

A Multimodal Framework for Transport Planning

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Abstract

Planning high-quality public transport systems requires taking multiple modalities of transportation into account simultaneously. This includes public transport such as trains or buses, but also ridepooling, walking or biking. While planning of public transport lines and timetables has been studied extensively for unimodal systems, research on settings which include more than one modality is just at its beginning. In this paper, we categorize the modalities as line-based, ridepooling, and nonscheduled for which we develop a framework for multimodal planning. This can be applied for different line-based modalities, e.g., for planning bus and tram lines simultaneously, but also for modalities from different categories, e.g., for planning ridepooling together with bus lines while taking footpaths into account. Technically, we extend classic graph structures from unimodal public transport optimization to the multimodal case and show how established solution methods for the unimodal case carry over to the multimodal setting. In particular, we formulate multimodal line planning and periodic timetabling problems. We demonstrate the practical applicability of the framework through a case study on real-world data from Helsinki, Finland, considering five line-based modalities and walking. Our experiments show that incorporating multimodal aspects into the planning process consistently reduces average passenger travel time.

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1 Motivation

High-quality public transport systems are of great significance in a world that is striving towards carbon neutrality and improved access within and across regions. To enable passengers to replace journeys usually undertaken with individual transport in private vehicles,

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convenient and practical alternatives are necessary. One single modality, e.g., regional trains, is not enough to motivate passengers to use public transport since the first and last mile is not covered. In real-world scenarios, this is taken into account: public transport typically does not consist of only one modality. Besides line-based modalities, e.g., buses and trams or underground trains within a city, other means of transport such as walking, cycling or taxi rides are common. A ridepooling service can also be part of a passenger's journey.

However, so far, the planning process in public transport has mostly been focused on one single modality, typically, on buses or on trains. It is usually decomposed into its stages that are followed sequentially: For a given demand for this modality, first the stops are located, then the lines and their frequencies are determined, followed by timetabling, vehicle and crew scheduling, see, e.g., [5] for an early definition of the stages and [22, 24] for the sequential process.

Multimodal planning is hard since the different modalities are linked by two major aspects: First, passengers are not restricted when choosing which mode of transportation to use, i.e., a single passenger's journey can consist of different legs, each with a different modality. The resulting modal split thus depends on the attractiveness of each modality and how well they are coordinated. Second, different modalities may partially share the same infrastructure like streets or tracks, imposing further capacity restrictions on their combined traffic volume. Detecting shared infrastructure capacity is a subtle issue: two modalities may appear independent, while in fact competing for the same underlying infrastructure. We refer to Figure 1, discussed in more detail later, for an illustration.

While literature on public transport planning is at least 50 years old [4], multimodal planning has just recently become of interest. In [3], frequency setting in the context of multimodal line planning has been considered. Intermodal timetabling including flight services has been studied in [13]. Models for integrating line planning and strategic ridepooling have been provided in [8]. A multi-objective analysis for different transport modalities on one single edge is given in [27].

In freight transport, multimodal planning is more common, see the survey [9]. Note that for *evaluating* the passengers' travel times in an existing multimodal transport supply, *multimodal routing* is needed. This has been extensively researched. We refer to [1, 2, 6, 10] and references therein. Multimodal equilibria for public transport including ridepooling have been tackled in [16]. The authors of [21] consider a fixed multimodal system and plan the operation of an on-demand service optimizing different justice metrics.

In this paper, we propose a general framework to extend the traditional unimodal sequential approach to multimodal planning, where we can decide for each planning step if it is performed for each modality in a unimodal way, or if it is performed for multiple modalities at the same time. The data structures are developed so that known algorithms for unimodal planning can be applied to the multimodal case, in particular for line planning and timetabling. Our contribution is the following:

- We propose data structures and a framework for multimodal planning following the sequential approach.
- We show that the framework covers many problems with practical relevance as special cases.
- In our case study we show that multimodal planning improves public transport supplies.

The remainder of this paper is structured as follows. First, we introduce different categories of transport modalities in Section 2, grouping modalities with similar properties together, and differentiate the categories by outlining the key aspects that set them apart.

