# Integrating Line Planning for Construction Sites into Periodic Timetabling via Track Choice

Berenike Masing<sup>1</sup>  $\square$   $\square$ Zuse Institute Berlin, Germany

Niels Lindner  $\square$ Freie Universität Berlin, Germany

#### Christian Liebchen 🖂 🕩

Technical University of Applied Sciences Wildau, Germany

#### – Abstract

We consider maintenance sites for urban rail systems, where unavailable tracks typically require changes to the regular timetable, and often even to the line plan. In this paper, we present an integrated mixed-integer linear optimization model to compute an optimal line plan that makes best use of the available tracks, together with a periodic timetable, including its detailed routing on the tracks within the stations. The key component is a flexible, turn-sensitive event-activity network that allows to integrate line planning and train routing using a track choice extension of the Periodic Event Scheduling Problem (PESP). Major goals are to maintain as much of the regular service as possible, and to keep the necessary changes rather local. Moreover, we present computational results on real construction site scenarios on the S-Bahn Berlin network. We demonstrate that this integrated problem is indeed solvable on practically relevant instances.

2012 ACM Subject Classification Applied computing  $\rightarrow$  Transportation; Mathematics of computing  $\rightarrow$  Combinatorial optimization

Keywords and phrases Periodic Timetabling, Line Planning, Track Choice, Mixed-Integer Programming, Construction Sites, Railway Rescheduling

Digital Object Identifier 10.4230/OASIcs.ATMOS.2023.5

Acknowledgements We would like to thank DB Netz AG for providing us with data and sharing their experience and insights.

#### 1 Introduction

#### 1.1 Motivation

In particular in agglomerations, metro and local fast train systems are among the transportation systems with the highest capacity, and commonly considered very environmentally friendly. Keeping them in a safe and efficient state requires continuous maintenance measures, some involving construction sites. Track blockages are a likely consequence, and often risk to restrain capacity such that not the complete service of the annual timetable can be operated. In the combination of numerous such construction sites and valid periods of few weeks or even only days, the resulting efforts of the planning divisions are particularly challenging.

In [12], an optimization model has been proposed, which covers parts of this planning task. Since the infrastructure which remains available for the operation typically will face a very high load, efficient planning of track occupation becomes key. Based on this motivation, track choice has been integrated into the basic model of periodic timetabling in [23], and it has been extended in [12] to deal with conflicts that arise for non-negligible turning or waiting times inside stations, as they are natural in construction site scenarios.

© Berenike Masing, Niels Lindner, and Christian Liebchen;

licensed under Creative Commons License CC-BY 4.0 23rd Symposium on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems (ATMOS 2023).

Editors: Daniele Frigioni and Philine Schiewe; Article No. 5; pp. 5:1–5:15

OpenAccess Series in Informatics **OASICS** Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

corresponding author

#### 5:2 Track Choice PESP with Integrated Line Planning

Yet, in [12], the line plan had been assumed to be already given on a macroscopic station level as part of the input. But this way, major decisions have already been taken, and even an implicit qualified guessing of a possible timetable, including routings within stations, might have been considered. In other words, in particular for construction sites, a separation of line planning and timetabling might be too restrictive. This is why in the present paper, we broaden the scope even further: We enrich that model to also make decisions of line planning. We restrict ourselves only to parts of the network in such a way, that the model remains solvable but is of relevant size for an infrastructure manager.

The paper is structured as follows: We review briefly literature on line planning, periodic timetabling and their integration in Section 1.2. Section 2 is the theoretical core. Starting with a description of the input in Section 2.1, we construct our main modeling ingredient, the *extended turn-sensitive event-activity network* in Section 2.2. We discuss operational requirements in Section 2.3. This leads to the definiton of our central problem, the *Integrated Line Planning and Turn-Sensitive Periodic Timetabling Problem with Track Choice*, which we formulate as a mixed-integer program in Section 2.4. Finally, we extend the problem and its MIP model to the construction site context. Section 3 is devoted to an experimental application of our model to real-world scenarios on the S-Bahn Berlin network. After describing these instances in Section 3.1, we present computational results in Section 3.2, and conclude the paper in Section 3.3.

#### 1.2 Literature Overview

The standard mathematical model for periodic timetable optimization is the Periodic Event Scheduling Problem (PESP) introduced in [21]. The literature on PESP is numerous, we refer to the monographs [8, 14, 10, 15] and to recent algorithmic advances [6, 1, 13, 11, 2]. Several extensions of PESP for the application of railway timetabling have been singled out. These include, e.g., flexible event timings [3], robustness [5], flexible routings via track choice [23], rescheduling for construction sites [22], and recently, [12].

Line planning is a planning step that usually directly precedes timetabling. We refer to [20, 18] for an overview. The integration of line planning and periodic timetabling is an ongoing research topic and is a showcase of the eigenmodel approach [19, 16]. An integrated model that produces one component of the event-activity network per line in a line pool is presented in [17]. An iterative approach using satisfiability methods is described in [4].

Our contribution consists of a highly integrated model that unifies periodic timetabling, line planning and also parts of vehicle scheduling by exploiting track choice. As our primary goal is to apply the model to construction sites, we do not work with an arbitrary line pool, but rather work with certain sets of alternatives per regular line. We describe our model in detail in the subsequent section.

## 2 A Model for Integrated Line Planning and Periodic Timetabling with Track Choice

Before addressing the specific problem of construction sites, let us discuss how to integrate track choice, but also line planning into a general periodic timetabling context. We will first outline which preliminary assumptions we make, how to model this with the help of an *extended turn-sensitive event-activity network* and present a basic model for integrated line planning with periodic timetabling LPTT.

### 2.1 Input Description

We will use  $\mathcal{V}(\cdot)$  to refer to the node set and  $\mathcal{A}(\cdot)$  for the set of arcs both in the context of graphs, as well as paths.

The station-link graph S is a digraph set on a macroscopic level, where  $\mathcal{V}(S)$  represent stations and an arc  $a = (v, w) \in \mathcal{A}(S)$  indicates that there are tracks that link stations v and w with tracks, such that a train can drive from v to w without a change in direction.

Let I denote the set of *infrastructure points*, which encompasses the pocket tracks and platforms in the transportation network, i.e., places which may be occupied by a train for turning or waiting operations, while the vehicle itself remains idle. For correct planning, we need to capture direction information, namely from which direction a train enters a platform or pocket track, and in which it departs. The physical counterparts correspond to track segments, each with two ends, which we label by + and -, respectively.

