Simulating Path Integration with Continuous Attractor Neural Networks

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Abstract

This paper explores how grid cell-inspired neural dynamics can simulate path integration during human navigation. We combine smartphone GPS traces, trajectory interpolation, and a two-dimensional continuous attractor neural network (2D CANN) to model path integration in York and Leeds, United Kingdom. The model reproduces grid cell-like firing and estimates trajectories that closely match mapped movement. We further introduce a logarithmic psychophysical transformation of velocity to examine how perceived speed may distort cognitive spatial structure, especially under mixed transportation modes. The results suggest that velocity-driven path integration can computationally account for asymmetric distance estimation and cognitive collage-like distortions in urban environments.

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1 Introduction

The metric structure of spatial mental representation, and the Euclidean form built upon it, is central to cognitive geography. The status of Euclidean space in human spatial cognition has long motivated debate [16, 7, 18]. Researchers generally agree that cognitive space differs from physical space and often contains biases or distortions. Geographers and urban analysts have examined associations between geodesic and cognitive distance [15], as well as built-environment features that influence perceived distance or accessibility [12]. Recent findings question the relevance of Euclidean representation, highlighting the role of topological structures [3] and non-integrated cognitive collages [17].

Cognitive neuroscience has provided mechanistic evidence for the formation of Euclidean cognitive space. One of the most influential findings is the discovery of *grid cells* in the medial entorhinal cortex (MEC). Grid cells are activated at locations that tessellate Euclidean space in a hexagonal lattice. This firing pattern represents metric distance, Euclidean spatial relations, and coarse direction [8]. Subsequent work has shown that grid cells encode important properties of Euclidean space because their activity is driven by velocity signals. As an individual moves continuously, grid cells integrate velocity inputs and iteratively update their encoding of current position. This process, analogous to SLAM or inertial navigation, is known as *path integration* [5].

Path integration is highly relevant to cognitive maps in human geography and urban science. Grid cells in the human brain may register travel speed during everyday mobility and, through path integration, partially shape spatial layout understanding. Perceived travel

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speed is influenced by transportation mode, attention, and individual differences [13, 21]. Estimating cognitive spatial relationships consistent with principles of spatial cognition would be meaningful for spatial theory in GIS and for human-centered geospatial applications. Despite this importance, relatively few studies have explored this topic.

Recent computational neuroscience now provides toolkits for simulating multiple neural cell types, including grid cells. By constructing a recurrent neural network, inputting signals such as velocity, and iteratively simulating the dynamic system until it reaches a stable state, specific firing patterns can emerge. This recurrent dynamic system is an attractor neural network (ANN), and the grid-cell system is commonly modeled as a continuous variant, namely a continuous attractor neural network (CANN). We use these tools to model grid cells and path integration, and to examine how these neural processes may systematically distort cognitive maps.

2 Methods

This initial study uses GPS trajectories recorded by a smartphone during explorations of York and Leeds, two major cities in northern England. The trajectories covered urban and rural areas of both cities. GPS positions were recorded with the smartphone application *FootPrint* and exported as time series of geolocated coordinates. Time was recorded as local timestamps, and the application-reported location error averaged 8.94 m (SD = 7.89 m). York is smaller than Leeds and its attractions are more spatially concentrated. Trajectories combine walking and bus travel.

Mobility in urban environments includes at least two forms: staying at a place for an activity and moving toward the next place [14]. Staying may occupy much of the recording period without movement. We removed stationary periods to reduce data volume and accelerate the neurodynamic simulation. Because users do not generate dense moving points while stationary, we calculated time differences between consecutive GPS records. The 90th percentile was used as a threshold, and longer gaps were compressed to it. After compression, the trajectories approximate continuous movements without stays. The mean time difference between consecutive GPS records was 74.43 seconds (SD = 113.89 s), still too coarse for precise velocity estimation. We therefore used `RatInABox` [6] to interpolate intermediate points between adjacent GPS records at 0.02 s resolution using cubic splines, and then estimated the corresponding velocity time series. This fine temporal resolution is required for numerical stability of the CANN differential-equation integration; the spline interpolation produces smooth velocity signals between GPS fixes while keeping the overall trajectory shape anchored to the actual GPS data. The resulting trajectories have high spatiotemporal resolution and dense sampling, with 90,173 records for York and 133,485 for Leeds.

