Test Selection for Diagnosing Multimode Systems

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Abstract

This work considers the problem of selecting residuals for consistency-based diagnosis of multimode systems. The system operation mode is assumed to be given by a set of known discrete variables. The number of operation modes grows exponentially with the number of binary variables, thus methods enumerating the modes are not feasible. Here a method is proposed to select a small subset of residuals for diagnosing multimode systems. The selection is based on the fault signature of the residuals for the different modes of operation. To avoid the exponential growth of the number of modes, the multimode fault signature matrix is used to compute the diagnosability of the residuals. The approach is inspired and exemplified by a dynamically configurable battery pack. The result is a small set of residuals with the maximum diagnosability in all operation modes.

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Software: `https://github.com/MattiasKrysander/Multimode-Test-Selection`

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1 Introduction

Residual selection is crucial in diagnosis system design using pre-compiled tests for detecting and isolating faults. The selection of residuals is a combinatorial problem, and the number of possible combinations grows exponentially with the number of residuals. Furthermore, the number of residuals grows exponentially in the redundancy of the model. The problem is even more challenging in multimode systems where the system can operate in different operational modes. Thus, there is a need to develop computationally efficient algorithms to select a small set of residuals that can detect and isolate faults in all operation modes. Set minimal and minimum cardinality solutions could be desirable but due to the highly combinatorial nature of the problem, it is not always possible to find optimal solutions. Here we propose a greedy algorithm to select a small set of residuals for diagnosing multimode systems with maximum diagnosability in all operation modes. The selection is based on a fault signature matrix that describes the fault influence on the residuals for the different operation modes.

Structured residuals and fault signature matrices are discussed in [8], where two isolation patterns are introduced: weakly and strongly isolating structures. Weakly isolating structures can isolate faults when the column-matching approach is applicable, whereas strongly isolating structures can isolate faults using the consistency-based approach. In the presence of non-ideal residuals, such as those caused by model errors and uncertainties, only strongly isolating structures are applicable, which is the approach adopted in this work. Furthermore,[17] establishes a connection between the fault signature matrix used in the Fault Detection and Isolation (FDI) community and consistency-based diagnosis performed in the DX community. The foundation of the present work aligns with these principles. A recent comprehensive review of control system structural analysis diagnosability is provided in [9].

The problem of test selection for diagnosing single-mode systems has been studied in the literature. For instance, [3] formulates the test selection problem as an integer programming problem. Additionally, research has explored the combination of sensor placement and residual selection, as discussed in [16]. This problem has been formulated and solved as a mixed-integer optimization problem in [14]. Residual selection has also been studied in the context of distributed systems. For example, [13] addresses this problem using binary integer linear programming. Additionally, multimode systems have been considered in the literature [1, 12]. In [1], a solution to the double challenge of system decomposition and diagnostic test selection, aiming to minimize subsystem interconnections while maximizing diagnosability is presented. In [12], an online method is proposed to detect mode changes and select residuals dynamically based on the current mode hypothesis. In contrast to the latter work, the approach adopted in this work is offline, where residuals are selected before system operation. In [15], a reconfigurable battery system similar to the one employed in this paper is presented, but it utilizes a different heuristic approach to manage the complexity of mode switches. Test selection methods for single-mode systems have been developed to evaluate residual fault sensitivity using training data, enabling robust residual selection. Examples of such approaches can be found in [5] and [11]. However, these works do not consider systems with multimode operation.

The contributions of this work is an algorithm to compute the diagnosability of residuals in multimode systems and for selecting a small set of residuals for diagnosing multimode systems with maximum diagnosability in all operation modes. A Python package is developed and is available at `https://github.com/MattiasKrysander/Multimode-Test-Selection`. It is based on `dd` library [4] for efficiently handling Boolean functions and logic inference. The methods are based on the fault signatures of the residuals for the various modes of operation, without accounting for the quantitative impact of faults. The approach is not limited to any specific residual generation method, both model-based and data-driven residuals can be used if the fault signature of the residual for the different operation modes is known. The fault influence and decoupling are important since the diagnosability is derived from a consistency-based diagnosis framework. Since the selection method relies only on the qualitative fault signatures of the residuals, structural methods can also be applied to compute potential residuals along with their expected fault influence. This enables residual selection based on the potential residuals, followed by the development of only the necessary residual generators as determined by the selection process. The approach is inspired by and demonstrated through a modular, dynamically configurable battery pack. This kind of battery pack has all the challenges that make the problem interesting, such as multiple operation modes, many sensors making the model redundancy large, and many components that need supervision. The selection of residuals is non-trivial because the number of residuals to choose from is large and the number of operation modes is exponential in the number of binary operation mode variables. The result is a small set of residuals with the maximum diagnosability in all operation modes.