In Section 3 we introduce some known graph-structures commonly used in unimodal public transport optimization and extend them to the multimodal case. Section 4 gives some examples of possible applications for the multimodal models proposed in this work. We demonstrate in Section 4.1 by a case study using data from the city of Helsinki, Finland, that multimodal planning methods significantly enhance the quality of the public transport system. Finally, we conclude in Section 5.

2 Categories of Transport Modalities

Clearly, not all modes of transportation function in the same way: Bus or train systems have fixed timetables, while walking, cycling or ridepooling are not scheduled beforehand. Walking or cycling do not require action of a public transport provider, while line-based or ridepooling systems need to be planned, set in place and operated. To model different types of transport modalities, we propose three different *modality categories*, grouping modalities with similar properties together.

- **Line-based:** In a line-based transport system such as buses, trains, trams, or metros, vehicles travel along fixed lines. The departure and arrival times are determined by a timetable, which is planned in advance. A line-based system is operated according to the timetable, regardless of the actual demand.
- **Ridepooling:** Ridepooling works as follows: A provider has a fleet of vehicles. The passengers request trips, i.e., to be taken from an origin to a destination at a specified time (window). If the request is accepted, the ridepooling service provider sends a vehicle to pick up the passenger and drop them off at their destination. Passengers' trips can be pooled, meaning that different trips with a similar direction can be combined and served by the same vehicle. This can lead to small detours in the passengers' journeys, but also reductions in the overall necessary number of vehicles and driving distance. The vehicle routes depend on the passenger demand that takes place at a specific day. The planning decisions for ridepooling services are the areas where it is offered and the number of vehicles operating in each area. It is on the same level as the line planning step for line-based modalities, as it determines the rough layout of the system. This is known as *Strategic Ridepooling* and was introduced in [8]. The operative aspects, such as the routes of vehicles and their arrival and departure times, depend on the actual demand.
- **Nonscheduled:** Means of transport such as walking, cycling or the use of private cars are not scheduled and do not require any action by a provider during the operational phase. Passengers can travel using a nonscheduled modality, wherever the underlying road system allows it. The planning problem determines where footpaths or bike lanes should be built, see, e.g., [12] for bicycle network design. The availability of such modalities is independent of passenger demand or utilization and there are no capacity constraints.

The networks to be described in the next section can cover all three categories of modalities. There are two ways these modalities can be included in multimodal planning:

- (a) The modality can be assumed to be given and fixed, and hence (only) used for passenger routing.
- (b) The modality can be designed/planned within the optimization.

Planning a modality has mostly been researched for line-based modalities (e.g., bus line planning, train timetabling), and is at its start for ridepooling. It is more common to assume different types of modalities as given and fixed and include them when routing the passengers. In the case study in Section 4.1 we plan five different line-based modalities and use one fixed nonscheduled modality (walking) for determining optimal passenger routes.

3 Networks and Planning Problems

In this section, we develop the networks needed for the different planning stages of a multimodal public transport system. While theoretically all planning stages can be included in our framework, we concentrate on line planning and timetabling here. We start by introducing the *infrastructure network*, which contains information on the underlying infrastructure, its capacity, and the modalities that can use it. Additionally, we assume that the *public transport networks* for each of the single modalities are given. They are the basis for unimodal public transport planning in the literature. To detect and account for (partially) shared infrastructure between different modalities or between edges of the same modality, a relation between the infrastructure network and the public transport networks must be established. We also define one common *multimodal public transport network*, allowing passengers to transfer from one modality to another at common stops.

We proceed with the *change&go network* for line planning (see [25]) and the *event-activity-network* for timetabling (see [26]). Both are defined in their unimodal version per modality and in their multimodal version for joint planning of all modalities. As research on strategic ridepooling planning is still in its early stages, these two graph structures are presented for only line-based and nonscheduled modalities in this work, though a general extension of the change&go network to ridepooling modalities is possible and demonstrated in [7].

3.1 Infrastructure Network

To capture shared infrastructure capacity between different modalities, we first define the *infrastructure network* as the most basic underlying graph structure. It contains information on the capacity of the infrastructure segments which all modalities must respect jointly.