To model path integration supported by grid cells, we use the computational neuroscience toolbox `CANNs` to simulate a continuous attractor neural network that generates grid-cell-like firing [10]. We briefly introduce the continuous ANN theory of grid cells here. Although researchers generally agree that grid-cell firing is driven by velocity input and neural encoding of current position, the mechanism by which velocity produces a hexagonal firing pattern remains debated. One prominent theory argues that this pattern emerges from an internally interconnected recurrent neural network. Neurons send excitatory or inhibitory signals to one another, and as the system evolves iteratively, cell activation reaches a stable dynamic equilibrium. If the system deviates from this equilibrium, internal interactions drive it toward a stable state, which may or may not match the previous one. Mathematically, the

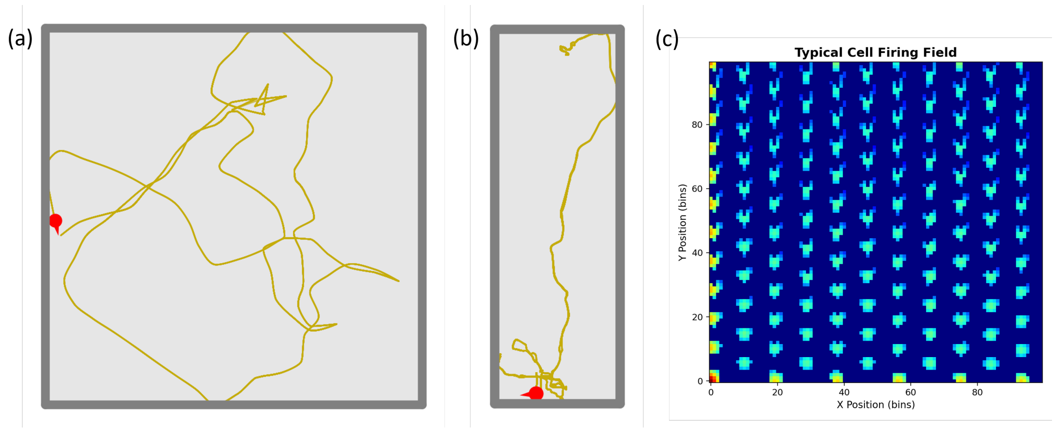

Figure 1 Interpolated trajectories in York (a) and Leeds (b), displayed as if they occurred in an experimental arena. Panel (c) shows the simulated firing pattern of a single grid cell in the CANN system defined in this study, generated in a controlled square arena (side length 2.2 m, movement speed 0.3 m/s). Colour intensity indicates firing rate (brighter means stronger activation); the distinct peaks form the canonical hexagonal lattice.

Table 1 Important parameters used in the CANN grid-cell simulation.

Parameter	Value	Explanation
τ	0.01	Time constant controlling response and decay of neural activity.
α	0.10	Coupling strength between velocity input and activity-bump displacement.
W_ℓ	3.0	Spatial scale of local recurrent connectivity.
λ_{net}	17.0	Internal spatial periodicity controlling the grid-like firing scale.

neural network forms a dynamic system, and stable states are solutions to the corresponding differential equations. The set of all stable solutions is called an attractor. External inputs can break the equilibrium by injecting directed excitatory or inhibitory signals, after which the system forms a new stable state. Computational neuroscientists therefore model grid cells as a neural network with attractor dynamics, that is, as an ANN [11]. In terms of neural connectivity, this ANN forms a torus-like structure. Neurons are arranged as a square sheet in which each cell excites nearby cells and inhibits more distant cells [2], while opposite edges are treated as adjacent. When mammals move through an environment, the excitatory and inhibitory signals shift directionally, changing cell activation. As a result, each neuron in the sheet can generate a hexagonal lattice of spatial activation.

