

Integrated Line Planning and Strategic Ridepooling: Minimizing the Passengers' Travel Time

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Abstract

Public transport systems increasingly combine fixed-line services with demand-responsive transportation such as ridepooling. In this paper, we present an integrated optimization model for line planning and strategic ridepooling that explicitly incorporates passenger routing into the planning process. Building on previous work on integrated line planning and strategic ridepooling as well as Change&Go networks, we develop a model for the integration of line planning and strategic ridepooling that minimizes the total travel time of all passengers while respecting a given budget. The resulting mixed-integer optimization model simultaneously determines line frequencies, ridepooling capacities, and passenger routes, capturing interactions between the two different transportation modes – one line-based mode and one mode without fixed lines and timetables – and still accounts for travel times, transfer times, and capacity decisions within a single optimization model. Focusing on an applicable model, we show how the data structure of an existing line planning model with integrated passenger routing in a Change&Go network can be improved, rendering the problem solvable for medium-sized instances. We consider the optimization model as a bi-criteria problem with the objective functions of travel time and operating cost, and compute pareto-optimal solutions with a budget constraint method. Our computational experiments on benchmark instances demonstrate the applicability of the model and show that integrating passenger routing into the strategic planning process yields multimodal transport systems that dominate solutions from approaches based on fixed routings.

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1 Motivation

Line Planning is a well-known problem in public transport optimization, with many papers published on the topic, including the over 100 years old [12] and more recent surveys such as [14] or [15]. The task is to choose lines and their frequencies such that everybody can travel and a budget restricting the costs is respected. In this paper we aim to minimize the overall travel time of the passengers. One model which minimizes the passengers' travel times, including their transfer times between different lines, has been presented in [16].

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In addition to line-based public transport systems, demand responsive services such as ridepooling are becoming more and more popular. Typically, a ridepooling service provider has a fleet of vehicles available to transport passengers, who can request to be taken from their origin to a destination. A passenger who requested a trip is then picked up by a vehicle, which drives them to their destination. The trips can be *pooled*, meaning that different trips with similar routes can be covered by the same vehicle, which is shared by the passengers. This pooling aspect leads to a reduction of the overall number of vehicles and vehicle driving time and distance, compared to a traditional taxi service or private transportation in individual cars. In [4], the role of ridepooling in multimodal public transport settings is discussed. Some example of these services include the company *MOIA* [11] which offers ridepooling in the city of Hamburg, Germany, with an all-electric fleet of vehicles, complementing the transportation infrastructure of line-based public transport and private vehicles. In the city of Birmingham, United Kingdom, *Max On-Demand* [10] offers on-demand transportation in different zones in and around the Birmingham city center. The transport association *Verkehrsbund Rhein-Neckar (VRN)* covers parts of the German federal states Baden-Württemberg, Rhineland-Palatinate and Hesse, including the entire Palatinate Forest and both cities of Kaiserslautern and Landau. As an addition to an extensive system of line-based public transportation, VRN also offers a demand-responsive transport option *VRNflexline* [20] in many different areas, especially in rural areas such as villages which are often not well-connected to the line systems.

Known as the *dial-a-ride-problem (DARP)*, the operative aspects of ridepooling services of assigning passenger requests to vehicles and scheduling their journeys have been studied extensively, both in online and in offline settings. The operative business is important, but not topic of our research in this paper. For a literature review we refer to [17, 19], see also [3] for some references.

In [3], *strategic* ridepooling with public transport line planning is integrated into one optimization model that jointly determines line plans and ridepooling services, thereby explicitly accounting for the interaction between fixed-line public transport and strategically planned ridepooling operations. However, in [3], the passengers' routes have been fixed in advance to receive traffic loads that then have to be covered either by a bus line or a ridepooling area. This is common in older models also for line planning, but is, in particular when planning multiple modes with different velocities, a simplification that may yield undesirable results. In this paper we hence extend [3] and include routing of passengers into the design of a multimodal public transport system, i.e., we integrate line-based transport with demand-responsive ridepooling services taking passengers' routes into account. Our contribution hence is two-fold:

- We present a new model for integrating strategic ridepooling with line planning *and* passengers' routing. We illustrate that integrating the routes of the passengers is crucial in particular when different modalities are planned. The numerical study shows the usefulness of the approach.
- We improve the underlying data structures for the Change&Go network making our approach applicable to medium-sized instances. Line planning with integrated passenger routing is a special case and hence benefits from this algorithmic improvement making the model presented in [16] eventually applicable for real-world instances.

The remainder of this paper is structured as follows: In Section 2 we give an introduction to strategic ridepooling and its integration with line planning, and highlight the benefits of including the routing of passengers into the model. In Section 3 we repeat the model for line planning from [16] which aims at minimizing the overall travel time of the passengers. We

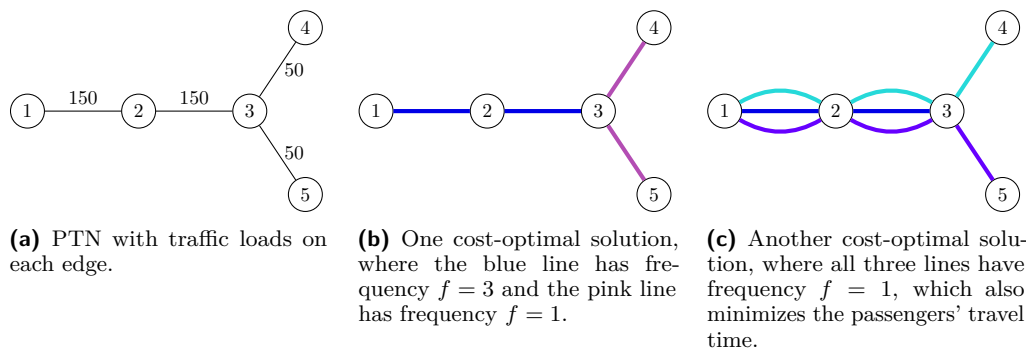
demonstrate that two model reduction techniques compared to the original model presented in [16] lead to significant improvements in performance. In Section 4 a new model for strategic ridepooling is presented, which has the objective of minimizing the passengers' travel time. Analogous to the lines and frequencies in line planning, this strategic ridepooling model chooses ridepooling areas and determines for each a number of vehicles to be available to the passengers. Both models are combined in Section 5 to form an integrated model for planning multiple modalities. Numerical experiments are presented in Section 6. We conclude in Section 7.

2 Importance of Passengers' Routes in Multimodal Settings

Ridepooling services can be a valuable addition to a line-based public transport system. Line-based systems are typically very good at transporting many passengers quickly and efficiently, but in situations with only a low passenger demand, the large vehicles used in these systems often contribute to a huge waste of resources, both economical and ecological. Contrarily, ridepooling services are more useful in lower demand settings, where ridepooling vehicles offer more flexibility since they can react spontaneously to passenger requests and hence avoid driving around unnecessarily without passengers. Hence, it is useful to have both modes of public transport as part of a public transport system, each deployed in areas where they are most beneficial to the overall system. Multi-modal transport is hence beneficial.