► **Definition 3.1** (Infrastructure Network). Let $ISN = (N, \Lambda)$ be an undirected (multi-)graph representing the infrastructure. The vertices of the graph $\nu \in N$ are called *nodes*. They can represent crossings and stops or stations. The edges $\lambda \in \Lambda$ are called *links* and are given with

- $\text{length}(\lambda)$: the physical length of the link,
- $\text{capacity}(\lambda)$: the maximal capacity of the link, i.e., the maximal amount of traffic that can be sent over it during one period.

Let $\mathcal{M}$ be a set of (line-based, ridepooling, and nonscheduled) modalities operating in a given infrastructure network $ISN = (N, \Lambda)$. We denote by $\mathcal{M}(\nu)$ the set of modalities, for which the node ν is a *stop*, i.e., an entry point for passengers to this modality. For a line-based or ridepooling modality, these are the stops or stations where the vehicles halt and passengers can board and alight, and for a nonscheduled modality, every infrastructure node reachable with this modality is also called a stop. Similarly, $\mathcal{M}(\lambda)$ denotes the modalities that can use the infrastructure link λ .

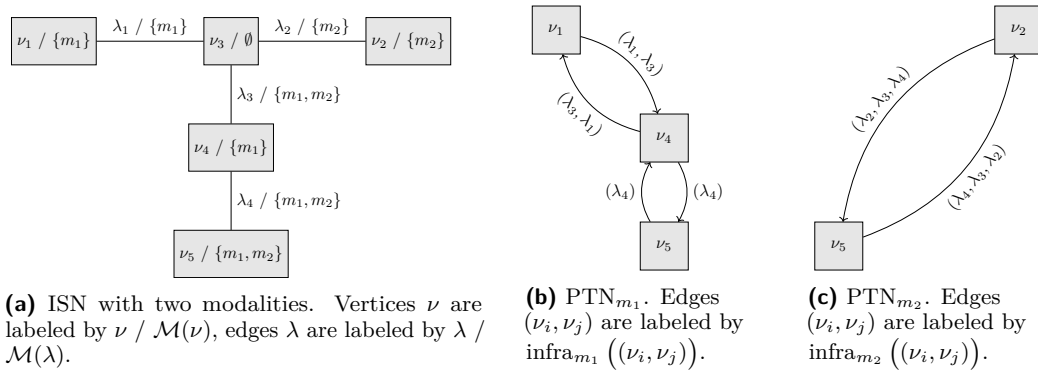
Suppose that we want to plan a public transport system in ISN with modalities $\mathcal{M}$ for a time period $T > 0$, e.g., T could represent one hour. The passenger demand in the network is represented by an *OD matrix* $\mathcal{OD} \in \mathbb{N}^{|N| \times |N|}$ where for a pair of nodes $\nu_i, \nu_j \in N$ the entry $\mathcal{OD}_{\nu_i, \nu_j}$ represents the number of passengers who wish to travel from ν_i to ν_j with any of the modalities from $\mathcal{M}$ in the planning period T . For every OD pair (ν_i, ν_j) with $\mathcal{OD}_{\nu_i, \nu_j} > 0$ we assume that $\mathcal{M}(\nu_i) \neq \emptyset$ and $\mathcal{M}(\nu_j) \neq \emptyset$, otherwise the passengers have no possibility to start or end their journey with any modality at the desired start or end node.

3.2 Public Transport Network

For planning a single modality, not all information from the infrastructure network is relevant: All parts of the underlying infrastructure network that are inaccessible to this modality need not be considered, e.g., bike lanes cannot be used by a bus. For nonscheduled modalities, all parts of the infrastructure network that cannot be reached using this modality can be omitted. For instance, if some infrastructure links represent a track system, then they are not relevant for passengers traveling on foot. To obtain a more suitable abstraction, one commonly relies on the public transport network, as established in the literature.

► **Definition 3.2** (Public Transport Network). For a modality $m \in \mathcal{M}$ the *public transport network* (PTN) of modality m is the directed simple graph $\text{PTN}_m = (V_m, E_m)$, where V_m contains all stops of modality m and E_m are all direct connections between them.