As a representative two-dimensional continuous attractor neural network (2D CANN), the grid-cell system can be modeled with `CANNs`, a library for simulating attractor neural networks and their time-dependent evolution. We define a neural sheet with 40×40 cells. To obtain stable results, we set the main parameters according to official guidance [10], as summarized in Table 1. The 2D CANN tracks the firing state of each cell during movement and decodes an inferred trajectory from neural activity. We treat this trajectory as a computational representation of the cognitive map.

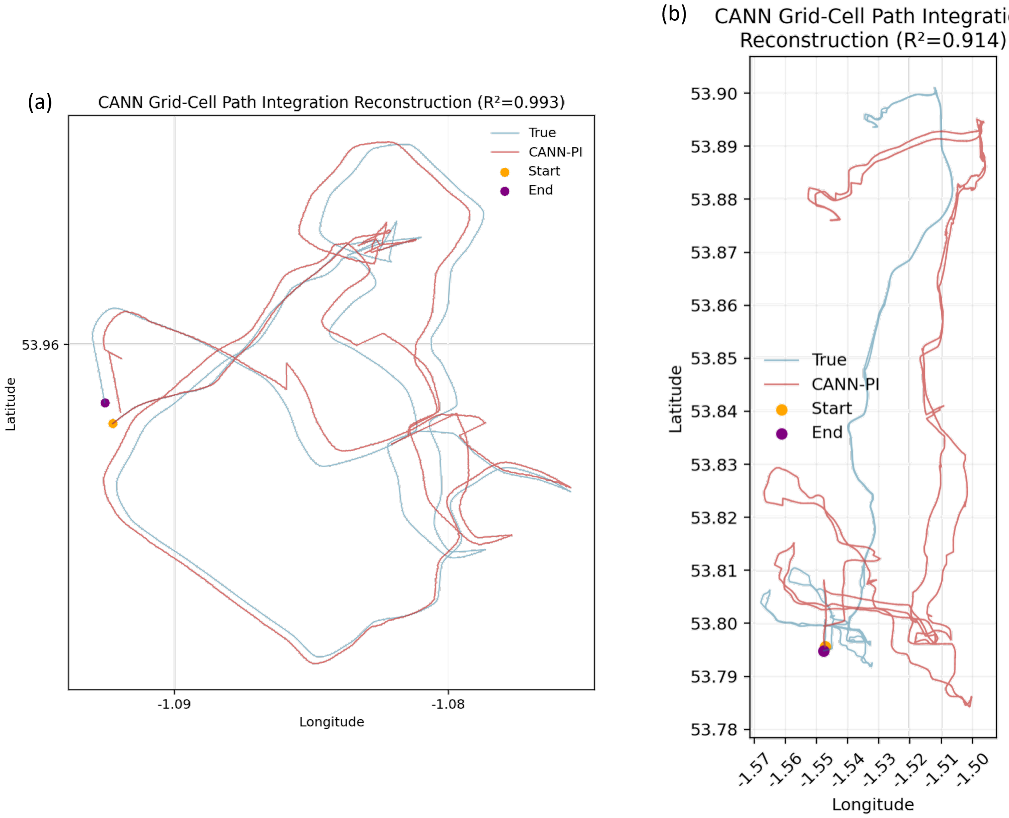

Figure 2 Path integration results of the CANN grid-cell system in York (a) and Leeds (b).

3 Results

The grid-cell network defined as a 2D CANN reproduces the canonical grid-cell firing pattern and supports path integration. The simulated grid cell shows the typical hexagonal lattice firing pattern (Figure 1c). When the inferred trajectory is decoded from the network and compared with mapped mobility, the system behaves like inertial navigation. It delineates the displacement vector from the starting point with high accuracy in York (Figure 2a). To quantify trajectory similarity, we compute cumulative displacement coordinates from the displacement increment series (Δx, Δy) of both the decoded and actual trajectories, then regress the decoded coordinates against the actual coordinates (x and y pooled) using univariate linear regression. This captures shape fidelity while remaining insensitive to overall scale differences, which is appropriate because the grid-cell path-integration system is primarily concerned with relative spatial relationships between locations rather than absolute metric distances. The resulting R^2 scores are 0.994 in York and 0.914 in Leeds. Notably, the estimated Leeds trajectory is distorted: round trips between the city center and Harewood House are slightly shortened. This suggests that the grid-cell CANN system itself can contribute to spatial cognitive distortion.

