rangle$	$\langle a^\dagger \rangle$	
$\mathcal{L}_{\text{IB}}$	✓		✓	✓	✓				$\sim_{\text{IB}}$
$\mathcal{L}_{\text{SB}}$	✓		✓	✓		✓			$\sim_{\text{SB}}$
$\mathcal{L}_{\text{PB}}$	✓		✓	✓			✓		$\sim_{\text{PB}}$
$\mathcal{L}_{\text{FRB}}$	✓		✓	✓	✓			✓	$\sim_{\text{FRB}}$
$\mathcal{L}_{\text{IB:brm}}$	✓	✓	✓	✓	✓				$\sim_{\text{IB:brm}}, \sim_{\text{WHPB}}, \sim_{\text{WHPB:brm}}$
$\mathcal{L}_{\text{SB:brm}}$	✓	✓	✓	✓		✓			$\sim_{\text{SB:brm}}, \sim_{\text{WHPB}}, \sim_{\text{WHPB:brm}}$
$\mathcal{L}_{\text{PB:brm}}$	✓	✓	✓	✓			✓		$\sim_{\text{PB:brm}}, \sim_{\text{WHPPB}}, \sim_{\text{WHPPB:brm}}$
$\mathcal{L}_{\text{FRB:brm}}$	✓	✓	✓	✓	✓			✓	$\sim_{\text{FRB:brm}}, \sim_{\text{WHHPB}}, \sim_{\text{WHHPB:brm}}$

Let us reconsider the counterexamples right after Theorem 17. It holds that: $a \parallel_\emptyset b + a.b$ satisfies $\langle a \rangle \langle b \rangle \{b\}$ while $a \parallel_\emptyset b$ does not; $a.(b+c) + (a \parallel_\emptyset b) + a.b$ satisfies $\langle a \rangle (\neg \langle c \rangle \text{true} \wedge \langle b \rangle \{b\})$ while $a.(b+c) + (a \parallel_\emptyset b)$ does not; $E \models \neg \langle a \rangle \neg \langle a; b \rangle \text{true}$ in the forthcoming Figure 3 (where only different actions with the same subscript are causally related) while F does not.

Over stable configuration structures that are *image finite*, i.e., such that each of its configurations has finitely many outgoing transitions labeled with the same action, the no-iso and var-iso bisimilarities are characterized by the fragments of $\mathcal{L}_{\text{brm}, \text{pom}}$ shown in Table 1, where α_S is a pomset term in which “;” does not occur. This can be proven by directly addressing only the no-iso ones along with their brm-variants and then exploiting Corollary 22. Note that true, negation, and conjunction are common to all fragments, while M is needed only for the brm-variants. The used modalities depend on the specific bisimilarity.

► **Theorem 24.** *Let $C_i = (\mathcal{E}_i, \mathcal{C}_i, \ell_i)$, $i \in \{1, 2\}$, be two image-finite stable configuration structures and $B \in \{\text{IB}, \text{SB}, \text{PB}, \text{FRB}, \text{IB:brm}, \text{SB:brm}, \text{PB:brm}, \text{FRB:brm}\}$. Then $C_1 \sim_B C_2$ iff $C_1 \models \phi \iff C_2 \models \phi$ for all $\phi \in \mathcal{L}_B$.* $\dashv$

► **Theorem 25.** *Let $C_i = (\mathcal{E}_i, \mathcal{C}_i, \ell_i)$, $i \in \{1, 2\}$, be two image-finite stable configuration structures with $VIB \in \{\text{WHPB}, \text{WHPB:brm}\} \wedge NIBrmB \in \{\text{IB:brm}, \text{SB:brm}\}$ or $VIB \in \{\text{WHPPB}, \text{WHPPB:brm}\} \wedge NIBrmB \in \{\text{PB:brm}\}$ or $VIB \in \{\text{WHHPB}, \text{WHHPB:brm}\} \wedge NIBrmB \in \{\text{FRB:brm}\}$. Then $C_1 \sim_{VIB} C_2$ iff $C_1 \models \phi \iff C_2 \models \phi$ for all $\phi \in \mathcal{L}_{NIBrmB}$.* $\dashv$