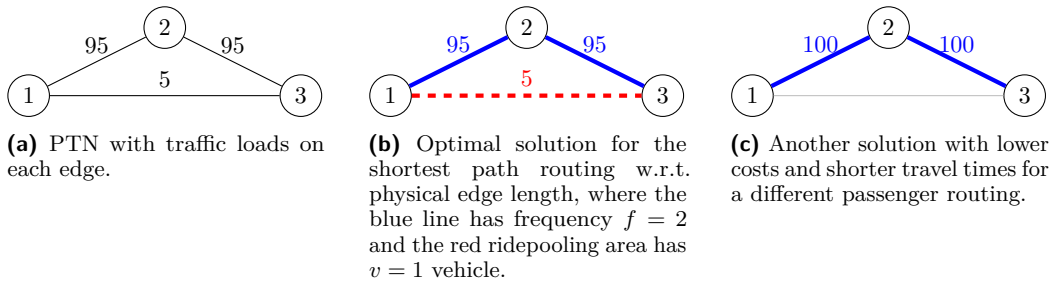
In order to model multi-modal transport, [3] combine line planning and strategic ridepooling in an integrated model minimizing the cost of the line concept and the ridepooling service. However, the model assumes that passengers are pre-routed such that only traffic loads are given. The number of transfers and the travel time of the passengers are hence not considered. We now improve this model by taking the passenger routing explicitly into account and instead of the cost we minimize their travel times.

Taking the routing into account is already relevant for only one mode of transport, as can be seen in the following example: Suppose we have a PTN as in Figure 1a, and a line modality with vehicle capacity 50. The traffic loads on the edges originate from three OD-pairs $(1, 3)$, $(1, 4)$ and $(1, 5)$, each with 50 passengers. Suppose the cost of operating a line with frequency f is f times the number of edges in the line. Then, from a cost perspective, both solutions in Figures 1b and 1c are optimal, but the latter would be preferable for the passengers since it does not require any transfers. In this example, the difference between the two solutions is only in the transfers.



■ **Figure 1** Example: Transfers should be considered in the optimization.

For multimodal planning, taking the routing into account is even more important, since the modes might travel at vastly different speeds, which has a big impact on the passengers' preferences and route choices. The following example shows this effect: Take the PTN as depicted in Figure 2a, with the OD-pairs $(1, 2)$, $(2, 3)$ with 95 passengers and $(1, 3)$ with 5 passengers. For the traffic loads in the PTN, the passengers have been routed along shortest paths according to the physical lengths of the edges. This leads to the solution in Figure 2b with one line and one ridepooling area. However, if, for instance, the line vehicles are high-speed trains which can travel fast and are not affected by traffic congestion, unlike a ridepooling vehicle which travels on roads, then, the passengers traveling between stations 1 and 3 would prefer the train over the ridepooling service, rendering the latter unnecessary. The new solution in Figure 1c is even cheaper than the solution from Figure 1b since it uses the same line with the same frequency but no ridepooling vehicles.



■ **Figure 2** Example: A pre-determined routing does not always lead to the best possible solution, even if passengers are routed on shortest paths.

Hence, in this paper, we propose a new model for strategic ridepooling which incorporates the passenger routing in the optimization step. The sum of all passengers' travel times can be minimized, while the overall cost is bounded by a budget constraint, or vice versa. We show that solutions of the integrated model from [3] yield feasible solutions for the new model, provided the budget constraint is chosen appropriately.

3 Line Planning with Minimal Travel Time

We briefly repeat the model for line planning from [16] and emphasize two techniques for reducing the model size compared to the original formulation, in order to be able to solve large instances without a specialized algorithm like column generation. We start with some notation also needed in Section 4.

Let $\text{PTN} = (V, E)$ be a public transport network represented as an undirected graph with a set of stops or stations V as its nodes and a set of direct connections E as edges. Let $\mathcal{OD} = (\text{OD}_{o,d})_{o,d \in V} \in \mathbb{N}^{V \times V}$ be the OD-matrix describing the expected passenger demand within a given planning period T . A line is a path in the PTN. Let $\mathcal{L}$ be a set of potential lines, the so-called *linepool*. We assume that the cost of a line depends linearly on its frequency $f_l \in \mathbb{N}$, i.e., the number of times it is operated within the given planning period T . For a line $l \in \mathcal{L}$ let $C_l \geq 0$ be the cost per frequency of this line. The task in the line planning problem is to assign a service frequency $f_l \in \mathbb{N}$ to each line $l \in \mathcal{L}$. Note that frequency $f_l = 0$ means that line l is not selected. A vector of frequencies $f \in \mathbb{N}^{\mathcal{L}}$ for a given linepool $\mathcal{L}$ is called a *line concept*.

The goal is to find a line concept that minimizes the sum of travel times. Since the exact travel times depend on the timetable, which is usually determined after the lines and their frequencies have been chosen, we assume constant speed driving on the lines and add a penalty $t^{\text{trans-}\mathcal{L}}$ for each transfer.

The passenger routing is often modeled using a Change&Go network, introduced by [16], which is a graph that has a node for each possible line at each served station and an additional node for each station. The arcs of the Change&Go network represent the driving along a line and the transfers between lines at a station. This is integrated into the line planning model, which then simultaneously determines the line frequencies and the passengers' routes as an (integral) multi-commodity flow.

The first reduction technique concerns the modelling of transfers in the Change&Go network. Instead of adding a transfer arc for each pair of lines at each station, transfers are modeled via a platform node with an arc from and to each line node at the station. This technique is described in [7, 5] and also used in the recent computational comparison of line planning models [18]. It is applicable when the transfer penalty is independent of the ordered pair of lines but depends only on making a transfer.

Formally, for a given PTN = (V, E) and line pool $\mathcal{L}$, the *Change&Go network* is the graph CGN = (V, E) defined as follows: The nodes are $\mathcal{V} = \mathcal{V}^p \cup \mathcal{V}^d$, where $\mathcal{V}^p = \{(s, 0, \mathcal{L}) : s \in V\}$ are the platform nodes and $\mathcal{V}^d = \{(s, l, \mathcal{L}) : l \in \mathcal{L}, s \in l\}$ are the driving nodes. The arcs are $\mathcal{E} = \mathcal{E}^p \cup \mathcal{E}^d$, where

$$\mathcal{E}^p = \{((s, 0, \mathcal{L}), (s, l, \mathcal{L})) \in \mathcal{V}^p \times \mathcal{V}^d, ((s, l, \mathcal{L}), (s, 0, \mathcal{L})) \in \mathcal{V}^d \times \mathcal{V}^p : s \in V, l \in \mathcal{L}, s \in l\}$$

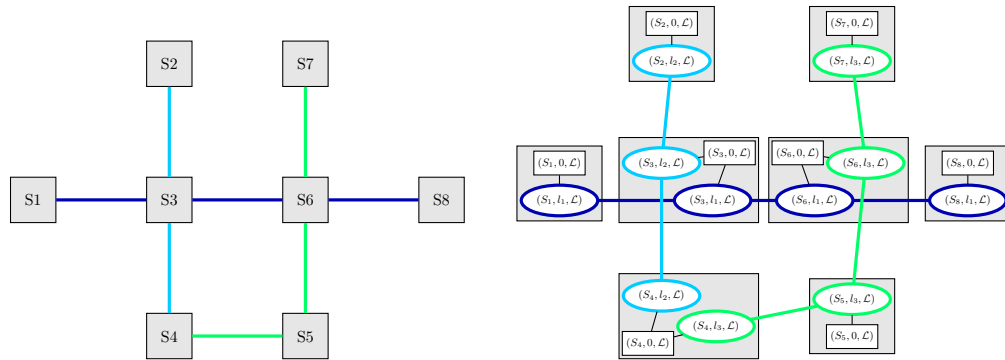
are the platform arcs and $\mathcal{E}^d = \bigcup_{l \in \mathcal{L}} \mathcal{E}^l$ with

$$\mathcal{E}^l = \{((s, l, \mathcal{L}), (t, l, \mathcal{L})), ((t, l, \mathcal{L}), (s, l, \mathcal{L})) \in \mathcal{V}^d \times \mathcal{V}^d : e = \{s, t\} \in l\}, \quad l \in \mathcal{L},$$

are the driving arcs. Note that the Change&Go network is a directed graph, where for each driving and platform arc, its backward arc also exists. Figure 3a shows a PTN with three lines, and Figure 3b shows the corresponding Change&Go network.