▶ Definition 1 (Infrastructure graph  $\mathcal{I}$ ). The infrastructure graph  $\mathcal{I}$  is a digraph with  $\mathcal{V}(\mathcal{I}) = I$ . Two infrastructure points v and w are connected by a track-link  $(v, w) \in \mathcal{A}(\mathcal{I})$  if a train can drive from v directly to w without going over other infrastructure points. To each track-link (v, w) we assign a direction label  $\phi(v, w) = (z_v, z_w)$ , where  $z_v, z_w \in \{+, -\}$  correspond to the labeled ends of the physical tracks when driving from v to w. We denote by  $\phi(v, w)^{out} = z_v$  and  $\phi(v, w)^{in} = z_w$  the out- and in-labels for  $\phi(v, w) = (z_v, z_w)$ , respectively.

The direction labels of the arcs on  $\mathcal{I}$  can be used to formally describe direction changes:

▶ **Definition 2** (Direction Change). Let p be a path in the infrastructure graph  $\mathcal{I}$ . We say that p contains a direction change if there is a consecutive pair of edges  $(u, v), (v, w) \in \mathcal{A}(p)$  where the in-label of (u, v) is equal to the out-label (v, w), i.e., if  $\phi(u, v)^{in} = \phi(v, w)^{out}$ .

Moreover, each infrastructure point  $v \in \mathcal{V}(\mathcal{I})$  belongs to a unique station in  $\mathcal{S}$ . Our planning will be based on a set of *planned trips*  $\mathcal{T}$ :

▶ Definition 3 (Planned Trips  $\mathcal{T}$ ). A planned trip  $\tau \in \mathcal{T}$  is a directed, possibly closed, path in S such that a train can travel along its station sequence without a change in direction. More precisely, there must exist a train path without direction change in  $\mathcal{I}$  such that its projection to S corresponds to  $\tau$ .

Let  $R(\tau) \subseteq \mathcal{A}(\mathcal{I})$  be the set of reachable track-links of planned trip  $\tau \in \mathcal{T}$ : An arc (v, w) is in  $R(\tau)$  if there is a path p on  $\mathcal{I}$  with  $(v, w) \in \mathcal{A}(p)$ , which does not change direction and whose projection onto S is  $\tau$ .

Intuitively, the planned trips encode the maximal station-sequence that can be covered by a vehicle. We will plan routings such that possibly only subsections of the planned trips are covered.

▶ **Example 4.** Consider the schematic infrastructure depicted in Figure 2, where the black rectangles show platforms, lines correspond to tracks and black triangles are switches. A possible + and - labeling of the track segments is displayed by the green markers. The corresponding station-link graph S arising from Figure 2 can be found in Figure 1. It also shows two planned trips  $\tau_0$  and  $\tau_1$ , marked in purple and pink, respectively. The infrastructure graph arising from Figure 2 with its track-links and corresponding direction labels can be found in Figure 3.

In practice, there might be restrictions on which planned trips are allowed to be linked with each other: E.g., some parts of the network might have to be operated by a certain train type. We therefore assume that there is some information about which planned trips

#### 5:4 Track Choice PESP with Integrated Line Planning

may be coupled – i.e. trips that can be operated in sequence by the same train unit – given as the set of allowed couplings between planned trips  $C \subseteq \mathcal{T} \times \mathcal{T}$ . If  $(\tau_0, \tau_1) \in C$  then a train is allowed to serve  $\tau_1$  after  $\tau_0$ .

Lastly, we define  $\overline{f} : \mathcal{A}(S) \to \mathbb{N}$  as the *intended arc frequency* –  $\overline{f}_{vw}$  indicates the frequency with which the station-link  $(v, w) \in \mathcal{A}(S)$  should preferably be served.

### 2.2 Extended Turn-Sensitive event-activity Network



**Figure 4** An excerpt of an extended turn-sensitive event-activity network of the two planned trips  $\tau_0$  and  $\tau_1$  for an allowed coupling  $(\tau_0, \tau_1)$ . Nodes are marked by their direction label, those with purple border are events from  $\tau_0$ , while those in pink correspond to  $\tau_1$ . Departure and arrival events are filled in white and gray, respectively. Arcs in black correspond to driving, blue to waiting and orange to turning activities.

An instance of the Periodic Event Scheduling Problem (PESP) is based on an *event-activity network*  $\mathcal{N}$ . Typically, events represent departures or arrivals of trips, and activities model relations between events, e.g., driving, waiting or turning of vehicles, or passenger activities such as transfers [9]. In our setting, we will consider the following network:

▶ Definition 5 (Extended Turn-Sensitive Event-Activity Network (adapted from [12])). Given a set of planned trips  $\mathcal{T}$  and a set of allowed couplings  $\mathcal{C}$ , we construct the extended turn-sensitive event-activity network  $\mathcal{N}$  as the digraph generated by the following arc set  $\mathcal{A}(\mathcal{N})$ :

- For each planned trip  $\tau \in \mathcal{T}$ , we add a
  - driving activity  $((\tau, v, dep, z_v), (\tau, w, arr, z_w))$  for each reachable track-link  $(v, w) \in R(\tau)$  where  $\phi(v, w) = (z_v, z_w)$ ,
  - waiting activity  $((\tau, v, arr, z), (\tau, v, dep, z'))$  if there are two reachable track-links  $(u, v), (v, w) \in R(\tau)$  such that  $z = \phi(u, v)^{in} \neq \phi(v, w)^{out} = z'$ ,
- For each allowed coupling  $(\tau, \tau') \in C$  we add an activity from  $(\tau, v, arr, \phi(u, v)^{in})$  to  $(\tau', v, dep, \phi(v, w)^{out})$  for all reachable track-links  $(u, v) \in R(\tau)$  and  $(v, w) \in R(\tau')$ . The activity is a turning activity if  $\phi(u, v)^{in} = \phi(v, w)^{out}$  and a waiting activity otherwise.

The set of arcs  $\mathcal{A}(\mathcal{N})$  consequently consists of activities which can be performed by trains. We call the nodes  $\mathcal{V}(\mathcal{N})$  of the thus constructed digraph events and distinguish between arrival and departure events, based on their label dep and arr, respectively.

**Example 6.** A section of an extended turn-sensitive event-activity network based on the infrastructure graph  $\mathcal{I}$  from Example 4 is depicted in Figure 4. It is obtained from the planned trips  $\tau_0$  and  $\tau_1$  and the allowed coupling  $(\tau_0, \tau_1)$ .