5 Backward Ready Multisets and Diamonds for the Incr-Iso Family

In this section we discuss how to pass from the no-iso to the incr-iso family of bisimilarities. The intuition is that, when relating two configurations, in addition to verifying backward ready multiset equality – which is sufficient to pass from the no-iso to the var-iso family – in the simultaneous presence of autoconcurrency and non-local conflicts we also need to verify the existence of suitable diamond-shaped and half-diamond-shaped subconfiguration structures. When two transitions depart from a configuration thus opening a shape resembling the initial part of a diamond, whether it closes (diamond) or not (half diamond) gives us information about concurrency or conflict, respectively, among events not belonging to that configuration. Working with diamonds and half diamonds we obtain the same distinguishing power achieved by building an incremental isomorphism between the events matched so far.

Let us formalize what we mean by diamond and half diamond. The former is the result of interleaving two concurrent events starting from a given configuration. In the case of a half diamond of a configuration X , we not only require the existence of two transitions from X that open something that never closes, but we also require that at least one of these transitions does not belong to any diamond opening at the same configuration X .

► **Definition 26.** Let $C = (\mathcal{E}, \mathcal{C}, \ell)$ be a stable configuration structure. Given two configurations $X, Y \in \mathcal{C}$, the diamond that opens at X and closes at Y is defined by $d(X, Y) = \{(X, X', X'', Y) \mid X \xrightarrow{a}_C X', X \xrightarrow{b}_C X'', X' \xrightarrow{b}_C Y, X'' \xrightarrow{a}_C Y\}$. We denote by $\text{diam}(X) = \{d(X, Y) \mid Y \in \mathcal{C}\}$ the set of diamonds opening at X . $\lrcorner$

► **Definition 27.** Let $C = (\mathcal{E}, \mathcal{C}, \ell)$ be a stable configuration structure. Given a configuration $X \in \mathcal{C}$, a half diamond that opens at X is a triple (X, X', X'') such that $X \xrightarrow{a}_C X'$, $X \xrightarrow{b}_C X''$, $\nexists Y \in \mathcal{C}. (X' \xrightarrow{b}_C Y \wedge X'' \xrightarrow{a}_C Y)$, $\nexists D_1, D_2 \in \text{diam}(X). (X' \in D_1 \wedge X'' \in D_2)$. We denote by $\text{hdiam}(X)$ the set of half diamonds opening at X . $\lrcorner$

Let us indicate with $\sim_{\text{IB:brm:diam}}$ and $\sim_{\text{FRB:brm:diam}}$ the variants of $\sim_{\text{IB:brm}}$ and $\sim_{\text{FRB:brm}}$ where related configurations satisfy the following additional condition:

- If $\text{diam}(X_1) \neq \emptyset \wedge \text{diam}(X_2) \neq \emptyset$ then $\text{hdiam}(X_1) \neq \emptyset \iff \text{hdiam}(X_2) \neq \emptyset$.

We now provide some examples that illustrate the effect of this condition. In the forthcoming Figures 3–6, configurations are represented by numbers, parallel transitions have the same action label, the subscripts associated with identical actions identify the events originating those actions, and only different actions with the same subscript are causally related.

► **Example 28.** We start by noting that premise $\text{diam}(X_1) \neq \emptyset \wedge \text{diam}(X_2) \neq \emptyset$ is necessary for avoiding to rule out pairs of configurations that are trivially equivalent like for example a and $a + a$. If we were to check only the existence of half diamonds, then we would distinguish a and $a + a$, as the latter opens one half diamond while the former does not. By first requiring that both of them open at least one diamond each, the overall condition is vacuously satisfied and hence they are not told apart. $\lrcorner$

► **Example 29.** A more interesting example is given by the configuration structures E and F in Figure 3, taken from [25, Example 4.7], which are described by the following processes according to [16]:

$$((a_1 \cdot b_1) \parallel_{\emptyset} (a_2 \cdot b_2) \parallel_{\emptyset} (a_3 \cdot b_3)) \parallel_{\{a_1, a_3, b_1, b_2, b_3\}} ((a_1 + a_3) \parallel_{\emptyset} (b_1 + b_2) \parallel_{\{b_2\}} (b_2 + b_3))$$