To connect the information from the infrastructure network to the public transport network, we assume that $V_m = \{\nu \in N : m \in \mathcal{M}(\nu)\} \subseteq N$. Furthermore, an edge $e = (\nu_1, \nu_2) \in E_m$ in the public transport network represents, that there is a direct connection from ν_1 to ν_2 in the infrastructure network without passing another stop of this modality. Therefore, we assume that together with the $\text{PTN}_m = (V_m, E_m)$, we are given a map $\text{infra}_m : E_m \rightarrow \Pi_{\text{ISN}}$, mapping each PTN edge $e = (\nu_1, \nu_2)$ to a path P in the infrastructure network starting at ν_1 and ending at ν_2 while $m \in \mathcal{M}(\lambda)$ holds for all $\lambda \in P$.



■ **Figure 1** An infrastructure network and corresponding public transport networks.

While the infrastructure network provides much more detailed information about the specific layout of the underlying road or track network, the public transport network of a modality omits irrelevant details, capturing only the information that is relevant to this modality. However, these details are important to correctly model infrastructure with limited capacity shared between different modalities. As an example, consider the ISN shown in Figure 1a with the two modalities m_1, m_2 . Figures 1b and 1c show the PTNs of the two modalities together with their underlying infrastructure paths. When focusing on the PTN graphs, one could overlook that m_1 and m_2 both use the same infrastructure link λ_4 between ν_4 and ν_5 . The maximal amount of traffic along λ_4 during one period is not allowed to exceed $\text{capacity}(\lambda_4)$ which can be expressed as

$$w_{m_1} \cdot Z_{m_1} + w_{m_2} \cdot Z_{m_2} \leq \text{capacity}(\lambda_4), \quad (1)$$

where Z_m denotes the number of units (e.g. vehicles, trains, bikes or pedestrians) of modality m during one period and w_m is the *infrastructure capacity weight* of modality m , expressing

how much traffic one unit of this modality produces. For nonscheduled modalities like walking or biking, the infrastructure capacity weight can be set to zero, but for line-based modalities like trains, where a certain safety distance between consecutive vehicles is required, it is important to respect the shared capacity.

For each edge $e \in E_m$ in the PTN_m , the amount of time necessary to traverse the edge using modality $m \in \mathcal{M}$ can be determined using the edge length as given by the lengths of the ISN links of the underlying infrastructure path $\text{infra}_m(e)$ and the speed of the modality.

► **Definition 3.3 (PTN Travel Times).** For each modality $m \in \mathcal{M}$ let $\text{speed}(m)$ be the speed at which passengers travel with this modality. Then, for $e \in E_m$ define its travel time by

$$t_e := \frac{1}{\text{speed}(m)} \sum_{\lambda \in \text{infra}_m(e)} \text{length}(\lambda).$$

If m is a ridepooling modality, the speed should be adjusted by the average detour factor, reflecting that vehicles rarely travel the shortest path when multiple trips are pooled. Note that one could also use edge-specific speed values or even travel times derived from empirical data rather than physical length.

For the multimodal case on the PTN-level, we define the multimodal public transport network by stacking the unimodal public transport networks.

► **Definition 3.4 (Multimodal Public Transport Network).** For an infrastructure network $\text{ISN} = (N, \Lambda)$ with modalities $\mathcal{M}$ define the *multimodal public transport network* as a directed simple graph $\text{MPTN} = (V, E)$ with

- $V := \{(\nu, 0) : \nu \in N\} \cup \left(\bigcup_{m \in \mathcal{M}} \{(\nu, m) : \nu \in V_m\}\right)$, and
- $E := E^{\text{trans}} \cup \left(\bigcup_{m \in \mathcal{M}} \{((\nu_1, m), (\nu_2, m)) : (\nu_1, \nu_2) \in E_m\}\right)$,
where $E^{\text{trans}} := \{((\nu, 0), (\nu, m)), ((\nu, m), (\nu, 0)) : \nu \in V_m\}$.

For each modality there are the nodes from its public transport network, and, additionally, there are transfer nodes at each infrastructure network node. Analogously, all modality-specific public transport edges are also in the MPTN, and all public transport network nodes are connected to the transfer node corresponding to the same infrastructure network nodes via transfer edges in both forward and backward directions.