The residual selection problem is formally defined in Section 2. A motivating case study of a modular dynamically reconfigurable battery pack is presented in Section 3 where multimode residuals are exemplified. Section 4 introduces the concept of fault signature of residuals in multimode systems. Then the diagnosability of residuals in multimode systems is discussed in Section 5. The residual selection algorithm is presented in Section 6. The algorithm's results applied to the battery pack case study are presented in Section 7 and the algorithm's computational complexity is discussed in Section 8. Finally, the paper is concluded in Section 9.

2 Problem Formulation

The work considers a system with a set of faults F. The system can operate in different modes and the set of all valid operation modes is $\mathbb{M}$, not to be confused with fault modes. This work assumes that the operation mode is measured or controlled. The system is monitored by sensors that produce residuals by applying them to a set of residual generators, R. The residuals are used to detect and isolate faults. The applicability of the residuals and their fault influence depends on the system's operation mode. The fault influence of the residuals is represented by the fault signature matrix, S_R, where each entry (r_j, f_i) is a Boolean function over the set of valid operation modes $\mathbb{M}$. This function evaluates to $\mathbf{T}$ (true) if residual r_j is sensitive to the fault f_i in mode m, and to $\mathbf{F}$ (false) otherwise.

Maximum fault diagnosability is achieved by utilizing all residuals R. However, all residuals are typically not needed for maximum diagnosability, here detectability and single-fault isolability will be considered. The challenge lies in selecting an appropriate subset, given the many possibilities, which grow exponentially with the number of residuals. This challenge becomes even more difficult in multimode systems, where the complexity of the problem increases significantly. The main problem addressed in this paper is how to identify a small subset of residuals, $R_s \subseteq R$, with the same diagnosability as all residuals across all operation modes.

3 Case Study: A Multimode Modular Battery Pack

A motivating case study that will be used to illustrate concepts and results of the paper is a multimode modular dynamically reconfigurable battery pack. It includes n modules in series, numbered from 1 to n, as illustrated in Fig. 1. Module k in Fig. 1(a) includes a battery cell, modeled as an equivalent circuit model, along with a full-bridge converter consisting of four MOSFET switches, i.e., S_1-S_4, and two sensors, i.e., a voltage and a current sensor, measuring $v_{\text{cell},k}$ and $i_{\text{cell},k}$, respectively. Two pack sensors measure the output voltage v_{pack} and the output current i_{pack}, respectively, as shown in Fig. 1(b). Faults for each battery cell and each sensor will be considered.

The term multimode describes the system's ability to operate in different modes, determined by different switch positions. Each module has three valid modes depending on how its cell is connected to the battery pack. In forward mode, the cell adds to the pack voltage, in backward mode it subtracts, and in bypass mode, the cell is disconnected from the circuit. The module is said to be turned on if it is in forward or backward mode.

A model of the battery system has been implemented in the Fault Diagnosis Toolbox (FDT) [6, 7]. The model of module j is

```
v_p_der_j == i_cell_j / Cp_j - v_p_j / (Rp_j * Cp_j),...
v_ocv_j == OCV_fun( SOC_j ),...
SOC_der_j == -1/(3600*Q_j)*i_cell_j,...
DiffConstraint("SOC_der_j","SOC_j"),...
v_cell_j == v_p_j + R0_j * i_cell_j + v_ocv_j + f_cellj,...
DiffConstraint("v_p_der_j","v_p_j"),...
```

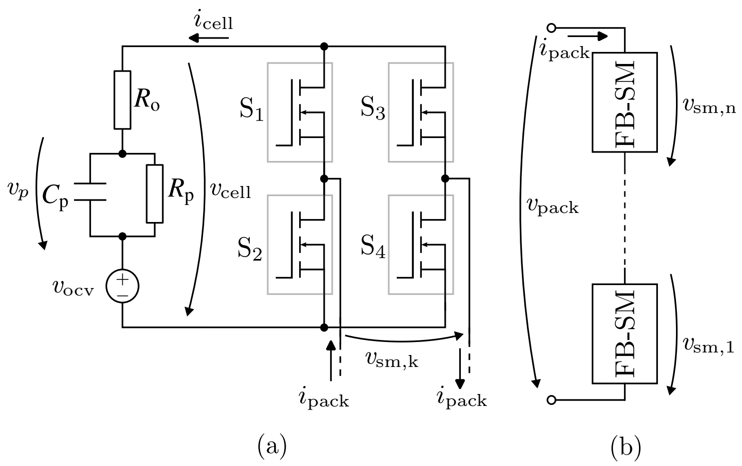

Figure 1 A modular dynamically reconfigurable battery system. (a) A battery pack with n modules. (b) A battery module.

```
7 v_sm_j == v_sm_fun( forward_j, backward_j, v_cell_j),...
8 i_cell_j == i_cell_fun( forward_j, backward_j, i_pack),...
9 y_i_j == i_cell_j + f_ij,...
10 y_v_j == v_cell_j + f_vj
```