For a driving arc $e \in \mathcal{E}^d$ denote by $E(e)$ its underlying edge in the PTN. We define the arc weights to determine the length of a passenger's path:

$$t_e = \begin{cases} t_{E(e)}^{\text{drive-}\mathcal{L}} & \text{if } e \in \mathcal{E}^d, \\ \frac{1}{2} t_{E(e)}^{\text{trans-}\mathcal{L}} & \text{if } e \in \mathcal{E}^p. \end{cases}$$



(a) PTN with three lines.

(b) Change&Go network, note that each undirected arc in the picture represents a forward and a backward arc in the Change&Go network.

■ **Figure 3** Networks for Line Planning.

If a passenger wishes to travel from PTN-node $o \in V$ to $d \in V$, then in the CGN they are routed from platform node $(o, 0, \mathcal{L}) \in \mathcal{V}$ to platform node $(d, 0, \mathcal{L}) \in \mathcal{V}$. Their travel time can be computed using the knowledge of the travel times on the arcs along their path. Suppose

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p is a passenger's path from their origin to their destination in the CGN. Then the travel time on the path p is the sum of the travel times on the edges in the path:

$$t_p = \sum_{e \in p} t_e. \quad (1)$$

Note that each transfer arc in the CGN has travel time $\frac{1}{2}t^{\text{trans-}\mathcal{L}}$, but since each transfer consists of two transfer arcs (from driving node to platform node to another driving node), the time needed for each transfer is $t^{\text{trans-}\mathcal{L}}$. Furthermore, the initial entry onto and final exit from the line system each contribute $\frac{1}{2}t^{\text{trans-}\mathcal{L}}$ the passenger's travel time. Hence, in (1), it is assumed that the passengers also spend $t^{\text{trans-}\mathcal{L}}$ waiting to enter their first line vehicle.

For given line frequencies f any feasible routing of the passengers corresponds to a feasible (integral) multi-commodity flow in the Change&Go network, where each arc's capacity is determined by the line frequency. Most formulations model this either with path variables or with arc flow variables for every origin-destination pair. The second model reduction technique is to aggregate all passengers with the same origin, which reduces the number of variables and constraints. This is a standard technique for multi-commodity flow problems, see, e.g., [8], which was only recently applied to line planning with passenger routing [2, 1]. This formulation is integrated into the problem of choosing a frequency for each line from the linepool, resulting in an integer programming formulation with the constraints that all passengers must be transported and a given cost budget B is not exceeded and the objective of minimizing the overall sum of the passengers' travel times. To this end, let Θ be the node-arc-incidence-matrix of the CGN:

$$\Theta_{v,e} = \begin{cases} 1 & \text{if } e \text{ enters } v, \\ -1 & \text{if } e \text{ leaves } v, \\ 0 & \text{otherwise.} \end{cases}$$

For each origin $o \in V$ and CGN arc $e \in \mathcal{E}$, let $x_e^{(o)} \geq 0$ be the number of passengers with origin o who travel along arc e . For each origin $o \in V$ define $b^{(o)} = (b_v^{(o)})_{v \in V}$ with:

$$b_v^{(o)} = \begin{cases} -\sum_{d \in V} \text{OD}_{o,d} & \text{if } v = (o, 0, \mathcal{L}), \\ \text{OD}_{o,d} & \text{if } v = (d, 0, \mathcal{L}) \text{ for some } d \in V \setminus \{o\}, \\ 0 & \text{else.} \end{cases}$$

Let $\text{cap}^{(\mathcal{L})} \in \mathbb{N}$ be the capacity of a line vehicle. The frequencies $f_l \in \mathbb{N}$ of the lines $l \in \mathcal{L}$ need to be chosen large enough such that all passengers on their driving arcs can be transported, while overall, the cost of the line system is not above the given budget B .

This leads to the following integer linear program:

$$\begin{aligned} \min \quad & \sum_{o \in V} \sum_{e \in \mathcal{E}} t_e \cdot x_e^{(o)} & (\text{CG-L}) \\ \text{s.t.} \quad & \sum_{o \in V} x_e^{(o)} \leq \text{cap}^{(\mathcal{L})} \cdot f_l & \forall l \in \mathcal{L}, e \in \mathcal{E}_l \\ & \Theta x^{(o)} = b^{(o)} & \forall o \in V \\ & \sum_{l \in \mathcal{L}} C_l \cdot f_l \leq B \\ & f_l \in \mathbb{N} & \forall l \in \mathcal{L} \\ & x_e^{(o)} \in \mathbb{N} & \forall o \in V, e \in \mathcal{E} \end{aligned}$$

Any flow $x^{(o)} \in \mathbb{N}^{\mathcal{E}}$ of passengers starting at o that satisfies the flow conservation constraints $\Theta x^{(o)} = b^{(o)}$ can be decomposed into flows for all OD-pairs. After removing cycles, this can be decomposed into integral path flows, corresponding to paths used by passengers with origin o and destination d .

The two presented model reduction techniques have a significant effect on the performance on practical instances: To illustrate this, we applied Gurobi Version 13.0.2 [6] on an AMD Ryzen 7 5825U laptop with 16 GiB memory RAM to two instances from LINTIM [13]. For the Mandl instance [9], depicted in Figure 6 using the original formulation from [16] took ≈ 14 seconds. When replacing pairwise transfer arcs by platform-mediated transfers, the computation time was reduced to ≈ 10 seconds, with origin aggregation ≈ 2 seconds, and, when both techniques are combined, only ≈ 1 second.² For larger models such as the Athens metro instance, without origin aggregation, the solver exceeded the memory, whereas for aggregated flows, the optimum could be computed in ≈ 6 minutes, and with additional reduction of the transfer arcs, in ≈ 1 minute.

Line planning problems are known to be NP-complete, see e.g. [16].

4 Strategic Ridepooling with Minimal Travel Time

Ridepooling services are becoming more and more common. They typically function as follows: A provider has a fleet of vehicles which are available to the passengers, who request trips, i.e., to be taken by a ridepooling vehicle from their origin to their destination. The ridepooling service provider then assigns vehicles to service passenger requests, similar to a taxi service. But the passengers' trips can be pooled, meaning that it is allowed for the vehicles to be shared by multiple passengers, who might travel along small detours to allow for more efficient pooling of trips, such that the overall necessary number of vehicles and distance driven by vehicles can be reduced.

The operative aspects of ridepooling services have been very well researched. Contrarily, strategic ridepooling is a rather new problem in public transport optimization. Disregarding operational details such as specific vehicle routes, passenger requests and their assignments to available ridepooling vehicles, strategic ridepooling is the problem of deciding where a ridepooling service should be established and how much capacity (i.e. how many ridepooling vehicles) is needed to service the expected passenger demand. This is similar to the line planning problem, where the specific timetable is disregarded and only the layout and capacity of the line system is determined. Summarizing, strategic ridepooling considers ridepooling from a planning perspective rather than an operative one.