The multimodal public transport network represents each modality through its public transport network, and, additionally, all possible transfer points between modalities. The travel times on the edges from the modality-specific public transport networks PTN_m are the same. For the transfer edges let t^{trans} be a time penalty for each transfer between different modalities and for $e \in E^{\text{trans}}$ define

$$t_e := \frac{1}{2} t^{\text{trans}},$$

since a passenger has to use two transfer edges to transfer between two different modalities. A path between two stations $\nu_1, \nu_2 \in N$ can be represented as a path between the two corresponding transfer nodes $(\nu_1, 0)$ and $(\nu_2, 0)$. The travel time of a passenger path can be computed as the sum of travel times of its edges. Note that the time of such a path then includes the transfer penalty t^{trans} one more time than there are transfers in the path, since the path begins and ends with a transfer edge. However, this is a constant which can be subtracted to get the actual passenger travel time.

3.3 Change&Go Network

For passengers traveling in a line-based transport system, not only transfers between different modalities are relevant, but also transfers within the same modality, since they are time consuming and often seen as inconvenient. In the following, denote by $\mathcal{M}^{\mathcal{L}} \subseteq \mathcal{M}$ the set of line-based modalities. A line l of a line-based modality $m \in \mathcal{M}^{\mathcal{L}}$ is a path in the modality's public transport network PTN_m . For each line-based modality $m \in \mathcal{M}^{\mathcal{L}}$ let $\mathcal{L}_m$ be a line pool, i.e., a set of possible lines of this modality where some of them will be chosen to be operated together with a frequency per period.

The *change&go network* was introduced by [25] and can be used to solve the line planning problem with the objective of minimizing the travel time or for routing passengers with respect to fixed line frequencies. Figures 2a and 2b visualize the construction process for a line pool with three lines.

► **Definition 3.5** (Change&Go Network). For a line-based modality $m \in \mathcal{M}^{\mathcal{L}}$ with $\text{PTN}_m = (V_m, E_m)$ and a line pool $\mathcal{L}_m$, define the directed simple graph $\text{CGN}_m = (\mathfrak{N}_m, \mathfrak{E}_m)$ as the *change&go network* of modality m with line pool $\mathcal{L}_m$.

Each stop from the PTN_m is represented by a platform node in the CGN_m . For each line $l \in \mathcal{L}_m$ and each stop $\nu \in V_m$ contained in the line, the CGN_m has a corresponding driving node, i.e., $\mathfrak{N}_m := \mathfrak{N}_m^p \cup \mathfrak{N}_m^d$ with

- platform nodes $\mathfrak{N}_m^p := \{(\nu, 0, m) : \nu \in V_m\}$ and
- driving nodes $\mathfrak{N}_m^d := \{(\nu, l, m) : l \in \mathcal{L}_m, \nu \in l\}$.

There are two different types of arcs in the CGN_m : Driving arcs connecting consecutive driving nodes of the same line and platform arcs connecting driving nodes to the platform node that corresponds to the same stop $\nu \in V_m$. Formally, let $\mathfrak{E}_m := \mathfrak{E}_m^p \cup \mathfrak{E}_m^d$ with

- platform arcs $\mathfrak{E}_m^p := \{((\nu, 0, m), (\nu, l, m)), ((\nu, l, m), (\nu, 0, m)) : l \in \mathcal{L}_m, \nu \in l\}$ and
- driving arcs $\mathfrak{E}_m^d := \bigcup_{l \in \mathcal{L}_m} \mathfrak{E}_m^l$, where
- $\mathfrak{E}_m^l := \{((\nu_1, l, m), (\nu_2, l, m)) : (\nu_1, \nu_2) \in l\}$ are the driving arcs of every line $l \in \mathcal{L}_m$.