The battery is modeled in rows 1-6 where `i_cell_j` is the cell current, `v_cell_j` the cell voltage, and `SOC_j` its state-of-charge. The open circuit voltage of the cell is interpolated from its state-of-charge in function `OCV_fun`. The full-bridge converter is modeled in rows 7-8 where `v_sm_j` is the output voltage of the module and `i_pack` is the battery pack current. The Boolean variables `forward_j` and `backward_j` determine the operation mode of the module. For example, the cell current and pack current are related by the function `i_cell_fun` given by

```
function i_cell = i_cell_fun( forward, backward, i_pack)
if forward
    i_cell = i_pack;
elseif backward
    i_cell = -i_pack;
else
    i_cell = 0;
end
```

The voltage of the module `v_sm_j` is governed by a similar function `v_sm_fun`. In addition to the modules, the battery pack has sensors measuring the total output voltage and current of the pack

```
v_pack == v_sm_1 + v_sm_2 + ... + v_sm_n
y_v_pack == v_pack + f_vpack
y_i_pack == i_pack + f_ipack
```

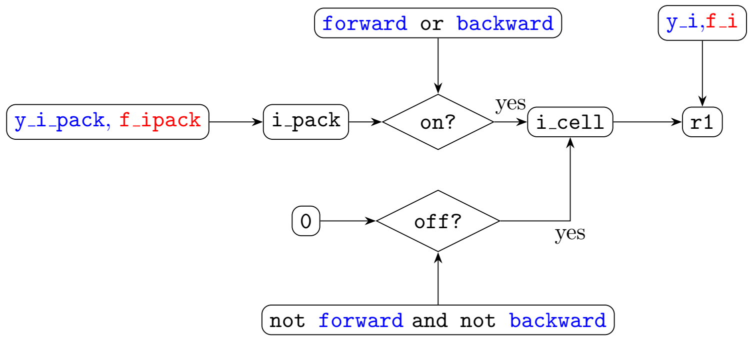

Figure 2 Graph representation of the residual generation function `ResGen_1`. In particular it shows the influence of the faults `f_i` and `f_ipack` on residual `r1`.

For each module j, faults for the cell `f_cellj`, current sensor `f_ij`, and voltage sensor `f_vj` are considered as well as for the pack voltage and current sensors `f_vpack` and `f_ipack` respectively. In the FDT, faults are represented as general fault signals, although they can equivalently be interpreted as assumptions on the components. For instance, `y_i_pack == i_pack + f_ipack` could also be interpreted as

$$\text{OK}(\text{Pack current sensor}) \rightarrow \texttt{y\_i\_pack == i\_pack}. \quad (1)$$

4 Fault Signature of Residuals

The input to the residual selection algorithm proposed in this work is a multimode extension of the fault signature matrix. The underlying concept will be introduced and examined in detail in this section.

Consider a one-module battery pack with a pack current sensor to illustrate the concept of residuals and fault signature. In this example, no sensor for measuring pack voltage is included. Using the FDT, three multimode residuals are generated for the battery pack. One residual `r1` is comparing the current measurements as follows

```
function [r1] = ResGen_1(y_i,y_i_pack,forward,backward)
i_pack = y_i_pack;
i_cell = y_i;
r = i_cell - i_cell_fun(forward, backward, i_pack);
```

A computational graph is shown in Figure 2, where known inputs are in blue and faults are in red. The fault `f_i` will influence the residual independent of the mode. The fault `f_ipack` will only influence the residual if the module is turned on.

Next, the fault signature of a residual in a multimode system is formally defined.

▶ **Definition 1** (Multimode fault signature of a residual). *In a multimode system, the signatures of fault f of residual r is a Boolean function $S_r^f : \mathbb{M} \rightarrow \mathbb{B}$ of operation modes such that it is* $\mathbf{T}$ *if fault f influences residual r in mode $m \in \mathbb{M}$ and* $\mathbf{F}$*, otherwise. The* multimode fault signature of residual r *is the collection of the fault signatures of all faults F in the system, i.e.,* $S_r = \{S_r^f | f \in F\}$.