The presented event-activity network is the natural extension of the turn-sensitive eventactivity network introduced by Masing, Lindner and Liebchen [12]: Instead of allowing turning activities only at terminal stations and thus fixing the entire course of the line in advance, we add them at any intermediate station, so that we allow short-turning of lines in the sense of the previous paper. The main difference is in the setup of the event-activity network based on the planned trips and allowed couplings. They are responsible for the line planning aspect, as then (partial) trips can be flexibly linked together such that lines can be extended, shortened and rerouted. Moreover, we will see that this construction permits also multiple vehicle circulations along the same planned trips. As in [12], any simple path in  $\mathcal{N}$ corresponds to an activity sequence which can be performed by a train, meaning that our model covers aspects of vehicle scheduling as well:

▶ Definition 7 (Vehicle Circulation and Vehicle Schedule). A vehicle circulation is a simple directed cycle in the extended turn-sensitive event-activity network  $\mathcal{N}$ . A vehicle schedule Q is a collection of vehicle circulations that are pairwise vertex-disjoint.

We will use the notation  $\mathcal{V}(Q) := \bigcup_{q \in Q} \mathcal{V}(q)$  and  $\mathcal{A}(Q) := \bigcup_{q \in Q} \mathcal{A}(q)$ . Denote by  $\sigma(i) \in \mathcal{V}(S)$  the station that is associated to the event  $i \in \mathcal{V}(\mathcal{N})$ . The arc frequency of Q on  $(s,t) \in \mathcal{A}(S)$  is defined as  $f_{st}^Q := |\{(i,j) \in \mathcal{A}(Q) : \sigma(i) = s, \sigma(j) = t\}|$ , *i.e.*, the number of driving activities from station s to t in Q.

The intuition behind this definition is straight forward: Any vehicle circulation in a vehicle schedule corresponds to a sequence of activities a vehicle performs and thus induces its closed path through the infrastructure graph  $\mathcal{I}$ . Since we want to assign each event in  $\mathcal{V}(\mathcal{N})$  to at most one circulation, we require them to be pairwise vertex-disjoint.

In the model, which we are about to we present in Section 2.4, we will use  $\mathcal{N}$  as a basis and the goal will be to find the most compatible vehicle schedule Q as to cover as much of the intended arc frequencies  $f_a$  as possible while respecting certain operational requirements. Where the corresponding turns are performed, and thus, how many of the stations of the planned trips are covered, remains part of the optimization process.

#### 5:6 Track Choice PESP with Integrated Line Planning

### 2.3 Operational Duration Requirements

From an operational point of view, there are certain requirements for a timetable: First of all, there are minimum and maximum durations for activities. For instance, a turnaround should always take at least some minutes for the driver to comfortably move from one end of the train to the other; in a busy station, the dwell time should be at least a minute, in order for the expected passenger load to have enough time to board and alight, etc. Let  $\ell_a$  and  $u_a$  be the lower and upper bounds for each arc  $a \in \mathcal{A}(\mathcal{N})$  corresponding to such minimum and maximum duration requirements of the activity. We will assume that  $0 \leq \ell_a < T$  and  $0 \leq u_a - \ell_a < T$ .

▶ **Definition 8** (Periodic Timetable). Let  $T \in \mathbb{N}$  be the period time and  $\mathcal{N}$  a turn-sensitive event-activity network with associated activity bounds  $\ell, u$ . A periodic timetable  $\hat{\pi}$  of a vehicle schedule Q on  $\mathcal{N}$  is an assignment of timestamps  $\hat{\pi} : \mathcal{V}(Q) \to [0, T[$  such that

$$\forall (i,j) \in \mathcal{A}(Q) : \quad \ell_{ij} \leq \ell_{ij} + (\hat{\pi}_j - \hat{\pi}_i - \ell_{ij}) \mod T \leq u_{ij}. \tag{1}$$

Observe that if a vehicle schedule has already been fixed, then Definition 8 boils down the standard definition of a periodic timetable on an event-activity network in the context of the Periodic Event Scheduling Problem (PESP) [21].

Apart from duration requirements on the activities, there are certain security requirements which need to be fulfilled. Obviously, two trains may not be scheduled to be at the same track at the same time. Moreover, buffer times are needed for a safe operation, e.g., at least one minute must pass between the departure of a train the and arrival of a subsequent train. Let  $h, \varepsilon \ge 0$  be such security times, where h denotes the minimum time needed between two arrivals of different trains at the same infrastructure point, while  $\varepsilon$  describes the minimum time needed between the departure of a train and the arrival of the next.

In the context of PESP, such security requirements are usually modelled by adding arcs, called *headway* activities with corresponding lower and upper bounds (see, e.g., [9, 10]). For our purposes, it will be helpful to consider headway arcs separately from the event-activity network  $\mathcal{N}$ :

▶ **Definition 9** (Headway Network  $\mathcal{H}$ ). Let  $\mathcal{A}_v^{stat} \subseteq \mathcal{A}(\mathcal{N})$  be the set of waiting and turning activities at infrastructure point  $v \in \mathcal{V}(\mathcal{I})$  and let

$$\mathcal{P} = \bigcup_{v \in \mathcal{V}(\mathcal{I})} \{ ((i_1, j_1), (i_2, j_2)) \in \mathcal{A}_v^{stat} \times \mathcal{A}_v^{stat} \mid (i_1, j_1) \neq (i_2, j_2) \}.$$

We define the headway network  $\mathcal{H}$  as the graph induced by the arc set

$$\mathcal{A}(\mathcal{H}) := \{ (i_1, i_2) \mid ((i_1, j_1), (i_2, j_2)) \in \mathcal{P} \} \cup \{ (j_1, i_2) \mid ((i_1, j_1), (i_2, j_2)) \in \mathcal{P} \}.$$

A visualization of the headway network  $\mathcal{H}$  is given in Figure 5.

We consider a periodic timetable to be  $(\varepsilon, h)$ -conflict-free if two vehicles using the same infrastructure point for waiting or turning do not occupy it at the same time and fulfill the security requirements with respect to  $\varepsilon$  and h. We refer to [12] for a more precise definition, as well as a in-depth discussion on modeling possibilities.

Our aim is to answer the question of how much of the intended arc frequency can be achieved by an operable vehicle schedule. Thus, we chose a fairly simple objective, focusing on the line-planning aspect, where we minimize the aggregated frequency gap:

▶ **Problem Formulation 1.** For a set of planned trips  $\mathcal{T}$  and allowed couplings C, let  $\mathcal{N}$  be its derived extended turn-sensitive event-activity network by Definition 5. Suppose that activity bounds  $\ell, u : \mathcal{A}(\mathcal{N}) \to \mathbb{N}$ , a period time  $T \in \mathbb{N}$ , as well as security and buffer times

 $h, \varepsilon \ge 0$  be given. Let further  $\overline{f}$  be the intended arc frequency. The goal of the Integrated Line Planning and Turn-Sensitive Periodic Timetabling Problem with Track Choice is to find a vehicle schedule Q and an  $(\varepsilon, h)$ -conflict-free periodic timetable  $\hat{\pi}$  for Q such that the aggregated frequency gap

$$\sum_{a \in \mathcal{A}(\mathcal{S})} \max(0, \overline{f}_a - f_a^Q)$$

is minimized.