To model strategic ridepooling with minimal traveling time, take, again, a public transport network $\text{PTN} = (V, E)$, a planning period $T \in \mathbb{N}$ and a passenger demand OD-matrix $\text{OD} \in \mathbb{N}^{V \times V}$. Define a *ridepooling area* $r \subseteq E$ to be a connected set of edges in the PTN. Let $\mathcal{R}$ be a *ridepooling pool*, i.e., a set of ridepooling areas. Suppose that the cost of operating a ridepooling vehicle for the duration of the planning period T is $C_{\mathcal{R}} \geq 0$. The task in strategic ridepooling is to assign a number of vehicles $v_r \in \mathbb{N}$ to each ridepooling area $r \in \mathcal{R}$ to be available to the passengers in the planning period. Note that $v_r = 0$ is allowed, i.e., not every ridepooling area needs to be chosen. The number of vehicles for each area should be chosen such that all passengers can be transported and the expected overall travel time is minimized while a given cost budget B is not exceeded.

² These times include the whole execution of the program, including reading the input data and building the model.

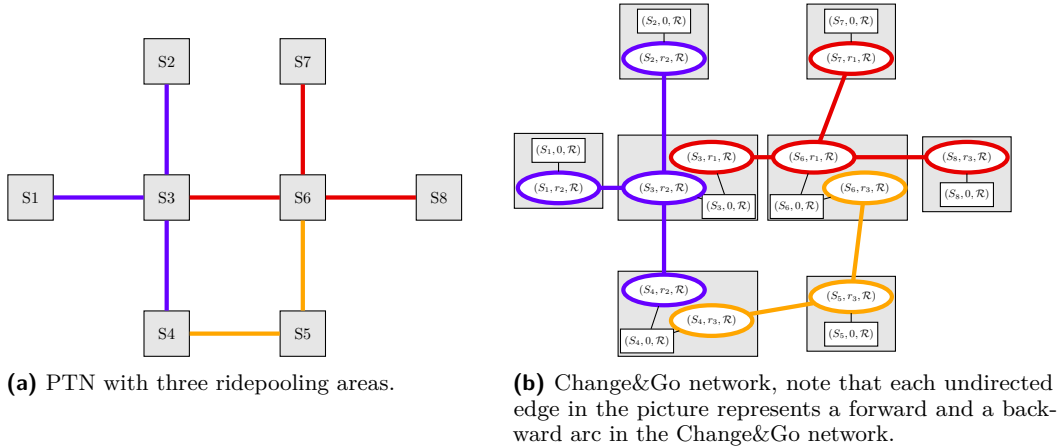
Similarly to the line planning case, define a Change&Go Network $\text{CGN} = (\mathcal{V}, \mathcal{E})$ for a given PTN (V, E) and ridepooling pool $\mathcal{R}$:

- $\mathcal{V} = \mathcal{V}^p \cup \mathcal{V}^d$ where:
 - Platform nodes: $\mathcal{V}^p = \{(s, 0, \mathcal{R}) : s \in V\}$
 - Driving nodes: $\mathcal{V}^d = \{(s, r, \mathcal{R}) \in V \times \mathcal{R} : s \in r\}$
- $\mathcal{E} = \mathcal{E}^p \cup \mathcal{E}^d$ where:
 - Platform arcs: $\mathcal{E}^p = \{((s, 0, \mathcal{R}), (s, r, \mathcal{R})) \in \mathcal{V}^p \times \mathcal{V}^d \text{ and } ((s, r, \mathcal{R}), (s, 0, \mathcal{R})) \in \mathcal{V}^d \times \mathcal{V}^p\}$
 - Driving arcs of ridepooling area $r \in \mathcal{R}$:
 $\mathcal{E}^r = \{((s, r, \mathcal{R}), (t, r, \mathcal{R})), ((t, r, \mathcal{R}), (s, r, \mathcal{R})) \in \mathcal{V}^d \times \mathcal{V}^d : \{s, t\} \in r\}$
 - Driving arcs: $\mathcal{E}^d = \bigcup_{r \in \mathcal{R}} \mathcal{E}^r$

Note the slight abuse of notation: A station $s \in V$ is said to be contained in a ridepooling area $r \in \mathcal{R}$ (i.e. $s \in r$), if there exists an edge $e = \{s, v\} \in E$ incident to s with $v \in r$.

The exact vehicle routes, and therefore also the exact travel times of the passengers, are not yet known, since the vehicles react spontaneously to the passenger requests that arise in practice. However, in strategic ridepooling, these operative details are disregarded. Instead, assume that for each edge $e \in E$ in the PTN, the time needed $t_e^{\text{drive-}\mathcal{R}} \geq 0$ for a ridepooling vehicle to traverse the edge is known. Furthermore, let $t^{\text{trans-}\mathcal{R}} \geq 0$ be an estimation for the time needed to transfer between two ridepooling vehicles from two different ridepooling areas. Then, the driving times of the arcs in the Change&Go network $\text{CGN} = (\mathcal{V}, \mathcal{E})$ with ridepooling are defined as follows:

$$t_e = \begin{cases} t_{E(e)}^{\text{drive-}\mathcal{R}} & \text{if } e \in \mathcal{E}^d, \\ \frac{1}{2} t^{\text{trans-}\mathcal{R}} & \text{if } e \in \mathcal{E}^p. \end{cases}$$



■ **Figure 4** Networks for Strategic Ridepooling.

Again, passengers' routes between PTN nodes $o \in V$ and $d \in V$ through the ridepooling public transport system correspond to routes in the CGN between platform nodes $(o, 0, \mathcal{R}) \in \mathcal{V}$ and $(d, 0, \mathcal{R}) \in \mathcal{V}$. The travel time along a path p is computed analogously:

$$\text{travel time} = \sum_{e \in p} t_e. \quad (2)$$

Each transfer contributes $t^{\text{trans-}\mathcal{R}}$ to the overall travel time along the path. The initial entry into the first ridepooling vehicle of the journey also comes with a waiting time $t^{\text{trans-}\mathcal{R}}$.

It should be noted that the actual routes of the vehicles are not known at this stage of the planning process. Some passengers might make small detours to allow the ridepooling vehicle to pick up other passengers whose journeys have a similar direction. This assignment of vehicles to passenger requests is part of the optimization of the operative aspects of the ridepooling service, which is not considered in strategic ridepooling. The travel times as computed in (2) are therefore only estimations, not unlike they are in the line planning case, where the timetable is not yet known.

Instead of finding a frequency for each line in a given linepool, in strategic ridepooling the task is to determine a number of ridepooling vehicles for each ridepooling area $r \in \mathcal{R}$ from a given ridepooling pool. The number of vehicles should be chosen such that it suffices to cover the expected passenger demand without leaving many vehicles unused, since the cost for each vehicle $C_{\mathcal{R}} \geq 0$ needs to be paid regardless of their utilization. Let $r \in \mathcal{R}$ be a ridepooling area and for each arc $e \in \mathcal{E}$ let $p_e \geq 0$ be the number of passengers who wish to travel along this arc. The overall required passenger travel time in the ridepooling area r can then be computed:

$$\text{travel time in area } r = \sum_{e \in \mathcal{E}^d: E(e) \in r} t_e \cdot p_e = \sum_{e \in \mathcal{E}_r \cap \mathcal{E}^d} t_{E(e)}^{\text{drive-}\mathcal{R}} \cdot p_e. \quad (3)$$

Let $\text{cap}^{(\mathcal{R})} \in \mathbb{N}$ be the capacity of a ridepooling vehicle. A vehicle assigned to a ridepooling area for the duration of the planning period T comes with T units of available driving time. Hence, for the number of vehicles v_r in the area $r \in \mathcal{R}$, it is required that:

$$\frac{1}{\text{cap}^{(\mathcal{R})}} \cdot \sum_{e \in \mathcal{E}^d: E(e) \in r} t_e \cdot p_e \leq T \cdot v_r. \quad (4)$$

With this observation, the number of vehicles can be described as linear constraints in a strategic ridepooling model analogous to the line planning model CG-L. It should be noted that the number of vehicles as given by 3 and 4 is only an estimation. In 3, the overall passenger time in the area $r \in \mathcal{R}$ consists only of the passengers' direct travel time. It is possible to include a parameter to incorporate expected detours or add time penalties for passenger pick-ups and drop-offs. Similarly, 4 assumes that vehicles can operate at full capacity at all times, although, in practice, this is rarely possible. Again, additional parameters could be introduced to incorporate this. For the remainder of this work, we use 3 and 4, but it is possible to include more parameters to take the aforementioned aspects into consideration.