For residual `r1` in Figure 2, the multimode fault signature is given by $S_{\texttt{r1}}^{\texttt{f\_ipack}} = \texttt{forward} \vee \texttt{backward}$ and $S_{\texttt{r1}}^{\texttt{f\_i}} = \mathbf{T}$. The fault signature of a residual does not depend on whether a module is connected in the forward or backward mode. It is only dependent on if the module

Table 1 The multimode fault signature matrix of the residuals for the dynamically configurable battery pack with one module and a pack current sensor.

residual	f_ipack	f_i	f_cell	f_v
1	`on`	**T**	**F**	**F**
2	**F**	**T**	**T**	**T**
3	`on`	**F**	**T**	**T**

is turned on. Since the battery example will be used throughout the paper, we will simplify the notation by introducing a Boolean variable $\texttt{on} = \texttt{forward} \vee \texttt{backward}$ such that the fault signature can be written as $S_r^{\texttt{f\_ipack}} = \texttt{on}$.

▶ Remark. Note that the fault signature of a residual is not dependent on the gain from the fault to the residual, it is only a Boolean function indicating if the fault influences the residual. For example, the sign of the fault sensitivity of `f_ipack` is different in forward and backward directions but the fault signature is the same.

As mentioned before, in a single mode case the fault signature of a set of residuals R is often collected in a so-called fault signature matrix with rows and columns corresponding to residuals and faults respectively. An entry (r_i, f_j) is either **T** if fault f_j influences residual r_i or **F** otherwise. The signature can depend on the system operation mode in the multimode case and is extended to multimode systems in the following definition.

▶ **Definition 2** (Multimode fault signature matrix). *Given a system of valid operation modes $\mathbb{M}$, a set of faults F, and a set of residuals R; the* multimode fault signature matrix *is defined as a matrix S_R where each row corresponds to a residual $r \in R$ and each column corresponds to a fault $f \in F$ and the element in position (r, f) is S_r^f.*

The multimode fault signature will also be evaluated for subsets of residuals $R_s \subseteq R$ and then the notation S_{R_s} will be used.

The fault signature matrix of the three residuals derived for the one-module battery-pack is given in Table 1. The first residual is the one illustrated in Figure 2.

Residual selection will be based on fault signature matrices similar to the one in Table 1. The fault signature matrix will be used to evaluate the diagnosability of different subsets of residuals which will be the topic of the next section.

5 Fault Diagnosability of Residuals

This section explains how to compute the multimode diagnosability of a residual set given their fault signature matrix. The diagnosability will be defined as the ability to detect and isolate faults using the residuals which depends on how the residuals are integrated into diagnosis computations. This work is based on a consistency-based framework, thus adopting the following definition of diagnosis from [2].

▶ **Definition 3** (Consistency-based diagnosis). *Given a diagnostic model $\mathcal{M}$ and observations $\mathcal{O}$, a diagnosis is an assignment $\mathcal{D}$ of a behavioral mode to each considered fault such that $\mathcal{M} \cup \mathcal{O} \cup \mathcal{D}$ is consistent. A diagnosis is minimal if it is minimal considering the set of faults.*

Here the diagnostic model is the set of residuals R with the corresponding fault signature matrix S_R. Observations are the residuals that trigger alarms $R_a \subseteq R$ and the present mode of operation $m \in \mathbb{M}$. A behavioral mode is represented as subsets of present faults $F_b \subseteq F$. For this particular case, the diagnosis is given by the following proposition.

▶ **Proposition 4** (Diagnosis of multimode residuals). *Given a fault signature matrix S_R and the observations (R_a, m) a behavioral mode $F_b \subseteq F$ is a diagnosis if it is a minimal hitting set of the sets $\{f|S_r^f(m) = \mathbf{T}\}$ for all $r \in R_a$. A diagnosis is minimal if no proper subset is a diagnosis.*

In the diagnosability analysis, the best possible residual response is considered. Then the diagnosability result will be an overestimate of the true diagnosability.

▶ **Definition 5** (Ideal fault response). *A residual r has an ideal fault response for all modes $m \in \mathbb{M}$ if it triggers an alarm for the set of faults $\{f|S_r^f(m) = \mathbf{T}\}$ it is influenced by according to the fault signature matrix.*

A consequence of ideal fault response and the definition of diagnosis is that F_b is a diagnosis if F_b is the present fault mode. Even if it is possible to take multiple faults into account, we will in the continuation consider the single fault case for ease of notation. This is a common assumption using pre-compiled tests. The no-fault mode will be denoted by `NF` and a single fault f_i with a slight abuse of notation.