Observe that the term  $\max(0, \overline{f}_a - f_a^Q)$  measures the difference of an undersupply of frequency from a vehicle schedule along a station-link  $a \in \mathcal{A}(\mathcal{S})$ , but does neither punish nor favor an oversupply of service.

#### 2.4 Integrated Line Planning with Timetabling Model (LPTT)

We have set the stage to introduce our mixed-integer linear optimization model LPTT for the Integrated Line Planning and Turn-Sensitive Periodic Timetabling Problem with Track Choice as defined in Problem Formulation 1. One can regard it as the natural extension of the model introduced in [12] tweaked to include line-planning decisions: As in [12], the key idea behind it is to introduce binary variables  $h_{ij}$  as decision variables, indicating whether an arc  $(i, j) \in \mathcal{A}(\mathcal{N})$  is chosen, and to apply modified PESP constraints on the entire network. The bounds on arcs, which are not part of a chosen train routing, are then relaxed via the big-M method. The novel aspects of the model now are, firstly, that we use an extended version of the turn-sensitive event-activity network, which encodes turnarounds at any station and thus allows for more flexibility. Secondly, we introduce *frequency gap variables*  $c_a, a \in \mathcal{A}(\mathcal{S})$ , which capture the difference between the service provided by the chosen routing and the intended arc frequency  $\overline{f}_a$ . The frequency gap variables will be responsible for the line planning aspect of the model.

• Model 1 (LPTT
$$_{\mathcal{N},\overline{f}}$$
).

s.t.

$$min \qquad \lambda_{lp} \left( \sum_{a \in \mathcal{A}(\mathcal{S})} c_a \right) + \lambda_{turn} \left( \sum_{ij \in \mathcal{A}_{turn}(\mathcal{N})} h_{ij} \right) \tag{2}$$

$$y_{ij} + \ell_{ij}h_{ij} = \pi_j - \pi_i + Tp_{ij} \qquad (i,j) \in \mathcal{A}(\mathcal{N})$$
$$y_{ij} \leq u_{ij} - \ell_{ij} + (T - 1 - u_{ij} + \ell_{ij})(1 - h_{ij}) \qquad (i,j) \in \mathcal{A}(\mathcal{N})$$

$$y_{ij} \leqslant u_{ij} - \ell_{ij} + (T - 1 - u_{ij} + \ell_{ij})(1 - h_{ij}) \qquad (i, j) \in \mathcal{A}(\mathcal{N})$$

$$\sum_{j \in \delta^+(i)} h_{ij} = \sum_{j \in \delta^-(i)} h_{ji} \qquad i \in \mathcal{V}(\mathcal{N})$$
(5)

$$\sum_{j \in \delta^+(i)} h_{ij} \leqslant 1 \qquad \qquad i \in \mathcal{V}(\mathcal{N}) \tag{6}$$

$$c_{a} + \sum_{\substack{(i,j)\in\mathcal{A}(\mathcal{N}):\\(\sigma(i),\sigma(j))=a}} h_{ij} \ge \overline{f}_{a} \qquad \qquad a \in \mathcal{A}(\mathcal{S})$$
(7)

| (0(1),0(1)) - u   |  |      |
|---|--|------|
| $\pi_{i_2} - \pi_{i_1} + Tp_{i_1i_2} \leq (T - h)(3 - h_{i_1j_1} - h_{i_2j_2})$           | $((i_1,j_1),(i_2,j_2))\in\mathcal{P}$      | (8)  |
| $\pi_{i_2} - \pi_{i_1} + Tp_{i_1i_2} \ge h(h_{i_1j_1} + h_{i_2j_2} - 1)$                  | $((i_1, j_1), (i_2, j_2)) \in \mathcal{P}$ | (9)  |
| $\pi_{j_2} - \pi_{i_1} + Tp_{i_1j_2} \leq (T - \varepsilon)(3 - h_{i_1j_1} - h_{i_2j_2})$ | $((i_1,j_1),(i_2,j_2)) \in \mathcal{P}$    | (10) |
| $\pi_{j_2} - \pi_{i_1} + Tp_{i_1j_2} \ge \varepsilon(h_{i_1j_1} + h_{i_2j_2} - 1)$        | $((i_1, j_1), (i_2, j_2)) \in \mathcal{P}$ | (11) |
| $p_{i_1j_1} + p_{j_1i_2} - p_{i_1i_2} \leq 2(2 - h_{i_1j_1} - h_{i_2j_2})$                | $((i_1, j_1), (i_2, j_2)) \in \mathcal{P}$ | (12) |
| $p_{i_1j_1} + p_{j_1i_2} - p_{i_1i_2} \ge -(2 - h_{i_1j_1} - h_{i_2j_2})$                 |  | (13) |

(3)

| $y_{ij} \geqslant 0$                | $(i,j)\in\mathcal{A}(\mathcal{N})$   | (14) |
|-------------------------------------|--------------------------------------|------|
| $c_a \ge 0$                         | $a \in \mathcal{A}(\mathcal{S})$     | (15) |
| $p_{ij} \in \{0, 1, 2\}$            | $(i,j)\in\mathcal{A}(\mathcal{N})$   | (16) |
| $p_{ij} \in \{0,1\}$                | $(i,j) \in \mathcal{A}(\mathcal{H})$ | (17) |
| $h_{ij} \in \{0,1\}$                | $(i,j)\in\mathcal{A}(\mathcal{N})$   | (18) |
| $0 \leqslant \pi_i \leqslant T - 1$ | $i \in \mathcal{V}(\mathcal{N})$     | (19) |
|                                     |                                      |      |

Much as in the classical PESP model [21, 8], we introduce  $y_{ij} \ge 0$  as the periodic slack and  $p_{ij} \in \{0, 1, 2\}$  as the periodic offset on each arc  $(i, j) \in \mathcal{A}(\mathcal{N})$ . Note that we can restrict  $p_{ij}$  to be in  $\{0, 1\}$  for all arcs (i, j) whose upper bound is at most T, this includes all arcs  $(i, j) \in \mathcal{A}(\mathcal{H})$  (cf. Figure 5). We assign timestamps to each event and describe them by  $\pi_i$  for  $i \in \mathcal{V}(\mathcal{N})$ , such that (3) models the periodicity constraints with an added bound activation for each activity  $(i, j) \in \mathcal{A}(\mathcal{N})$ :

In (4) the periodic slack is bounded by  $u_{ij} - \ell_{ij}$  if the arc (i, j) is part of a chosen vehicle circulation – i.e., if  $h_{ij} = 1$ . In this case, (4) in combination with (3) describe the periodicity requirements of periodic timetables (1). If  $h_{ij} = 0$ , the bound is relaxed by big-M constraints to T-1. This, together with the bound activation term  $\ell_{ij}h_{ij}$  in (3) ensures that there exists a valid  $y_{ij}$  for any choice of  $\pi_i, \pi_j \in [0, T-1]$  if (i, j) is not part of a chosen circulation.