Let Θ be the node-arc-incidence matrix of the Change&Go network for $\mathcal{R}$ and $b = (b^{(o)})_{o \in V}$ defined according to the OD-matrix as in CG-L. Again, the passengers are modeled as a multi-commodity flow aggregated by the origins. The travel time is minimized, while a given cost budget B must not be exceeded. The variables $\beta_{r,e} \geq 0$ for $r \in \mathcal{R}$ and $e \in \mathcal{E}_r$ represent the number of passengers who travel on this arc in the Change&Go network.

$$\begin{aligned}
\min \quad & \sum_{o \in V} \sum_{e \in \mathcal{E}} t_e \cdot x_e^{(o)} & (\text{CG-R}) \\
\text{s.t.} \quad & \sum_{o \in V} x_e^{(o)} \leq \beta_{r,e} & \forall r \in \mathcal{R}, e \in \mathcal{E}_r \cap \mathcal{E}^d \\
& \frac{1}{\text{cap}(\mathcal{R})} \sum_{e \in \mathcal{E}_r \cap \mathcal{E}^d} \beta_{r,e} \cdot t_e \leq T \cdot v_r & \forall r \in \mathcal{R} \\
& \Theta x^{(o)} = b^{(o)} & \forall o \in V \\
& \sum_{r \in \mathcal{R}} C_{\mathcal{R}} \cdot v_r \leq B \\
& x_e^{(o)}, v_r \in \mathbb{N} & \forall o \in V, e \in \mathcal{E}, r \in \mathcal{R} \\
& \beta_{r,e} \geq 0 & \forall r \in \mathcal{R}, e \in \mathcal{E}
\end{aligned}$$

The NP-completeness of CG-R can be derived from the known NP-completeness of CG-L.

5 Integrated Planning with Minimal Travel Time

For the integrated planning of lines and ridepooling with minimal travel time, we define a joint Change&Go network. To this end, let $\text{CGN}^{(\mathcal{L})} = (\mathcal{V}^{(\mathcal{L})}, \mathcal{E}^{(\mathcal{L})})$ be the Change&Go network for an instance of CG-L with underlying linepool $\mathcal{L}$ and $\text{PTN}^{(\mathcal{L})} = (V^{(\mathcal{L})}, E^{(\mathcal{L})})$. Analogously, let $\text{CGN}^{(\mathcal{R})} = (\mathcal{V}^{(\mathcal{R})}, \mathcal{E}^{(\mathcal{R})})$ be the Change&Go network for an instance of CG-R with underlying ridepooling pool $\mathcal{R}$ and $\text{PTN}^{(\mathcal{R})} = (V^{(\mathcal{R})}, E^{(\mathcal{R})})$. The public transport networks need not be disjoint, i.e., $V^{(\mathcal{L})} \cap V^{(\mathcal{R})}$ and $E^{(\mathcal{L})} \cap E^{(\mathcal{R})}$ need not be empty. Define a joint network $\text{PTN} = (V, E)$ with $V = V^{(\mathcal{L})} \cup V^{(\mathcal{R})}$ and $E = E^{(\mathcal{L})} \cup E^{(\mathcal{R})}$.

For multimodal planning problems, the joint Change&Go network should also capture the fact that intermodal transfers contribute differently to the travel time than transfers within one mode. In order to model this, define the joint Change&Go network as a two-stage network which has for every station a station node, a modality node for every modality at that station, and a line or ridepooling area node for every possible line or ridepooling area, respectively, at that station.

- $\mathcal{V} = \mathcal{V}^{(\mathcal{L})} \cup \mathcal{V}^{(\mathcal{R})} \cup \mathcal{V}^{(\mathcal{S})}$ with station nodes $\mathcal{V}^{(\mathcal{S})} = \{(s, 0, 0) : s \in V\}$,
- $\mathcal{E} = \mathcal{E}^{(\mathcal{L})} \cup \mathcal{E}^{(\mathcal{R})} \cup \mathcal{E}^{(\mathcal{S})}$ with station arcs

$$\begin{aligned}
\mathcal{E}^{(\mathcal{S})} = & \{((s, 0, 0), (s, 0, \mathcal{L})), ((s, 0, \mathcal{L}), (s, 0, 0)) : s \in V^{(\mathcal{L})}\} \\
& \{((s, 0, 0), (s, 0, \mathcal{R})), ((s, 0, \mathcal{R}), (s, 0, 0)) : s \in V^{(\mathcal{R})}\}.
\end{aligned}$$

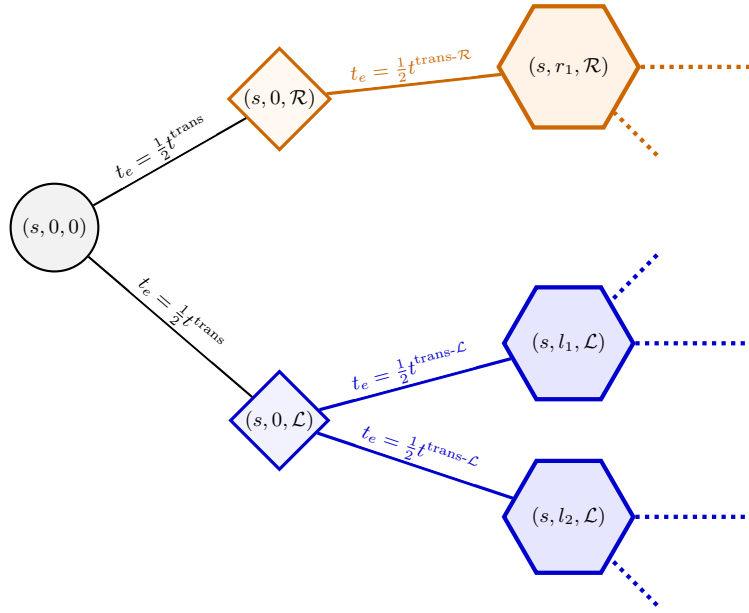
Figure 5 shows a section of joint Change&Go network with all nodes at one station $s \in V$. For the traveling times on the added station arcs, let $t^{\text{trans}} > 0$ be the penalty time for an intermodal transfer. Then, for $e \in \mathcal{E}^{(\mathcal{S})}$ define

$$t_e = \frac{1}{2} t^{\text{trans}}.$$

A passenger's route between two stations can then be represented by a path in the CGN between the two respective station nodes. A transfer between modes consists of one platform arcs from each mode and two station arcs, which contributes

$$\frac{1}{2} t^{\text{trans-}\mathcal{L}} + \frac{1}{2} t^{\text{trans-}\mathcal{R}} + 2 \cdot \frac{1}{2} t^{\text{trans}}$$

to the traveling time on the path.