▶ **Definition 6** (Structural detectability and isolability of multimode residuals). *A single fault f is structurally detectable in operation mode m with a set of residuals R if f is a minimal diagnosis given that all residuals have an ideal fault response. A fault f_i is structurally isolable from a fault $f_j (i \neq j)$, in mode m with a set of residuals R if f_i is a minimal diagnosis but not f_j given that all residuals have ideal fault response.*

To illustrate the implication of this definition on residual selection, consider a single-mode system with fault signature matrix

	f_1	f_2	f_3
r_1	**T**	**F**	**T**
r_2	**F**	**T**	**T**
r_3	**T**	**T**	**F**

(2)

The column-matching approach matches the residual response with the columns in the signature matrix to find the diagnoses. Then it would be sufficient to use the first two residuals to isolate all faults since all faults will have a unique signature. This isn't enough in the consistency-based approach as will be shown next. Assume that fault f_1 is present. Then residual r_1 will be triggered and the single faults f_1 and f_3 will be diagnosed. Hence, f_1 is not structurally isolable from f_3 with the first two residuals. Full single-fault isolability is obtained using all residuals.

Detectability and isolability will be unified in the following definition of diagnosability which is the main concept of this work.

▶ **Definition 7** (Structural diagnosability of multimode residuals). *Let the structural diagnosability be a Boolean function $D_R(f_i, f_j) : \mathbb{M} \to \mathbb{B}$, such that*

$$D_R(f, \mathit{NF})(m) = \begin{cases} \mathbf{T} & \textit{if } f_i \textit{ is structurally detectable in mode } m \\ & \qquad \textit{with ideal fault respons of } R \\ \mathbf{F} & \textit{otherwise} \end{cases} \tag{3}$$

$$D_R(f_i, f_j)(m) = \begin{cases} \mathbf{T} & \textit{if } f_i \textit{ is structurally isolable from } f_j (i \neq j) \textit{ in mode } m \\ & \qquad \textit{with ideal fault response of } R \\ \mathbf{F} & \textit{otherwise} \end{cases} \tag{4}$$

The following theorem shows how the fault signature matrix can be used to compute the diagnosability of a set of residuals.

▶ **Theorem 8.** *Given a fault signature S_R for a set of multimode residuals R, the structural detectability of the residuals in R is given by*

$$D_R(f_i, NF) = \bigvee_{r \in R} S_r^{f_i} \tag{5}$$

and the structural isolability by

$$D_R(f_i, f_j) = \bigvee_{r \in R} \left(S_r^{f_i} \wedge \neg S_r^{f_j}\right). \tag{6}$$

Proof. It is sufficient to consider an arbitrary fault pair (f_i, f_j). We will start to prove (6).

Select an arbitrary $m \in \mathbb{M}$ such that $D_R(f_i, f_j)(m) = \mathbf{T}$. Then there exists a residual $r \in R$ such that $S_r^{f_i}(m) = \mathbf{T}$ and $S_r^{f_j}(m) = \mathbf{F}$. This implies that $\bigvee_{r \in R}\left(S_r^{f_i} \wedge \neg S_r^{f_j}\right)(m) = \mathbf{T}$, thus $D_R(f_i, f_j) \models \bigvee_{r \in R}\left(S_r^{f_i} \wedge \neg S_r^{f_j}\right)$.

Now, select an arbitrary $m \in \mathbb{M}$ such that $\bigvee_{r \in R}\left(S_r^{f_i} \wedge \neg S_r^{f_j}\right)(m) = \mathbf{T}$. Then there exists a residual $r \in R$ such that $S_r^{f_i}(m) = \mathbf{T}$ and $S_r^{f_j}(m) = \mathbf{F}$. This implies that $D_R(f_i, f_j)(m) = \mathbf{T}$, thus $\bigvee_{r \in R}\left(S_r^{f_i} \wedge \neg S_r^{f_j}\right) \models D_R(f_i, f_j)$ and (6) is proved.

The proof of (5) follows from (6) by noting that no residual will trigger an alarm in `NF`, i.e., $S_r^{\emptyset} \equiv \mathbf{F}$ for all $r \in R$. Formula (5) follows by letting $f_j = \emptyset$ in (6) and the proof is complete. ◀

To exemplify diagnosibility matrices, consider the fault signature matrix in Table 1. The diagnosability of the residual 1 is according to Theorem 8 given by

	NF	f_ipack	f_i	f_cell	f_v
f_ipack	on	**F**	**F**	on	on
f_i	**T**	¬on	**F**	**T**	**T**
f_cell	**F**	**F**	**F**	**F**	**F**
f_v	**F**	**F**	**F**	**F**	**F**

(7)

and the diagnosability of all 3 residuals is given by

	NF	f_ipack	f_i	f_cell	f_v
f_ipack	**on**	**F**	**on**	**on**	**on**
f_i	**T**	**T**	**F**	**T**	**T**
f_cell	**T**	**T**	**T**	**F**	**F**
f_v	**T**	**T**	**T**	**F**	**F**

(8)

The detectability is given by the column `NF` in blue and the remaining part specifies the isolability. Full single fault diagnosability is given by a matrix with **T** in all entries except for the diagonal entries (f_i, f_i) which are **F**.