As opposed to the path-based approach in [12], we model our routing with flow conservation constraints (5) and ensure that each event is part of at most one vehicle circulation (6). The line-planning aspect is covered by (7), where  $c_a \ge 0$  measures the gap between how often the vehicle schedules covers an arc  $a \in \mathcal{A}(S)$  in comparison to the intended arc frequency  $\overline{f}_a$ . The constraints (8)-(13) ensure a  $(\varepsilon, h)$ -conflict-free timetable, meaning that for each pair of activities sharing the same infrastructure  $((i_1, j_1), (i_2, j_2)) \in \mathcal{P}$  their periodic intervals (including security and headway times) are disjoint – again, for details we refer to [12].

The objective function deserves a little discussion. Technically, to address Problem Formulation 1, the first term in the objective would be sufficient, as it describes exactly the aggregated frequency gap scaled by  $\lambda_{lp} > 0$ . For practical applications however, other additional terms describing circulation, travel or transfer times, or taking into consideration regularity or robustness, could – and should – be added. As a minimal extension, we propose to consider the set of turning activities  $\mathcal{A}_{turn}(\mathcal{N})$  and to add a term that penalizes the number of turning activities in the chosen vehicle schedule, scaled by the parameter  $\lambda_{turn} > 0$ . For our purposes,  $\lambda_{turn}$  should be significantly smaller than  $\lambda_{lp}$ , as then the focus is on the line planning aspect, but the second term then serves as a tie-breaker and ensures that long lines are favored over multiple short ones.

Note that we allow an oversupply of service on an arc  $a \in \mathcal{A}(\mathcal{S})$ : While there is no direct benefit of such with respect to the objective value, since  $c_a \ge 0$ , an oversupply might lead to a higher coverage and thus lower frequency gap on a different arc.

A vehicle schedule Q can be derived from the decision variables, such that  $\mathcal{A}(Q) := \{(i,j) \in \mathcal{A}(\mathcal{N}) : h_{ij} = 1\}$ . We can then obtain the periodic timetable  $\hat{\pi} : \mathcal{V}(Q) \to [0, T[$  of said schedule by setting  $\hat{\pi}_i = \pi_i$  for  $i \in \mathcal{V}(Q)$ .

A feature of the presented model is that feasibility is no issue: The trivial solution with  $c_a = f_a$  for all  $a \in \mathcal{A}(S)$  with all other variables set to zero – corresponding to not providing any train service – is always feasible. The trivial solution is thus the one with the maximal aggregated frequency gap. While this obviously is not the intended outcome, one could use the model in running-time-sensitive situations: A solver could be disrupted at any point, and would provide a conflict-free timetable, which – maybe not at full capacity – could be put into operation.

#### 2.5 Application to Construction Sites

Observe that the flexibility of the model LPTT, and thus the impact of the line-planning aspect, is highly dependent on the event-activity network  $\mathcal N$  and thus on our choice of planned trips  $\mathcal{T}$  as well as the allowed couplings  $\mathcal{C}$ . Clearly, the more planned trips and allowed couplings we base our model on, the more choices we get for meaningful line planning - however at the cost of the network size: When using the approach for line planning on a large scale, e.g., to plan the transportation network of a whole city, the corresponding event-activity network is likely to explode in size. It is also not well suited for this purpose, as it considers only the minimal operational requirements, but does not take into account relevant aspects in the context of long-term planning, such as robustness, regularity or passenger comfort, etc. For construction sites it is, however, well suited: Construction sites lead to some part of the infrastructure becoming unavailable. This has an impact not only on the construction site itself, but also on the surrounding area: Trains need to be rerouted, neighboring stations need to be used to make additional turnarounds, which can lead to capacity problems, such that some trains might need to be cancelled. If key elements of the infrastructure are under construction, large portions of the entire network may be affected by it. In any case, a planner has to adjust the timetable, but also make line planning decisions. As construction sites are (usually) only for short periods of time, the mentioned goals of long-term line planning become only secondary, while providing as much service as possible in the affected area becomes the priority.

Moreover, an important planning goal is to adhere to the regular timetable as much as possible, and regions far from the problematic area should remain unaffected. As such, we adjust the basic model LPTT for the purposes of construction sites:

Let  $\mathcal{T}^{\mathcal{R}}$  and  $\mathcal{C}^{\mathcal{R}}$  be the smallest set of planned trips and allowed couplings, respectively, such that the *regular vehicle schedule*  $\mathcal{R}$  is a vehicle schedule with a corresponding periodic timetable  $\overline{\pi} : \mathcal{V}(\mathcal{R}) \to [0, T[$  encoding the long-term regular service provided on a fully operational infrastructure network  $\mathcal{I}$ . Furthermore, let  $\mathcal{T}$  be a choice set of planned trips and  $\mathcal{C}$  a set of allowed couplings  $\mathcal{C}$  with  $\mathcal{T}^{\mathcal{R}} \subseteq \mathcal{T}$  and  $\mathcal{C}^{\mathcal{R}} \subseteq \mathcal{C}$ . Then  $\mathcal{R}$  is a vehicle schedule and  $\overline{\pi}$  is a periodic timetable with respect to the turn-sensitive event-activity network  $\mathcal{N}$ induced by  $\mathcal{T}$  and  $\mathcal{C}$ .

Further, we define four subgraphs of  $\mathcal{N}$ :

- the *blocked network*  $\mathcal{N}^X$  contains all activities, which cannot be performed due to the construction work,
- the *planning network*  $\mathcal{N}^P$  contains all potential activities, which can be operated and where re-scheduling from the regular timetable is allowed,
- the fixed network  $\mathcal{N}^F$  contains a selection of activities which are also part of the regular vehicle schedule.
- the construction network  $\mathcal{N}^C$  as the graph induced by the arc set  $\mathcal{A}(\mathcal{N}^F) \cup \mathcal{A}(\mathcal{N}^P)$ .