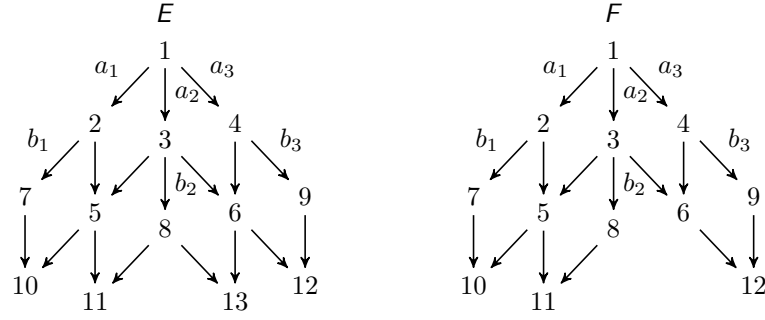
and:

$$((a_1 \cdot b_1) \parallel_{\emptyset} (a_2 \cdot b_2) \parallel_{\emptyset} (a_3 \cdot b_3)) \parallel_{\{a_1, a_3, b_1, b_2\}} ((a_1 + a_3) \parallel_{\{a_3\}} (a_3 + b_2) \parallel_{\{b_2\}} (b_1 + b_2))$$

where a relabeling mapping a_1, a_2, a_3 to a and b_1, b_2, b_3 to b should then be applied so as to get rid of action subscripts in the bisimulation game.

It is easy to see that E and F – where the latter features autoconcurrency as well as a conflict between actions a_3 and b_2 occurring in different parallel subprocesses – are not equated by any of the bisimilarities of the incr-iso family because:

- If 1_F performs a_3 thus evolving to 4_F , then 1_E can respond by performing a_3 (or equivalently a_1) thus evolving to 4_E . The initial bijection is $\{(a_3, a_3)\}$.
- If 4_F performs a_2 thus evolving to 6_F , then 4_E can respond by performing a_2 thus evolving to 6_E . The bijection becomes $\{(a_3, a_3), (a_2, a_2)\}$.



■ **Figure 3** E and F are not $\sim_{\text{HPB}/\text{HHPB}}$ -equivalent but are $\sim_{\text{WHPB}/\text{WHPPB}}$ -equivalent.

- If 6_E performs b_2 thus evolving to 13_E – note that E has become the challenger in the bisimulation game – then 6_F can only respond by performing b_3 thus evolving to 12_F . The bijection becomes $\{(a_3, a_3), (a_2, a_2), (b_2, b_3)\}$ but causality is not preserved because a_2 causes b_2 in E whereas a_2 does not cause b_3 in F .

It is worth noting that $\sim_{\text{PB}}$ and $\sim_{\text{WHPPB}}$ also distinguish these two configuration structures, because in E it is always possible to perform a pomset transition $a;b$ after any a -labeled transition while in F we cannot perform $a;b$ after a_3 . On the other hand, if we were to use a brm-based equivalence like $\sim_{\text{IB:brm}}$ or $\sim_{\text{FRB:brm}}$, we would not be able to distinguish E and F because 13_E and 12_F have the same backward ready multisets. However, we can distinguish them by using our diamond-based condition. It is easy to check that 1_F can perform an a -labeled transition and reach 3_F , which opens a diamond and a half diamond where only the transition from 3_F to 8_F belong to another diamond of 3_F , while all the configurations a -reachable from 1_E can only open diamonds. $\lrcorner$