We further apply this tool to explore potential cognitive-map distortions. A typical question is whether motorized transportation distorts the metric structure of an individual's cognitive space. We hypothesize that such an effect exists and mainly arises from biased speed estimation. Prior research shows that transportation mode and vehicle type can bias perceived movement speed [13]. Here, we adopt the Weber-Fechner law, a psychophysical rule of stimulus perception, to relate actual and perceived movement speed through a logarithmic function:

$$v_p = k \ln(v_a), \tag{1}$$

where

$$k = \frac{v_b}{\ln(v_b)}. \tag{2}$$

Here, v_p and v_a denote perceived and actual velocity, respectively. The mapping depends on coefficient k, which depends on a user-defined velocity benchmark v_b. The second equation implies that individuals estimate the benchmark velocity relatively accurately. We set $v_b = 5$ m/s, approximately 18 km/h, or cycling speed. This assumption is reasonable for the present exploratory analysis. For slower activities such as walking or running, higher speed usually implies greater physical effort, so people may overestimate movement speed. Conversely, in faster modes such as buses, private cars, or trains, people may not fully perceive speed because of attention, restricted visual exposure, or being inside an enclosed vehicle. They may therefore underestimate travel speed.

We transform all actual instantaneous velocities into perceived velocities with this logarithmic function and input the biased signals into the CANN dynamic system. The result is shown in Figure 3. Because visitors in York only walked through the city and their speed was relatively stable, the path-integration trajectory is slightly larger than the real trajectory because walking speed is overestimated. Nevertheless, its overall shape remains highly consistent with the mapped trajectory, with a squared R score of 0.942 (Figure 3a). The Leeds case differs because mobility included bus travel. Distances between the origins and destinations of bus trips are visibly shrunk (Figure 3b). Movements within Harewood House Trust and downtown Leeds are disproportionately enlarged, resembling "magnifying glass" bias in cognitive map distortion [4]. In this sense, the cognitive map simulated by CANN neural dynamics and a simple psychophysical rule may be closer to subjective spatial cognition than to geomatic survey space. Another important finding is that the real Leeds trajectory starts and ends at the train station, as indicated by the orange and purple points in Figure 3b. In the estimated trajectory, however, the two points are far apart. One reason is that the outbound trip from downtown to the University was on foot, whereas the return trip was by bus. This difference produced asymmetric distance estimation between the two locations, causing the estimated endpoint to deviate from the starting point. This representation of asymmetric distance estimation and positional ambiguity captures an important aspect of the cognitive collage phenomenon [17].

4 Discussion

This study introduces computational neuroscience theories and approaches to model grid cells and their tracking of locomotion. The simulations demonstrate the model's capacity to perform path integration and compute movement-based spatial estimates. A central contribution is that a simple psychological assumption about speed perception can produce a cognitive mapping representation broadly consistent with spatial cognitive intuition. The logarithmic function used here may oversimplify distance perception, and the cycling-speed benchmark is partly arbitrary. Nevertheless, with more systematic quantitative study of human distance perception, this approach may support more plausible computational cognitive maps.