We assume that  $\mathcal{A}(\mathcal{N}^X), \mathcal{A}(\mathcal{N}^P)$  and  $\mathcal{A}(\mathcal{N}^F)$  are pairwise disjoint.

We now can formally formulate the construction-site rescheduling problem:

▶ Problem Formulation 2. Consider an instance of the Integrated Line Planning and Turn-Sensitive Periodic Timetabling Problem with Track Choice on an extended turn-sensitive event-activity network  $\mathcal{N}$ . Let further  $\mathcal{R}$  be the regular vehicle schedule on the fully operational infrastructure network  $\mathcal{I}$  with the regular periodic timetable  $\overline{\pi}$ . Moreover, let  $\mathcal{N}^P$  be a planning,  $\mathcal{N}^F$  be a fixed, and  $\mathcal{N}^C$  their corresponding construction network. The goal is to find a vehicle schedule Q and a ( $\varepsilon$ , h)-conflict-free periodic timetable  $\hat{\pi}$  for Q on  $\mathcal{N}^C$  such that the

**ATMOS 2023** 

#### 5:10 Track Choice PESP with Integrated Line Planning

aggregated frequency gap

$$\sum_{a \in \mathcal{A}(\mathcal{S})} \max(0, f_a^{\mathcal{R}} - f_a^Q)$$

is minimized.

To address this construction-site rescheduling problem, we propose to simply use LPTT restricted to the construction network  $\mathcal{N}^C$  and impose the regular timetable on the fixed graph  $\mathcal{N}^F$ . The intended arc frequency can then be set to the arc frequency of the regular vehicle schedule  $f^{\mathcal{R}}$ :

▶ Model 2 (LPTT<sup>C</sup>).

LPTT<sub>$$\mathcal{N}^C, f^{\mathcal{R}}$$</sub>  
subject to the additional constraint  $\pi_i = \overline{\pi}_i \quad \forall i \in \mathcal{V}(\mathcal{N}^F)$  (20)

A solution to  $LPTT^{C}$  will then induce an operable vehicle schedule with periodic timetable  $\hat{\pi}$  via the decision variables  $h_{ij}$  as discussed in Section 2.4. Since we restrict LPTT to the construction network  $\mathcal{N}^C$ , the vehicle schedule does not use any activities affected by the construction site. The constraints (20) ensure that we adhere to the regular timetable. Observe however, that we do not enforce that activities in the fixed graph  $\mathcal{N}^F$  have to be used. This can lead to a vehicle schedule which does not use activities in the fixed graph, which translates to a cancellation of a train. While this might seem like an oversight at first glance, we have made this decision for two reasons: Most importantly,  $LPTT^{C}$  remains always feasible, such that any sub-optimal solution can still be put into operation. Lines completely unaffected by the construction site could be scheduled immediately. In contrast, if we were to enforce service on the fixed graph, feasibility can become an issue – one might not be able to find any vehicle schedule at all and would have to include more in the planning area for another attempt. How much and which parts of  $\mathcal{N}$  should be included in  $\mathcal{N}^P$  however, would not be clear. This leads us to the second reason: A resulting vehicle schedule omitting some of the fixed activities implies that this part of the network is particularly hard to link to or at too high costs for the planning area. In any case, such a result could then give an indication of how to adjust the planning network  $\mathcal{N}^P$  for better results.

# **3** Computational Experiments

While LPTT<sup>C</sup> can in theory solve the construction-site rescheduling problem, there are multiple issues that come into play when solving the model: Both line planning and periodic timetabling are computationally hard. Moreover, the event-activity network can become very large, and we have multiple integer values associated to every arc. To demonstrate that LPTT<sup>C</sup> can be used in practice nevertheless, we implemented the model and tested it on 8 real construction sites on the S-Bahn network in Berlin, based on infrastructure data and timetabling parameters provided by DB Netz AG.

### 3.1 Construction Site Instances

We selected 8 construction sites of the years 2021-2023, where train service was disrupted. The Berlin network is operated periodically in 20 minute intervals, but planned with a resolution of 0.1 min. We consequently chose as period time T = 200.

We based our planned trips  $\mathcal{T}$  and allowed couplings  $\mathcal{C}$  on both the regular annual timetable and on the original construction schedule  $\mathcal{O}$  as was put into practice during the construction period: Let  $\mathcal{T}^{\mathcal{O}}$  and  $\mathcal{C}^{\mathcal{O}}$  be the smallest set of planned trips and allowed couplings such that  $\mathcal{O}$  is a vehicle schedule. Then  $\mathcal{T}$  contains all trips in  $\mathcal{T}^{\mathcal{O}}$  in addition to all paths of  $\mathcal{T}^{\mathcal{R}}$  which induce (at least some) activities in the planning network. The allowed couplings are then selected as  $\mathcal{C} := \{(\tau, \tau') \in \mathcal{T} \times \mathcal{T} \mid (\tau, \tau') \in \mathcal{C}^{\mathcal{O}} \text{ or } (\tau, \tau') \in \mathcal{C}^{\mathcal{R}}\}$ . A schematic overview over the areas in the station-link graph affected by the planning and blocked networks can be found in Figure 6 in Appendix A . Some key properties, which give an indication of the problem size can be found in Table 1.

| Scenario      | $ \mathcal{V}(\mathcal{N}^C) $ | $ \mathcal{A}(\mathcal{N}^{C}) $ | $ \mathcal{T} $ | $ \mathcal{C} $ | $ \mathcal{A}(\mathcal{H}) $ |
|---------------|--------------------------------|----------------------------------|-----------------|-----------------|------------------------------|
| BBER-BBU      | 192                            | 196                              | 6               | 6               | 36                           |
| BGAS-BKW      | 523                            | 602                              | 12              | 14              | 556                          |
| BBUP          | 766                            | 1014                             | 12              | 12              | 2967                         |
| BBOS-BWIN-BTG | 1317                           | 1602                             | 28              | 50              | 1798                         |
| BBKS-BWT      | 1323                           | 2538                             | 18              | 66              | 9396                         |
| BOSB          | 1518                           | 2580                             | 22              | 38              | 14472                        |
| BSW           | 1896                           | 3130                             | 32              | 64              | 12369                        |
| BGB-BWES      | 2539                           | 4631                             | 36              | 76              | 13794                        |

**Table 1** Size metrics of the 8 construction site scenarios.

Note that we use the annual timetable and the original construction schedule  $\mathcal{O}$  just as a source for the creation of  $\mathcal{T}$  and  $\mathcal{C}$ . For the model itself however, it is not necessary to have an initial feasible solution at hand.