■ **Figure 5** Multimodal Change&Go Network within one station $s \in V$ with one station node $(s, 0, 0)$, for the line modality a platform node $(s, 0, \mathcal{L})$ and two driving nodes $(s, l_1, \mathcal{L})$ and $(s, l_2, \mathcal{L})$, for the ridepooling modality a platform node $(s, 0, \mathcal{R})$ and a driving node $(s, r_1, \mathcal{R})$. The station node is connected to the platform nodes via station arcs, the platform nodes are connected to their respective driving nodes via platform arcs, the driving nodes are connected to driving nodes of the same line or ridepooling area at different stations. The station and platform arcs are labelled with their travel times. Note that each undirected arc in the picture represents two directed arcs, specifically a forward and a backward arc, in the CGN.

For the integrated planning of a line system and a ridepooling service in a joint Change&Go network, the two models CG-L and CG-R can be combined. The passengers form a multi-commodity flow between the station nodes are again aggregated by the origins. Line frequencies and ridepooling vehicles have to be assigned in a way that offers enough capacity on the driving arcs to accommodate the passenger flow. The overall travel time is minimized, while the cost cannot exceed a given budget B .

$$\begin{aligned}
 \min \quad & \sum_{o \in V} \sum_{e \in \mathcal{E}} t_e \cdot x_e^{(o)} & (\text{CG-L-R}) \\
 \text{s.t.} \quad & \sum_{o \in V} x_e^{(o)} \leq \text{cap}^{(\mathcal{L})} \cdot f_l & \forall l \in \mathcal{L}, e \in \mathcal{E}_l^{(\mathcal{L})} \\
 & \sum_{o \in V} x_e^{(o)} \leq \beta_{r,e} & \forall r \in \mathcal{R}, e \in \mathcal{E}_r^{(\mathcal{R})} \cap \mathcal{E}^{(\mathcal{R})d} \\
 & \frac{1}{\text{cap}^{(\mathcal{R})}} \sum_{e \in \mathcal{E}_r^{(\mathcal{R})} \cap \mathcal{E}^{(\mathcal{R})d}} \beta_{r,e} \cdot t_e \leq T \cdot v_r & \forall r \in \mathcal{R} \\
 & \Theta x^{(o)} = b^{(o)} & \forall o \in V \\
 & \sum_{l \in \mathcal{L}} C_l f_l + \sum_{r \in \mathcal{R}} C_r v_r \leq B \\
 & x_e^{(o)}, f_l, v_r \in \mathbb{N} & \forall o \in V, e \in \mathcal{E}, l \in \mathcal{L}, r \in \mathcal{R} \\
 & \beta_{r,e} \geq 0 & \forall r \in \mathcal{R}, e \in r
 \end{aligned}$$

The NP-completeness of CG-L-R follows from the NP-completeness of CG-L and CG-R.

There are possible variations of this model CG-L-R: For instance, there could be separate budgets $B^{(\mathcal{L})}$ or $B^{(\mathcal{R})}$ for each of the modalities. The (intra- and intermodal) transfer times could vary between different stations, since often the size and layout of a station can impact the amount of time needed to transfer. Furthermore, while the basic model only has one line mode and one ridepooling mode, multiple modes of each kind are also possible. The joint Change&Go network can be extended to consist of not only two but more mode-specific sub-networks, each connected to the station nodes via station arcs at their platform nodes.

In [3], a model for strategic ridepooling and its integration with line planning was introduced. For a fixed passenger routing in the PTN (V, E) , given as edge loads $p_e \in \mathbb{N}$ for all $e \in E$. The so-called *vehicle frequencies* $\alpha_{r,e} \geq 0$ for $r \in \mathcal{R}$ and $e \in r$ are an estimation for the number of times a vehicle assigned to the ridepooling area r traverses the edge e within the planning period T . For more detailed explanation of their meaning and a procedure for determining these constants, we refer the reader to [3].

$$\begin{aligned}
\min \quad & \sum_{l \in \mathcal{L}} C_l \cdot f_l + \sum_{r \in \mathcal{R}} C_{\mathcal{R}} \cdot v_r & (\text{Cost-L-R}) \\
\text{s.t.} \quad & p_e \leq \sum_{\substack{l \in \mathcal{L}: \\ e \in l}} \text{cap}^{(\mathcal{L})} \cdot f_l + \sum_{\substack{r \in \mathcal{R}: \\ e \in r}} \text{cap}^{(\mathcal{R})} \cdot \alpha_{r,e} \cdot v_r & \forall e \in E \\
& f_l, v_r \in \mathbb{N} & \forall l \in \mathcal{L}, r \in \mathcal{R}.
\end{aligned}$$

We assume that the edge loads have been obtained from an OD-matrix $\mathcal{OD}$ by routing the passengers through the PTN (V, E) , and that the vehicle frequencies $\alpha_{r,e}$ for $r \in \mathcal{R}$ and $e \in E$ have been chosen as proposed in [3], i.e.,

$$\alpha_{r,e} = N_e \cdot \frac{T}{\sum_{e' \in E} N_{e'} t_{e'}}$$

where $L_e = \left\lceil \frac{p_e}{\text{cap}^{(\mathcal{R})}} \right\rceil$ and

$$N_e = \begin{cases} L_e & \text{if } L_e \text{ is even,} \\ L_e + 1 & \text{if } L_e \text{ is odd.} \end{cases}$$

We can show that, for suitable budget B , feasible solutions of Cost-L-R yield feasible solutions for CG-L-R.

► **Theorem 1** (Feasible solutions of Cost-L-R and CG-L-R). *Let (f^*, v^*) be a feasible solution of Cost-L-R and $B \geq \sum_{l \in \mathcal{L}} C_l f_l^* + \sum_{r \in \mathcal{R}} C_{\mathcal{R}} v_r^*$. Then there exists a feasible solution for CG-L-R with the same frequencies f^* and numbers of ridepooling vehicles v^* .*

For a formal proof, see Appendix A.

6 Numerical Experiments

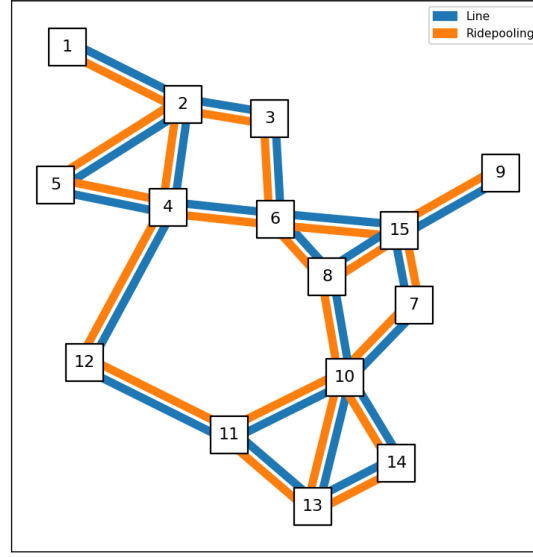
The model CG-L-R has been implemented in the open-source software toolbox LINTIM [13]. In the following, we present results from computational experiments. All IP formulations were solved in Gurobi 12.0.0 [6].

In this example, we use the small academic *mandl* graph [9], depicted in Figure 6, with a line modality and ridepooling allowed at all stations and on all edges. The linepool $\mathcal{L}$ is constructed as follows: First, for each pair of nodes a shortest path is computed. Then, all shortest paths which are contained in another shortest path are removed from the set

of paths. The remaining paths form the linepool. We assume that the cost of a line are computed as follows:

$$C_l = C_{\mathcal{L}}^{\text{fixed}} + \sum_{e \in l} C_{\mathcal{L}}^{\text{length}} \cdot t_e^{\text{drive-}\mathcal{L}}.$$

The ridepooling pool $\mathcal{R}$ contains one ridepooling area for each node, which consists of all edges incident to the node. Properties of the pools as well as cost parameters can be found in Table 1.



