Computability and Complexity of Julia Sets (Invited Talk)

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Studying dynamical systems is key to understanding a wide range of phenomena ranging from planetary movement to climate patterns to market dynamics. Various numerical tools have been developed to address specific questions about dynamical systems, such as predicting the weather or planning the trajectory of a satellite. However, the theory of computation behind these problems appears to be very difficult to develop. In fact, little is known about computability of even the most natural problems arising from dynamical systems.

In this talk I will survey the recent study of the computational properties of dynamical systems that arise from iterating quadratic polynomials on the complex plane. These give rise to the amazing variety of fractals known as Julia sets, and are closely connected to the Mandelbrot set. Julia sets are perhaps the most drawn objects in Mathematics due to their fascinating fractal structure. The theory behind them is even more fascinating, and the dynamical systems generating them are in many ways archetypal. I will present both positive and negative results on the computability and complexity of Julia sets.

In conclusion of the talk I will discuss possible future directions and challenges in the study of the computability and complexity of dynamical systems.