For each scenario we ran two tests, namely once without and once with an initial solution. We will refer to them by *cold start* and *warm start*. The initial solution was obtained from the original construction timetable.

As scalarization parameters for LPTT, we chose  $\lambda_{lp} = 100$  and  $\lambda_{turn} = 1$ , thus ensuring that no line gets shortened in favor of reducing a turn. Operating times are set as provided by DB Netz AG. On a technical note, we assume that driving times are fixed, i.e.,  $\ell_a = u_a$ if  $a \in \mathcal{A}(\mathcal{N}^C)$ , such that the adjustment of the timetable is shifted solely to turning and waiting activities. This has the consequence that safety constraints on most driving activities can be omitted, as they are implied by the stationary headway constraints given by the set  $\mathcal{P}$ . A notable exception are driving activities on single-track section, where conflicts can be resolved by standard headway activities.

We implemented the model and ran each instance with a wall time limit of one hour each, on an Intel i7-9700K CPU with the Gurobi Optimizer version 10.0.2 [7].

### 3.2 Results

An overview over our test results can be found in Table 2 in Appendix A, where we show the final objective value and the duality gaps for each of the instances. We also indicate how long it took to find the optimal solution – if at all. For a better comparison, we include the objective of the initial solution corresponding to the original construction schedule ( $\mathcal{O}$ ), the value of the natural LP relaxation (LP) as well as the objective of the trivial solution. The latter captures the cost of not providing any service, i.e., the value of the maximal frequency gap. We make the following observations:

First of all, the model is of use for realistic scenarios: Let us first focus on the cold started instances. After the run of one hour, each objective value is far from the maximal aggregated frequency gap as is provided by the trivial solution: The worst instance,

### 5:12 Track Choice PESP with Integrated Line Planning

namely BOSB, has a cost of only approximately 23% of the trivial solution. On average the objectives reach approximately 10% of the maximal aggregated frequency gap. We conclude that our model can, in fact, provide operable solutions within reasonable time.

- Secondly and unsurprisingly, finding qualitative solutions is difficult: While we were able to solve five of the scenarios to optimality with the initial solution provided, this was the case only for three of the cold started instances. The difficulty of finding good solutions is particularly obvious in the larger instances, e.g., BOSB and BGB-BWES: The cold started versions provided solutions not only significantly worse than the warm started ones, but also in comparison to the original timetable.
- However, when provided with a good starting solution, the model becomes fairly effective: We were able to find an improvement to the original construction timetable for all instances. The only exception was BBER-BBU, where the initial solution was already optimal. This suggests that the solver greatly benefits from a good input solution. An investigation of possible heuristic approaches seems promising for the future.
- A fourth observation is that while the size of the network gives an indication of the difficulty of the problem, it is not solely responsible: E.g., the instance BBUP is one of our smallest instances, but the optimality gap is close to 100%. In this instance, one platform of a highly frequented station is blocked with little turnaround possibilities, such that all trains passing through that station must use a single platform. This means that the station can still be served, as well as all neighboring stations, but at a lower frequency coverage. This leads us to the next observation:
- The LP-relaxation is of little use for dual bounds: Relaxing all integer variables to continuous variables essentially disables any duration and security requirements, resulting in fractional flows instead of vehicle circulations, such that e.g., in the BBUP scenario every frequency gap variable can be set to zero.
- Lastly, proving optimality is an issue: Even though we were able to find a certificate of optimality for five of the instances, this was only the case when we found a solution with the same objective as the LP-relaxation. For the non-optimal instances, the gap remains very large. In fact, for all instances the dual bounds remained at the value of the LP-relaxation.

### 3.3 Conclusions

We conclude that our model can be used to approach the construction-site rescheduling problem for practical purposes. For all real-world instances, we could provide a non-trivial operable schedule and timetable. Moreover, we were able to improve upon all of the original construction-site timetables in the sense that we were able to provide more service – with the exception of one, which was optimal in the first place. Our experiments reveal a few issues of the model: The most glaring one is the quality of the dual bounds in order to prove the optimality of a timetable. For the future, further investigation is required on how to obtain better quality bounds.

Maybe more promising is the search for heuristics in this context: Clearly, the goal of the model is to provide transportation planners with operable vehicle schedules and timetables. It somewhat defeats the purpose if the planner has to provide a qualitative starting solution to obtain a better one. However, our results from the warm started instances suggest that performance could be improved by giving the solver more guidance by heuristic approaches.

#### – References -

Ralf Borndörfer, Niels Lindner, and Sarah Roth. A concurrent approach to the periodic event scheduling problem. Journal of Rail Transport Planning & Management, 15:100175, 2020. Best Papers of RailNorrköping 2019. doi:10.1016/j.jrtpm.2019.100175.