The diagnosability matrix (8) shows that the 3 residuals have full single fault diagnosability except for the following two exceptions. The entries `on` in the first row show that fault `f_ipack` is not detectable nor isolable from any other fault in mode `on` = **F**. If `on` = **T** the fault is detectable and uniquely isolable. The two by two block in pink with entries **F** shows that `f_cell` and `f_v` are not isolable from each other in any mode.

6 Test Selection for Multimode Systems

This section describes the propsed algorithm to select a small subset of residuals with the same diagnosability as all residuals. The algorithm is based on the diagnosability of the set of residuals. The basic idea is to start with the empty set of residuals and then iteratively add the residual that improves the diagnosability the most.

Prior to presenting the algorithm, some convenient notation is introduced. Let the diagnosability matrix for a residual set R be denoted D_R. Furthermore, let $\neg D_R$ denote the elementwise negation of D_R. This means that $\neg D_R$ represents the diagnosability properties not provided by the residuals in R. Let g be a function from the set of diagnosability matrices to a scalar-valued goodness measure. A residual with the highest goodness measure will be selected. Different choices of improvement functions g will be discussed after the algorithm. The algorithm is described in Algorithm 1. The `dd` library [4] is used for efficient implementation of the fault signature and diagnosability matrices. Enumeration of all possible modes is avoided by encoding operation modes by Boolean functions.

Algorithm 1 Test Selection Algorithm.

Input: Fault signature matrix S_R for residuals R.
Output: Selected residuals $R_s \subseteq R$.
Initialize: Let the selected residuals be $R_s = \emptyset$ and the remaining residuals $R_r = R$.
while $R_r \neq \emptyset$ **do**
  for each remaining residual $r \in R_r$ **do**
    Compute diagnosability improvement of adding r to R_s: $\Delta D^r_{R_s} = D_{\{r\}} \wedge \neg D_{R_s}$.
  end for
  if no improvement is possible, i.e., $\max_{r \in R_r} g(\Delta D^r_{R_s}) = 0$ **then**
    break
  else
    Select a residual r^* with the largest improvement, i.e., $r^* = \arg\max_{r \in R_r} g(\Delta D^r_{R_s})$.
    Update the residual sets: $R_s = R_s \cup \{r^*\}$, $R_r = R_r \setminus \{r^*\}$.
  end if
end while

6.1 Improvement Functions

Two different improvement functions are proposed. The first counts the number of improved entries in the diagnosability matrix achieved by adding a residual. The diagnosability improvment of adding residual r to the set R_s is $\Delta D^r_{R_s}$. The improvement function g is in this case defined as

$$g(\Delta D^r_{R_s}) = |\{(f_i, f_j) \in (F \cup \texttt{NF}) \times F | \Delta D^r_{R_s}(f_i, f_j) \not\equiv \mathbf{F}\}|. \tag{9}$$

Algorithm 1 with improvement function (9) will be called `TestSelection`.

To give an example of how the improvement function is used in the residual selection consider the one-module battery pack and assume that no residuals have been selected, i.e., $R_s = \emptyset$. Consider the improvement function g for the first residual in Table 1. All entries of the diagnosability matrix of the empty set of residuals are $D_\emptyset(f_i, f_j) \equiv \mathbf{F}$. The diagnosability improvemnet $\Delta D^{\texttt{r1}}_\emptyset = D_{\{\texttt{r1}\}}$ which is given in (7). The improvement of adding residual 1 is given by the number of entries in the diagnosability matrix different from $\mathbf{F}$ which is 7. The improvement of adding residual 2 is 6 and adding residual 3 is 8. Hence the third residual is selected first.

The second improvement function will contrary to the function defined in (9) take into account the number of operation modes for which diagnosability is improved. Then the improvement function is defined as the number of assignments as

$$g(\Delta D^r_{R_s}) = \sum_{(f_i,f_j)\in(F\cup\texttt{NF})\times F} |\{m|\Delta D^r_{R_s}(f_i,f_j)(m) = \mathbf{T}\}|. \tag{10}$$

Algorithm 1 with improvement function (10) will be called `TestSelectionAssignment`.