■ **Figure 6** PTN of the *mandl* graph.

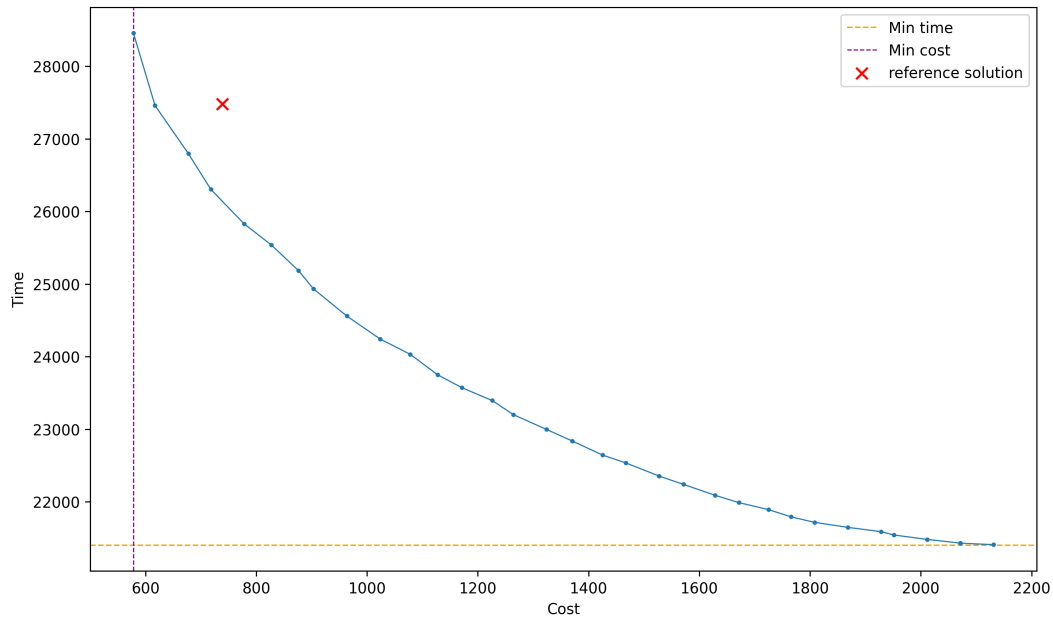
■ **Table 1** Input parameters and properties of the line- and ridepooling pools.

$C_{\mathcal{L}}^{\text{fixed}}$	$C_{\mathcal{L}}^{\text{length}}$	$C_{\mathcal{R}}$	$\text{cap}^{(\mathcal{L})}$	$\text{cap}^{(\mathcal{R})}$	$ \mathcal{L} $	$ \mathcal{R} $
8	4	60	50	6	22	15

■ **Table 2** Transfer times.

$t^{\text{trans-}\mathcal{L}}$	$t^{\text{trans-}\mathcal{R}}$	t^{trans}
8	4	2

For the passenger demand for each pair of nodes $v_1, v_2 \in V$ with $v_1 \neq v_2$ we have 5 passengers who wish to travel from v_1 to v_2 . The driving times on the edges differ between the two modalities and have been chosen such that the line modality takes about 1.2 times as long as the ridepooling modality. The platform and station times for each modality can be seen in Table 2. This means that while the line vehicles are cheaper and therefore preferable in terms of minimizing the operating cost, the ridepooling vehicles come with better traveling times for the passengers.

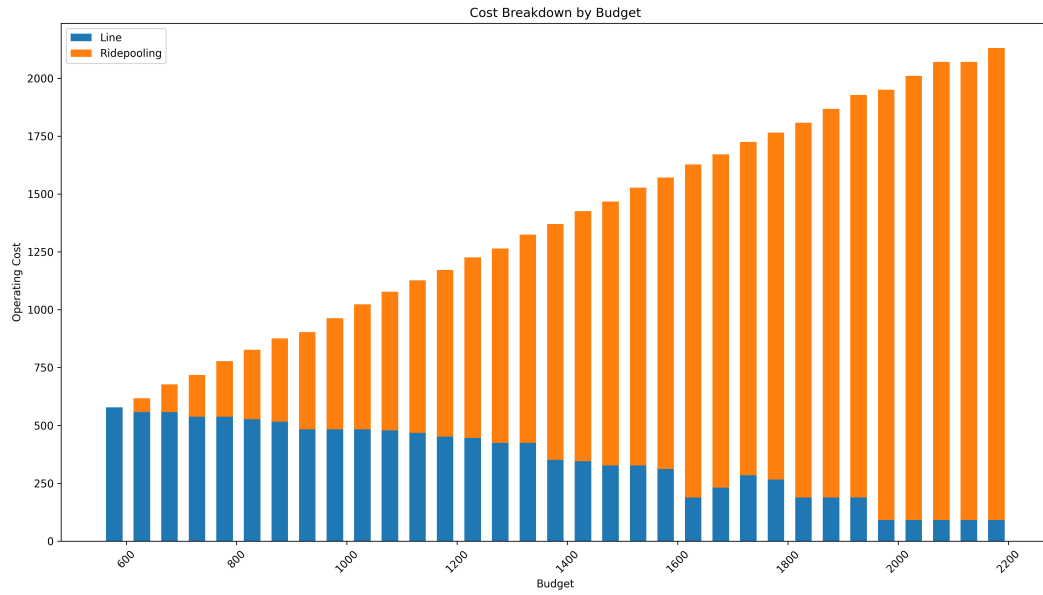


■ **Figure 7** Pareto front of CG-L-R with the optimal solution of Cost-L-R as the reference solution.

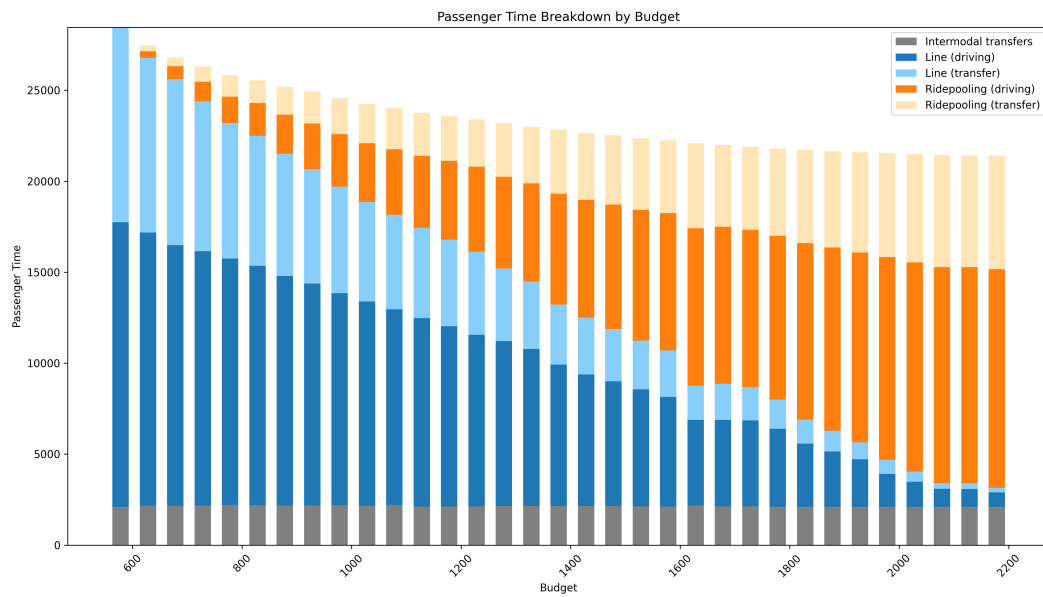
Both the cost and the travel time can be used as an objective function of CG-L-R. It can even be rewritten as a multi-objective optimization problem with the two objective functions of operating cost and travel time. By varying the budget in CG-L-R, we can compute a set of pareto-optimal solutions. All solutions have an optimality gap of at most 0.1%. The resulting pareto front is depicted in Figure 7. The reference solution is the optimal solution of Cost-L-R, where the passengers have been routed on shortest paths in the PTN, which yields a feasible solution for CG-L-R by Theorem 1, which is dominated by many of the optimal solution of Cost-L-R. The solution of Cost-L-R is even dominated in terms of cost. Since the routing is determined beforehand, without taking the cost into account, it is possible that for some passengers there exist alternative paths with equal travel time. Redistributing passengers between different paths of equal travel time can lead to a reduction in operating cost. Since ridepooling offers faster travel times for the passengers, but the line modality has lower costs, the low-budget solutions tend to have a higher portion of line-based transportation, while the size of the ridepooling portion of solutions increases for higher budgets, leading to lower travel times. Figure 8 shows how the two modalities contribute to the objective functions.