- 2 Enrico Bortoletto, Niels Lindner, and Berenike Masing. Tropical Neighbourhood Search: A New Heuristic for Periodic Timetabling. In DROPS-IDN/17107. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2022. doi:10.4230/OASIcs.ATMOS.2022.3.
- 3 Gabrio Caimi, Martin Fuchsberger, Marco Laumanns, and Kaspar Schüpbach. Periodic railway timetabling with event flexibility. *Networks*, 57(1):3–18, 2011. doi:10.1002/net.20379.
- 4 Florian Fuchs, Alessio Trivella, and Francesco Corman. Enhancing the interaction of railway timetabling and line planning with infrastructure awareness. *Transportation Research Part C: Emerging Technologies*, 142:103805, September 2022. doi:10.1016/j.trc.2022.103805.
- 5 Marc Goerigk. Exact and heuristic approaches to the robust periodic event scheduling problem. *Public Transport*, 7(1):101–119, March 2015. doi:10.1007/s12469-014-0100-5.
- 6 Marc Goerigk and Christian Liebchen. An Improved Algorithm for the Periodic Timetabling Problem. In Gianlorenzo D'Angelo and Twan Dollevoet, editors, 17th Workshop on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems (ATMOS 2017), volume 59 of OpenAccess Series in Informatics (OASIcs), pages 12:1-12:14, Dagstuhl, Germany, 2017. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik. ISSN: 2190-6807. doi:10.4230/OASIcs.ATMOS.2017.12.
- 7 Gurobi Optimization, LLC. Gurobi Optimizer Reference Manual, 2022. URL: https://www.gurobi.com.
- 8 Christian Liebchen. *Periodic timetable optimization in public transport*. PhD thesis, Dissertation.de, Berlin, 2006.
- 9 Christian Liebchen and Rolf H. Möhring. The modeling power of the periodic event scheduling problem: Railway timetables and beyond. In Frank Geraets, Leo Kroon, Anita Schoebel, Dorothea Wagner, and Christos D. Zaroliagis, editors, *Algorithmic Methods for Railway Optimization*, pages 3–40, Berlin, Heidelberg, 2007. Springer Berlin Heidelberg.
- 10 Thomas Lindner. Train Scheduling in Public Rail Transport. PhD thesis, Technische Universität Braunschweig, June 2000. URL: https://publikationsserver.tu-braunschweig. de/receive/dbbs\_mods\_00001135.
- 11 Berenike Masing, Niels Lindner, and Patricia Ebert. Forward and Line-Based Cycle Bases for Periodic Timetabling. Operations Research Forum, 4(3):53, June 2023. doi:10.1007/ s43069-023-00229-0.
- 12 Berenike Masing, Niels Lindner, and Christian Liebchen. Periodic Timetabling with Integrated Track Choice for Railway Construction Sites, 2022. URL: https://opus4.kobv.de/opus4-zib/ frontdoor/index/index/docId/8862.
- 13 Gonçalo P. Matos, Luís M. Albino, Ricardo L. Saldanha, and Ernesto M. Morgado. Solving periodic timetabling problems with SAT and machine learning. *Public Transport*, 13(3):625–648, October 2021. doi:10.1007/s12469-020-00244-y.
- 14 Karl Nachtigall. Periodic Network Optimization and Fixed Interval Timetables. Habilitation thesis, Universität Hildesheim, 1998.
- 15 Leon Peeters. *Cyclic Railway Timetable Optimization*. PhD thesis, Erasmus Universiteit Rotterdam, January 2003.
- 16 Julius Pätzold, Alexander Schiewe, Philine Schiewe, and Anita Schöbel. Look-Ahead Approaches for Integrated Planning in Public Transportation. In Gianlorenzo D'Angelo and Twan Dollevoet, editors, 17th Workshop on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems (ATMOS 2017), volume 59 of OpenAccess Series in Informatics (OASIcs), pages 17:1–17:16, Dagstuhl, Germany, 2017. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik. ISSN: 2190-6807. doi:10.4230/DASIcs.ATMOS.2017.17.
- Philine Schiewe. Integrated Optimization in Public Transport Planning, volume 160 of Springer Optimization and Its Applications. Springer International Publishing, Cham, 2020. doi: 10.1007/978-3-030-46270-3.
- Anita Schöbel. Line planning in public transportation: models and methods. OR Spectrum, 34(3):491–510, July 2012. doi:10.1007/s00291-011-0251-6.

#### 5:14 Track Choice PESP with Integrated Line Planning

- 19 Anita Schöbel. An eigenmodel for iterative line planning, timetabling and vehicle scheduling in public transportation. *Transportation Research Part C: Emerging Technologies*, 74:348–365, January 2017. doi:10.1016/j.trc.2016.11.018.
- 20 Anita Schöbel and Susanne Scholl. Line Planning with Minimal Traveling Time. In Leo G. Kroon and Rolf H. Möhring, editors, 5th Workshop on Algorithmic Methods and Models for Optimization of Railways (ATMOS'05), volume 2 of OpenAccess Series in Informatics (OASIcs), Dagstuhl, Germany, 2006. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik. ISSN: 2190-6807. doi:10.4230/OASIcs.ATMOS.2005.660.
- 21 Paolo Serafini and Walter Ukovich. A Mathematical Model for Periodic Scheduling Problems. SIAM Journal on Discrete Mathematics, 2(4):550–581, November 1989. doi:10.1137/0402049.
- 22 Sander Van Aken, Nikola Bešinović, and Rob M. P. Goverde. Designing alternative railway timetables under infrastructure maintenance possessions. *Transportation Research Part B: Methodological*, 98:224–238, April 2017. doi:10.1016/j.trb.2016.12.019.
- 23 Raimond Wüst, Stephan Bütikofer, Severin Ess, Claudio Gomez, Albert Steiner, Marco Laumanns, and Jacint Szabo. Improvement of maintenance timetable stability based on iteratively assigning event flexibility in FPESP. In Anders Peterson, Martin Joborn, and Markus Bohlin, editors, *RailNorrköping 2019*, Linköping Electronic Conference Proceedings; 69, pages 1160–1177, Linköping, September 2019. Linköping University Electronic Press. doi:10.21256/zhaw-18282.

### A Appendix



**Figure 5** Excerpt of the headway network  $\mathcal{H}$  with lower and upper bounds (orange and red arcs) induced by the activity-pair sharing the same infrastructure point  $((i_1, j_1), (i_2, j_2)) \in \mathcal{P}$  (in gray).

**Table 2** Overview over the solutions: obj corresponds to the objective value, gap to the optimality gap in percent, and time denotes the time to the optimal solution in seconds if found. For reference, we include the objective value of the initial solution ( $\mathcal{O}$ ), the natural LP-relaxation (LP), as well as the trivial solution (trivial).

|               | cold start |                   | warm start            |      |                   | 0      | LP   | $\operatorname{trivial}$ |       |
|---------------|------------|-------------------|-----------------------|------|-------------------|--------|------|--------------------------|-------|
| scenario      | obj        | $_{\mathrm{gap}}$ | $\operatorname{time}$ | obj  | $_{\mathrm{gap}}$ | time   | obj  | obj                      | obj   |
| BBER-BBU      | 806        | 0.00              | 0.0                   | 806  | 0.00              | 0.0    | 806  | 806                      | 9800  |
| BGAS-BKW      | 1210       | 0.00              | 0.2                   | 1210 | 0.00              | 0.1    | 1410 | 1210                     | 22000 |
| BBUP          | 3514       | 99.89             | x                     | 3010 | 99.87             | x      | 4808 | 4                        | 29000 |
| BBOS-BWIN-BTG | 3224       | 0.00              | 22.2                  | 3224 | 0.00              | 6.7    | 4034 | 3224                     | 58000 |
| BBKS-BWT      | 5426       | 70.11             | x                     | 1622 | 0.00              | 2349.3 | 2220 | 1622                     | 40000 |
| BOSB          | 10424      | 99.79             | x                     | 22   | 0.00              | 3100.9 | 822  | 22                       | 43800 |
| BSW           | 1660       | 12.29             | x                     | 1660 | 12.29             | x      | 5448 | 1456                     | 61000 |
| BGB-BWES      | 12640      | 68.06             | x                     | 6442 | 37.34             | x      | 8846 | 4036                     | 77000 |



**Figure 6** Overview over the 8 construction scenarios: Red corresponds to blocked areas (orange if partially blocked), and blue corresponds to the planning area.