Consider again the improvement function for adding residual 1 to the empty set of residuals. The diagnosability improvement is given in (7). With the introduced notation, there are 2 operation modes of the system, it is either turned on, i.e., $\texttt{on} = \mathbf{T}$ or turned off, i.e., $\texttt{on} = \mathbf{F}$. The evaluation of (10) is, to sum up the contributions from each entry. Entries that are $\mathbf{T}$ count as 2, entries that are $\mathbf{F}$ count as 0, and the rest count as 1. Thus, the improvement adding residual 1 is 10. The improvement adding residual 2 is 12 and residual 3 is 8, thus the second residual is selected. The computational complexity will be discussed in Section 8.

6.2 Diagnosability in any Operation Mode

For systems that frequently cycle through all operation modes or systems where the operation mode can be selected partially based on diagnosis requirements, it is possible to relax the diagnosability goal to possibly reduce the number of residuals. It could be sufficient to aim for maximum diagnosability first after cycling through all operation mode. The problem is formally stated as follows. The goal is to select a small set of residuals $R_s \subseteq R$ such that for each diagnosability property (f_i, f_j) satisfying $D_R(f_i, f_j) \not\equiv \mathbf{F}$ there exists a mode $m \in \mathbb{M}$ such that

$$D_{R_s}(f_i, f_j)(m) = \mathbf{T} \tag{11}$$

This problem is solved by Algorithm 1 with the improvement function `TestSelection` defined in (9) by using a modified fault influnce matrix $\tilde{S}_R$ defined as

$$\tilde{S}_R(f_i, f_j) \equiv \begin{cases} \mathbf{F} & \text{if } S_R(f_i, f_j) \equiv \mathbf{F} \\ \mathbf{T} & \text{otherwise.} \end{cases} \tag{12}$$

Algorithm 1 with improvement function (9) where the modified fault influnce matrix $\tilde{S}_R$ is used will be called `TestSelectionAnyMode`.

7 Method Demonstration

The proposed method is demonstrated on a modular battery pack. Consider the modular battery pack described in Section 3 with 2 modules and sensors measuring pack current and voltage. Multimode residuals have been generated and the fault signature matrix of those are given in Table 2. Two operation mode variables `on1` and `on2` indicate if the corresponding module is on. The residuals are generated using the Fault Diagnosis Toolbox [6, 7] and the enumeration of residuals corresponds to the underlying minimal structurally overdetermined sets. The fault signature matrix is generated using the method described in Section 5. The residuals are partitioned into sets relating to the same set of modules. The first three residuals, residual 1-3, only involve module 2, residuals 4, 11, and 12 only module 1, and the

Table 2 The multimode fault signature matrix for the residuals is derived from the dynamically configurable battery pack with 2 modules and sensors measuring pack current and voltage.

Residual	f_cell1	f_i1	f_v1	f_cell2	f_i2	f_v2	f_vpack	f_ipack
1	**F**	**F**	**F**	**F**	**T**	**F**	**F**	on2
2	**F**	**F**	**F**	**T**	**T**	**T**	**F**	**F**
3	**F**	**F**	**F**	**T**	**F**	**T**	**F**	on2
4	**F**	**T**	**F**	**F**	**F**	**F**	**F**	on1
11	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**F**
12	**T**	**F**	**T**	**F**	**F**	**F**	**F**	on1
7	**F**	**F**	on1	**F**	**F**	on2	**T**	**F**
8	**F**	**F**	on1	on2	on2	**F**	**T**	**F**
9	**F**	**F**	on1	on2	**F**	**F**	**T**	on2
15	on1	on1	**F**	**F**	**F**	on2	**T**	**F**
18	on1	**F**	**F**	**F**	**F**	on2	**T**	on1
16	on1	on1	**F**	on2	on2	**F**	**T**	**F**
17	on1	on1	**F**	on2	**F**	**F**	**T**	on2
20	on1	**F**	**F**	on2	on2	**F**	**T**	on1
21	on1	**F**	**F**	on2	**F**	**F**	**T**	on1∨on2

Table 3 Test selection applied to the two-module battery pack.

Algorithm	**Selected residuals**	**Improvement**
TestSelection	16, 3, 12, 1, 4, 7	32, 20, 20, 10, 10, 8
TestSelectionAnyMode	16, 3, 12, 1, 4, 7	32, 15, 11, 2, 2, 2
TestSelectionAssignment	2, 11, 21, 7, 1, 4, 3, 12	72, 72, 50, 15, 13, 10, 4, 4

rest include both modules. The order of the rows in the fault signature matrix can change the result of the residual selection because if there are residuals with equal improvement, the first one will be selected.