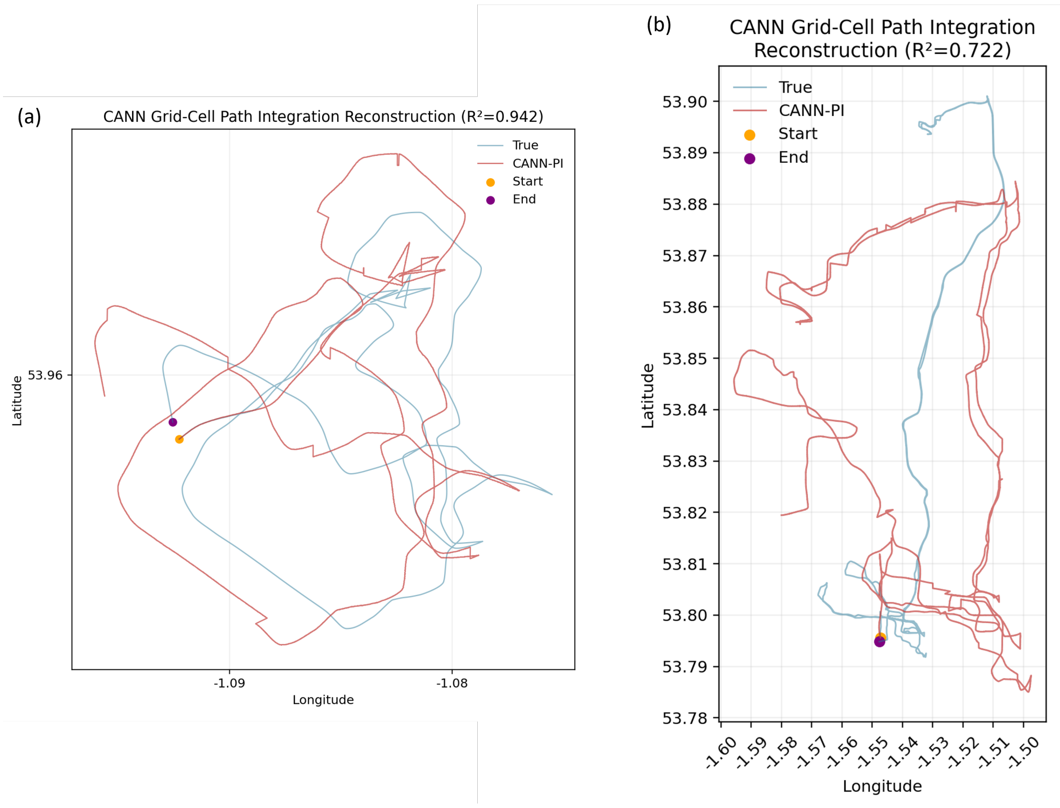

Figure 3 Path integration results based on logarithmically biased velocity signals in York (a) and Leeds (b).

Being driven by velocity signals is a defining feature of the grid-cell motion-tracking system. This velocity priority can explain several spatial cognitive phenomena. For example, when individuals travel between the same pair of places using different modes, such as walking one way and taking a bus the other, perceived speed biases may differ by direction. This can produce different distance estimates and generate the classic asymmetry of distance estimation in spatial cognition research [1]. Such asymmetry means that the same place may occupy different positions in cognitive space. Across a larger mobility network, non-integration may occur repeatedly. This provides a computational externalization and interpretation of the cognitive collage, characterized by locally near-Euclidean structure but non-uniform and fragmented global spatial relations.

Another issue concerns validation. Evaluating the alignment between "neurally estimated" cognitive distance and human judgments remains challenging. One feasible strategy is to follow multidimensional scaling (MDS) logic by asking survey participants to judge distances between pairs of places and comparing those judgments with the computational results. This could evaluate the logarithmic rule, but would not reveal how individual differences shape distorted cognitive mapping. A complementary strategy is to fit velocity perception parameters or functions to surveyed distance estimates. Future studies can investigate these questions in greater depth.

Undoubtedly, path integration is not the only component of cognitive mapping. In urban navigation, landmarks and other environmental features also play important roles. As travel time increases, pure path integration tends to generate larger errors in distance estimation, much as walking in darkness accumulates positional uncertainty. Landmarks provide sensory information, especially visual input, that can correct current position estimates and help integrate the overall map. At the neural level, path integration and sensory correction correspond to the hippocampal-entorhinal interactive system represented by grid cells and place cells [20, 19]. A major task for future work is to understand how visible elements, including landmarks and boundaries, correct path-integration biases and enable more accurate cognitive-map modeling. This issue is particularly important in urban environments. How do urban-specific elements, such as transit lines, street network topology, and building distribution, affect cognitive mapping differently from a rat moving in an experimental arena? Contemporary computational neuroscience suggests that cognitive maps are built upon two major pillars: movement-derived reference frames and sensory feature anchors [9]. Modeling urban cognitive maps with such interdisciplinary tools may therefore renew how behavioral geography and geographic information science understand space.

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