► **Example 30.** A reasoning similar to the one of the previous example applies to the configuration structures G and H in Figure 4, taken from [25, Example 4.8], whose corresponding processes are:

$$((a_1 \cdot b_1) \parallel_{\emptyset} (a_2 \cdot b_2) \parallel_{\emptyset} (a_3 \cdot b_3) \parallel_{\emptyset} (a_4 \cdot b_4)) \parallel_{\{a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4\}} ((a_1 + a_3) \parallel_{\{a_1\}} (a_1 + a_4) \parallel_{\{a_4\}} (a_2 + a_4) \parallel_{\emptyset} (b_1 + b_2) \parallel_{\{b_2\}} (b_2 + b_3) \parallel_{\{b_3\}} (b_3 + b_4))$$

and:

$$((a_1 \cdot b_1) \parallel_{\emptyset} (a_2 \cdot b_2) \parallel_{\emptyset} (a_3 \cdot b_3) \parallel_{\emptyset} (a_4 \cdot b_4)) \parallel_{\{a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4\}} ((a_1 + a_3) \parallel_{\{a_1\}} (a_1 + a_4) \parallel_{\{a_4\}} (a_2 + a_4) \parallel_{\{a_2\}} (a_2 + b_3) \parallel_{\{b_3\}} (b_3 + b_4) \parallel_{\emptyset} (b_1 + b_2))$$

where H features autoconcurrency and a non-local conflict. These configuration structures are distinguished by $\sim_{\text{PB}}$, $\sim_{\text{WHPPB}}$, $\sim_{\text{HPB}}$, $\sim_{\text{HHPB}}$, but identified by $\sim_{\text{IB:brm}}$ and $\sim_{\text{FRB:brm}}$. $\lrcorner$

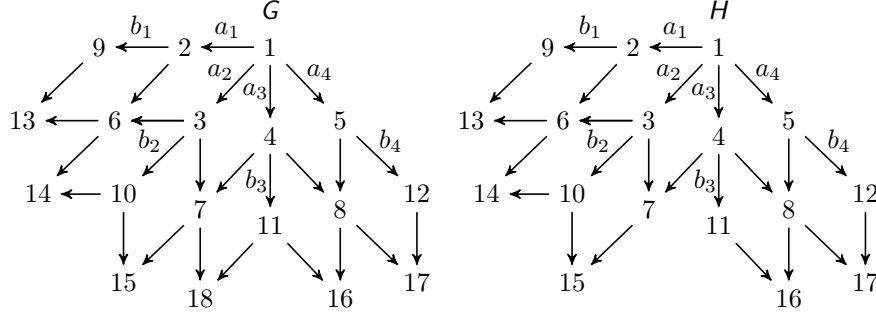
► **Example 31.** Consider the configuration structures I and J in Figure 5, taken from [25, Example 3.16], whose corresponding processes are:

$$((a_1 \cdot b_1) \parallel_{\emptyset} (a_2 \cdot b_2) \parallel_{\emptyset} (a_3 \cdot b_3) \parallel_{\emptyset} a_4) \parallel_{\{a_1, a_2, a_3, a_4, b_2, b_3\}} ((b_2 + b_3) \parallel_{\{b_2\}} (a_1 + b_2) \parallel_{\{a_1\}} (a_1 + a_3) \parallel_{\{a_1\}} (a_1 + a_4) \parallel_{\{a_4\}} (a_2 + a_4))$$

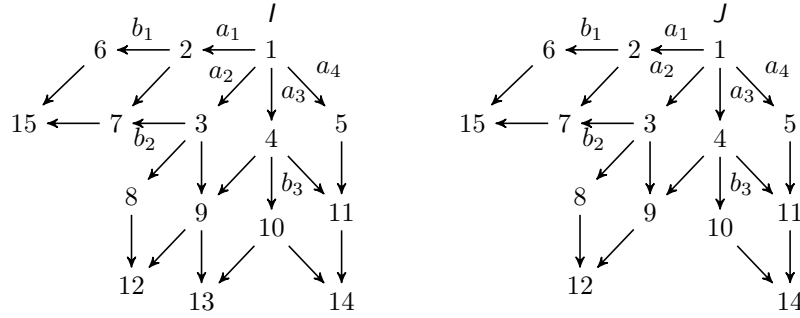
and:

$$((a_1 \cdot b_1) \parallel_{\emptyset} (a_2 \cdot b_2) \parallel_{\emptyset} (a_3 \cdot b_3) \parallel_{\emptyset} a_4) \parallel_{\{a_1, a_2, a_3, a_4, b_2, b_3\}} ((a_1 + b_2) \parallel_{\{a_1\}} (a_1 + a_3) \parallel_{\{a_1\}} (a_1 + a_4) \parallel_{\{a_4\}} (a_2 + a_4) \parallel_{\{a_2\}} (a_2 + b_3))$$