Table 3 shows the result of applying the different versions of test selection. The residuals are listed in the selection order together with the value of the improvement function for each selection. For this example the smallest solution found contains 6 residuals and the same solution is found both for the `TestSelection` and `TestSelectionAnyMode` algorithms. The `TestSelectionAssignment` algorithm selects 8 residuals, where residuals 2 and 11 are chosen first prioritizing residuals that improve the diagnosability in the most operation modes.

The diagnosability matrix for both the set of all residuals and the selected ones in Table 3 is given in Table 4. The diagnosability matrix shows that full single fault diagnosability with the exceptions that `f_ipack` is detectable and uniquely isolable if any module is turned on, and the cell fault `f_celli` and corresponding voltage sensor fault `f_vi` is isolable from each other if the corresponding module is turned on.

Diagnosability analysis can be applied to the model defined in Section 3 with the method proposed in [10]. The diagnosability of the model and the selected residuals are equal. This means that maximum diagnosability for all modes is achieved with the 6 residuals. The fault signature matrix of these residuals is given in Table 5. Two local residuals for each module and two residuals including both modules are selected to achieve maximum diagnosability. All the residuals are defined in all operation modes. Thus it is possible to run them continuously.

Table 4 The multimode fault diagnosability matrix for the residuals of the dynamically configurable battery pack. The variable on := on1 ∨ on2.

	NF	f_cell1	f_i1	f_v1	f_cell2	f_i2	f_v2	f_vpack	f_ipack
f_cell1	**T**	**F**	**T**	on1	**T**	**T**	**T**	**T**	**T**
f_i1	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**	**T**
f_v1	**T**	on1	**T**	**F**	**T**	**T**	**T**	**T**	**T**
f_cell2	**T**	**T**	**T**	**T**	**F**	**T**	on2	**T**	**T**
f_i2	**T**	**T**	**T**	**T**	**T**	**F**	**T**	**T**	**T**
f_v2	**T**	**T**	**T**	**T**	on2	**T**	**F**	**T**	**T**
f_vpack	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**T**
f_ipack	on	on	on	on	on	on	on	on	**F**

Table 5 Fault signature matrix for the selected residuals for the dynamically configurable battery pack.

Residual	f_cell1	f_i1	f_v1	f_cell2	f_i2	f_v2	f_vpack	f_ipack
16	on1	on1	**F**	on2	on2	**F**	**T**	**F**
7	**F**	**F**	on1	**F**	**F**	on2	**T**	**F**
4	**F**	**T**	**F**	**F**	**F**	**F**	**F**	on1
12	**T**	**F**	**T**	**F**	**F**	**F**	**F**	on1
1	**F**	**F**	**F**	**F**	**T**	**F**	**F**	on2
3	**F**	**F**	**F**	**T**	**F**	**T**	**F**	on2

This is particularly important for the residual generators with dynamic states. If these were shut off reinitialization of the states would be a problem to be solved. In the example, the residuals influenced by f_cellj have dynamics, i.e., residuals 3, 12, and 16.

8 Computational Complexity

The algorithm is linear in the number of residuals, quadratic in the number of faults, and worst case exponential in the number of operation modes variables. To empirically evaluate the algorithm's performance, the TestSelection-algorithm is applied to modular battery packs with 2, 4, and 6 modules. The results are given in Table 6. For the example, the computation time agrees well with the theoretical complexity, except that the computation time increases linearly with the number of mode variables.

It is also interesting to note that there are solutions with fewer residuals achieving maximum diagnosability for the 4 and 6-module cases. There is a solution with 10 residuals for the 4-module case and 14 for the 6-module case. In these cases, the smaller solutions are subsets of the selected ones. It is not surprising the algorithms do not find optimal solutions since they are greedy.

Table 6 The TestSelection-algorithm applied to modular battery packs with 2, 4, and 6 modules.

Modules/mode variables	Faults	Residuals	Selected Residuals	Time
2	8	15	6	44 ms
4	14	93	14	1.6 s
6	20	747	20	37 s

9 Conclusions

Residual selection for multimode systems has been described. An algorithm is proposed for selecting a small set of residuals with the same diagnosability as all residuals. It takes a multimode version of a fault signature matrix as input. This means that any residual generation technique, model-based or data-driven, is applicable as long as a correct fault signature matrix can be defined. The user can define or use one of the proposed improvement functions to map a certain diagnosability to a performance score. Properties that could be considered in the improvement function are e.g. the locality of tests searching for a distributed solution, and the computational complexity or robustness of the residual generators. A Python package has been developed and is available at `https://github.com/MattiasKrysander/Multimode-Test-Selection`. The algorithm is demonstrated with good results on a modular and dynamically reconfigurable battery pack. Packs of different sizes are analyzed and the results show that the algorithm is efficient and can be applied to systems with many potential residuals and operation modes.

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