7 Conclusion and Further Research

In this paper, we presented a new optimization model for the integrated planning of line-based public transport and strategic ridepooling with the objective of minimizing passengers' travel times. Building on existing approaches to line planning and strategic ridepooling, we introduced new formulations for both planning problems that explicitly incorporate passenger routing through Change&Go networks. These formulations were then combined into a unified multimodal optimization model that simultaneously determines line frequencies, ridepooling capacities, and passenger routes while respecting a given operating budget. By integrating passenger routing directly into the planning process, the proposed model is able to account



(a) Cost of solutions for CG-L-R.



(b) Travel time of solutions for CG-L-R.

Figure 8 Objective function values of the pareto solutions of CG-L-R from Figure 7 split by modality with the corresponding budget at the x-axis.

for transfer times, different travel speeds of the transportation modes, and the interaction between line-based services and demand-responsive ridepooling. This leads to solutions that are better aligned with the passengers' perspective than approaches relying on predetermined routings or minimizing operating costs alone.

The proposed model provides a first step towards a comprehensive strategic planning model for multimodal public transportation systems that jointly optimizes fixed-line and demand-responsive services from a travel-time perspective. The computational results

demonstrate the applicability of the approach and illustrate how the integrated optimization can exploit the strengths of both transportation modes to improve service quality under limited resources.

More experiments on larger and even real-life instances are part of current ongoing research. On the one hand, more model reduction techniques could be investigated: for example, the flow of passengers starting at origin o is limited to the arcs in the Change&Go network that are reachable from o . Precomputing these subnetworks reduces the number of variables further. However, at some point, these reduction techniques alone will not suffice any more. For such instances, decomposition approaches often offer promising alternatives to common IP solvers. Heuristic approaches could also be developed, providing feasible, and ideally also good solutions which could also be used as warm-starts for other algorithms aiming to solve instances to optimality, which can often improve both runtime and solution quality.

Of course, modes of transportation such as walking or cycling, which do not require any planning or operation by a public transport provider, are also an important part of a public transportation system, since passengers can also use them for parts of their journeys. Hence, they should also be considered in the planning process. We are working on also incorporating such modes into our multimodal planning models. Furthermore, the planning process of a public transport system does not end after the line planning stage: a timetable must be determined in a next step. It is an ongoing research question how to include ridepooling from a strategic perspective into the timetabling stage of the planning process.

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A Bonus Material

Proof of Theorem 1. The budget constraint is satisfied by the assumptions about the budget.

Define for $r \in \mathcal{R}$ and $e \in \mathcal{E}_r^{(\mathcal{R})}$ define $\beta_{r,e} = \alpha_{r,E(e)} \cdot \text{cap}^{(\mathcal{R})} \cdot v_r$. Then:

$$\begin{aligned}
 \sum_{e \in \mathcal{E}_r^{(\mathcal{R})}} \beta_{r,e} t_e &= \sum_{e \in \mathcal{E}_r^{(\mathcal{R})}} \alpha_{r,E(e)} \text{cap}^{(\mathcal{R})} v_r t_{E(e)} \\
 &= \text{cap}^{(\mathcal{R})} v_r \sum_{e \in \mathcal{E}_r^{(\mathcal{R})}} N_{E(e)} \frac{T}{\sum_{e' \in \mathcal{E}_r^{(\mathcal{R})}} N_{E(e')} t_{E(e')}} t_{E(e)} \\
 &= \text{cap}^{(\mathcal{R})} v_r T \frac{\sum_{e \in \mathcal{E}_r^{(\mathcal{R})}} N_{E(e)} t_{E(e)}}{\sum_{e' \in \mathcal{E}_r^{(\mathcal{R})}} N_{E(e')} t_{E(e')}} \\
 &= \text{cap}^{(\mathcal{R})} v_r T
 \end{aligned}$$

This shows the ridepooling vehicle constraint.

It remains to show that there exists a feasible passenger flow $x = (x_e^{(o)}) \in \mathbb{N}^{\mathcal{E} \times V}$. For $\bar{e} \in E$ a PTN-edge, define $\mathcal{E}(\bar{e}) = \{e \in \mathcal{E} : E(e) = \bar{e}\}$ the set of corresponding driving arcs in the Change&Go network. Since the traffic loads have been obtained by routing the demand given by $\mathcal{OD}$ in the PTN, we know that there exists a passenger flow in the CGN with:

$$\forall o \in V : \Theta x^{(o)} = b^{(o)} \quad (\text{OD-constraint})$$

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and for all $\bar{e} \in E$

$$p_{\bar{e}} = \sum_{o \in V} \sum_{e \in \mathcal{E}(\bar{e})} x_e^{(o)} \leq \sum_{\substack{l \in \mathcal{L}: \\ \bar{e} \in l}} \text{cap}^{(\mathcal{L})} f_l + \sum_{\substack{r \in \mathcal{R}: \\ \bar{e} \in r}} \text{cap}^{(\mathcal{R})} \alpha_{r, \bar{e}} v_r \quad (\text{Cap})$$

Since all transfer arcs in the CGN have unlimited capacity, the flow can be redistributed between parallel driving arcs $\mathcal{E}(\bar{e})$ for the same PTN edge $\bar{e} \in E$ without violating the flow constraints OD-constraint. It follows that there exists a passenger flow $x \in \mathbb{N}^{\mathcal{E} \times V}$ which satisfies OD-constraint, Cap, and, by redistributing between parallel driving arcs, also the following additional capacity constraints:

$$\forall l \in \mathcal{L}, e \in \mathcal{E}_l^{(\mathcal{L})} : \sum_{o \in V} x_e^{(o)} \leq \text{cap}^{(\mathcal{L})} \cdot f_l \quad (\text{Cap-}\mathcal{L})$$

$$\forall r \in \mathcal{R}, e \in \mathcal{E}_r^{(\mathcal{R})} : \sum_{o \in V} x_e^{(o)} \leq \alpha_{r, E(e)} \text{cap}^{(\mathcal{R})} v_r = \beta_{r, e} \quad (\text{Cap-}\mathcal{R})$$

This implies the claim. ◀