I and J can be distinguished only by the bisimilarities of the incr-iso family because:



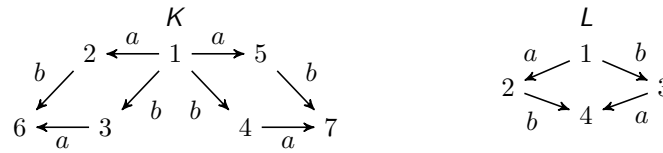
■ **Figure 4** G and H are not $\sim_{\text{HHPB/HHPB}}$ -equivalent but are $\sim_{\text{WHPPB/WHPPB}}$ -equivalent.



■ **Figure 5** I and J are not $\sim_{\text{HHPB/HHPB}}$ -equivalent but are $\sim_{\text{PB/WHPPB}}$ -equivalent.

- If 1_I performs a_3 thus evolving to 4_I , then 1_J can respond by performing a_3 (or a_1 , a_2 , or a_4) thus evolving to 4_J . The initial bijection is $\{(a_3, a_3)\}$.
- If 4_J performs a_2 thus evolving to 9_J , then 4_I can respond by performing a_2 thus evolving to 9_I . The bijection becomes $\{(a_3, a_3), (a_2, a_2)\}$.
- If 9_I performs b_3 thus evolving to 13_I , then 9_J can only respond by performing b_2 thus evolving to 12_J . The bijection becomes $\{(a_3, a_3), (a_2, a_2), (b_3, b_2)\}$ but causality is not preserved because a_3 causes b_3 in I whereas a_3 does not cause b_2 in J .

On the other hand, I and J are identified by $\sim_{\text{PB}}$ and $\sim_{\text{WHPPB}}$ and their backward ready multisets variants because 1_I and 1_J can both perform a single a -transition followed by either $a; b$ or $a \parallel b$. However, we can distinguish them with our diamond-based condition because if 1_J performs a_3 it evolves to 4_J , which opens a diamond and a half diamond, while all the configurations a -reachable from 1_I only open diamonds. $\lrcorner$



■ **Figure 6** K and L are identified by all the considered bisimilarities.

► **Example 32.** We must make sure that our diamond-based condition is not too discriminating. As an example, consider the configuration structures K and L in Figure 6, whose corresponding processes are $(a \parallel_{\emptyset} b) + (a \parallel_{\emptyset} b)$ and $(a \parallel_{\emptyset} b)$. The configurations 1_K and 1_L both

open diamonds, but the former also present the triples $(1_K, 2_K, 5_K)$, $(1_K, 2_K, 4_K)$, $(1_K, 3_K, 5_K)$, $(1_K, 3_K, 4_K)$ that resemble half diamonds. However, they do not comply with Definition 27 because all the configurations $2_K, 3_K, 4_K, 5_K$ appear in diamonds of 1_K . $\lrcorner$

► **Theorem 33.** *Let $C_i = (\mathcal{E}_i, C_i, \ell_i)$, $i \in \{1, 2\}$, be two stable configuration structures. Then $C_1 \sim_{\text{IB:brm:diam}} C_2$ iff $C_1 \sim_{\text{HPB}} C_2$.* $\lrcorner$

► **Theorem 34.** *Let $C_i = (\mathcal{E}_i, C_i, \ell_i)$, $i \in \{1, 2\}$, be two stable configuration structures. Then $C_1 \sim_{\text{FRB:brm:diam}} C_2$ iff $C_1 \sim_{\text{HHPB}} C_2$.* $\lrcorner$

6 Conclusions

In this paper we have revisited the spectrum of bisimilarities over stable configuration structures by dividing it into the no-iso, var-iso, and incr-iso families. We have shown that an additional backward ready multiset equality check – which is simpler to work with than isomorphisms – causes the discriminating power of no-iso bisimilarities to coincide with that of var-iso bisimilarities. Both families have been logically characterized via $\mathcal{L}_{\text{brm,pom}}$.

In the presence of autoconcurrency, backward ready multisets may not enough to obtain the discriminating power of incr-iso bisimilarities too. More precisely, recalling that $\sim_{\text{IB:brm}}$ and $\sim_{\text{FRB:brm}}$ are capable of distinguishing, e.g., between $a \parallel a$ and $a.a + a.a$ like $\sim_{\text{HPB}}$ and $\sim_{\text{HHPB}}$, the examples in Figures 3–5 clearly indicate that backward ready multisets are not enough in the simultaneous presence of autoconcurrency and non-local conflicts.

We have proven that, in order to reach the discriminating power of incr-iso bisimilarities also in the aforementioned case, it is necessary to examine diamonds and half diamonds. This yields an alternative characterization of $(\sim_{\text{HPB}} \text{ and } \sim_{\text{HHPB}})$ that, unlike the one in [1], uses binary relations instead of ternary ones and does not rely on event identification. The structural properties of diamonds and half diamonds clarify the boundary between the var-iso and incr-iso families, i.e., between weak and strong history preservations, thus potentially simplifying equivalence checking in a true concurrency setting.

As for future work, similar to [2, 26] we would like to find an extension of $\mathcal{L}_{\text{brm,pom}}$ whose fragments characterize the entire spectrum in Figure 2, i.e., also incr-iso bisimilarities.

References

- 1 C. Aubert and I. Cristescu. How reversibility can solve traditional questions: The example of hereditary history-preserving bisimulation. In *Proc. of the 31st Int. Conf. on Concurrency Theory (CONCUR 2020)*, volume 171 of *LIPIcs*, pages 7:1–7:23, 2020. doi:10.4230/LIPIcs.CONCUR.2020.7.
- 2 P. Baldan and S. Crafa. A logic for true concurrency. *Journal of the ACM*, 61:24:1–24:36, 2014. doi:10.1145/2629638.
- 3 M.A. Bednarczyk. Hereditary history preserving bisimulations or what is the power of the future perfect in program logics. Technical report, Polish Academy of Sciences, Gdansk, 1991.
- 4 M. Bernardo, A. Esposito, and C.A. Mezzina. Alternative characterizations of hereditary history-preserving bisimilarity via backward ready multisets. In *Proc. of the 28th Int. Conf. on Foundations of Software Science and Computation Structures (FOSSACS 2025)*, volume 15691 of *LNCS*, pages 67–87. Springer, 2025. doi:10.1007/978-3-031-90897-2_4.
- 5 M. Bernardo and S. Rossi. Reverse bisimilarity vs. forward bisimilarity. In *Proc. of the 26th Int. Conf. on Foundations of Software Science and Computation Structures (FOSSACS 2023)*, volume 13992 of *LNCS*, pages 265–284. Springer, 2023. doi:10.1007/978-3-031-30829-1_13.
- 6 G. Boudol and I. Castellani. Concurrency and atomicity. *Theoretical Computer Science*, 59:25–84, 1988. doi:10.1016/0304-3975(88)90096-5.
- 7 S.D. Brookes, C.A.R. Hoare, and A.W. Roscoe. A theory of communicating sequential processes. *Journal of the ACM*, 31:560–599, 1984. doi:10.1145/828.833.

- 8 Ph. Darondeau and P. Degano. Causal trees. In *Proc. of the 16th Int. Coll. on Automata, Languages and Programming (ICALP 1989)*, volume 372 of *LNCS*, pages 234–248. Springer, 1989. doi:10.1007/BFB0035764.
- 9 Ph. Darondeau and P. Degano. Causal trees: Interleaving + causality. In *Proc. of the LITP Spring School on Theoretical Computer Science: Semantics of Systems of Concurrent Processes*, volume 469 of *LNCS*, pages 239–255. Springer, 1990. doi:10.1007/3-540-53479-2_10.
- 10 R. De Nicola and F. Vaandrager. Three logics for branching bisimulation. *Journal of the ACM*, 42:458–487, 1995. doi:10.1145/201019.201032.
- 11 P. Degano, R. De Nicola, and U. Montanari. Observational equivalences for concurrency models. In *Proc. of the 3rd IFIP Working Conf. on Formal Description of Programming Concepts*, pages 105–134. North-Holland, 1986.
- 12 H. Fecher. A completed hierarchy of true concurrent equivalences. *Information Processing Letters*, 89:261–265, 2004. doi:10.1016/J.IPL.2003.11.008.
- 13 J.L. Gischer. The equational theory of pomsets. *Theoretical Computer Science*, 61:199–224, 1988. doi:10.1016/0304-3975(88)90124-7.
- 14 R.J. van Glabbeek and U. Goltz. Refinement of actions and equivalence notions for concurrent systems. *Acta Informatica*, 37:229–327, 2001. doi:10.1007/S002360000041.
- 15 R.J. van Glabbeek and G.D. Plotkin. Configuration structures, event structures and Petri nets. *Theoretical Computer Science*, 410:4111–4159, 2009. doi:10.1016/J.TCS.2009.06.014.
- 16 R.J. van Glabbeek and F. Vaandrager. Bundle event structures and CCSP. In *Proc. of the 14th Int. Conf. on Concurrency Theory (CONCUR 2003)*, volume 2761 of *LNCS*, pages 57–71. Springer, 2003. doi:10.1007/978-3-540-45187-7_4.
- 17 J. Grabowski. On partial languages. *Fundamenta Informaticae*, 4:427–498, 1981.
- 18 M. Hennessy and R. Milner. Algebraic laws for nondeterminism and concurrency. *Journal of the ACM*, 32:137–162, 1985. doi:10.1145/2455.2460.
- 19 A. Joyal, M. Nielsen, and G. Winskel. Bisimulation from open maps. *Information and Computation*, 127:164–185, 1996. doi:10.1006/INCO.1996.0057.
- 20 R.M. Keller. Formal verification of parallel programs. *Communications of the ACM*, 19:371–384, 1976. doi:10.1145/360248.360251.
- 21 R. Milner. *Communication and Concurrency*. Prentice Hall, 1989.
- 22 D. Park. Concurrency and automata on infinite sequences. In *Proc. of the 5th GI Conf. on Theoretical Computer Science (GI-TCS 1981)*, volume 104 of *LNCS*, pages 167–183. Springer, 1981. doi:10.1007/BFB0017309.
- 23 I. Phillips and I. Ulidowski. Reversing algebraic process calculi. *Journal of Logic and Algebraic Programming*, 73:70–96, 2007. doi:10.1016/J.JLAP.2006.11.002.
- 24 I. Phillips and I. Ulidowski. Reversibility and models for concurrency. In *Proc. of the 4th Int. Workshop on Structural Operational Semantics (SOS 2007)*, volume 192(1) of *ENTCS*, pages 93–108. Elsevier, 2007. doi:10.1016/J.ENTCS.2007.08.018.
- 25 I. Phillips and I. Ulidowski. A hierarchy of reverse bisimulations on stable configuration structures. *Mathematical Structures in Computer Science*, 22:333–372, 2012. doi:10.1017/S0960129511000429.
- 26 I. Phillips and I. Ulidowski. Event identifier logic. *Mathematical Structures in Computer Science*, 24(2):1–51, 2014.
- 27 L. Pomello. Some equivalence notions for concurrent systems. an overview. In *Advances in Petri Nets*, volume 222 of *LNCS*, pages 381–400. Springer, 1985. doi:10.1007/BFB0016222.
- 28 A.M. Rabinovich and B.A. Trakhtenbrot. Behavior structures and nets. *Fundamenta Informaticae*, 11:357–404, 1988.
- 29 J. Valdes, R.E. Tarjan, and E.L. Lawler. The recognition of series parallel digraphs. *SIAM Journal on Computing*, 11:298–313, 1982. doi:10.1137/0211023.
- 30 G. Winskel. Event structures. In *Advances in Petri Nets*, volume 255 of *LNCS*, pages 325–392. Springer, 1986. doi:10.1007/3-540-17906-2_31.