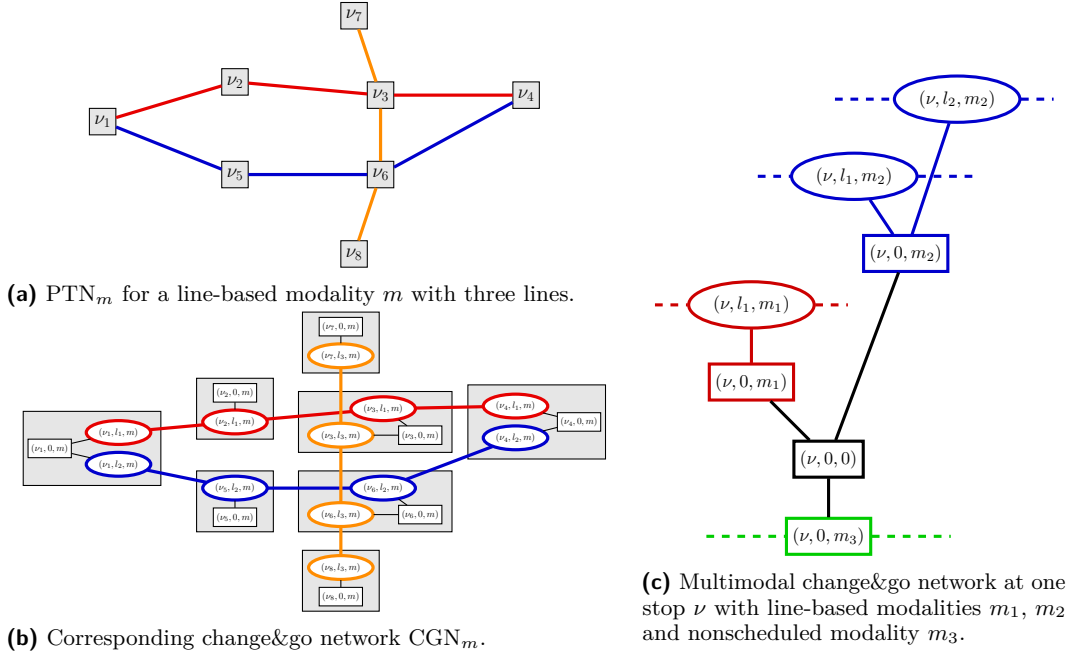
Multimodal Change&Go Network

In the multimodal case, transfers between modalities also need to be taken into account. Analogously to the multimodal public transport network for a set of unimodal public transport networks, we can define a multimodal change&go network for a set of unimodal change&go networks by stacking them as shown in Figure 2c. For the nonscheduled modalities we do not have a change&go network, since there are no transfers within a nonscheduled modality. Hence, they can still be represented by their public transport network instead. For ridepooling modalities we use the areas where it is offered instead of the lines in a line-based modality. In the following, we restrict ourselves to line-based and nonscheduled modalities, though the presented framework is capable of incorporating ridepooling as well. The multimodal change&go graph with ridepooling modalities is presented in [7].

► **Definition 3.6** (Multimodal Change&Go Network). For an infrastructure network $\text{ISN} = (N, \Lambda)$ with modalities $\mathcal{M}$, each with a $\text{PTN}_m = (V_m, E_m)$, and for each line-based modality $m \in \mathcal{M}^{\mathcal{L}}$ a line pool $\mathcal{L}_m$ and $\text{CGN}_m = (\mathfrak{N}_m, \mathfrak{E}_m)$, we define the *multimodal change&go network* $\text{MCGN} = (\mathfrak{N}, \mathfrak{E})$.

We use the nodes $\mathfrak{N}_m$ from the unimodal change&go networks for the line-based modalities. Additionally, for each infrastructure node $\nu \in N$, define a station node $(\nu, 0, 0)$. Each stop $\nu \in V_m$ of a nonscheduled modality $m \in \mathcal{M} \setminus \mathcal{M}^{\mathcal{L}}$ is represented by a nonscheduled platform node $(\nu, 0, m)$. Formally, define $\mathfrak{N} := (\bigcup_{m \in \mathcal{M}^{\mathcal{L}}} \mathfrak{N}_m) \cup \mathfrak{N}^N \cup (\bigcup_{m \in \mathcal{M} \setminus \mathcal{M}^{\mathcal{L}}} \mathfrak{N}_m)$ with

- station nodes $\mathfrak{N}^N := \{(\nu, 0, 0) : \nu \in N\}$ and
 - nonscheduled platform nodes $\mathfrak{N}_m = \{(\nu, 0, m) : \nu \in V_m\}$ for $m \in \mathcal{M} \setminus \mathcal{M}^{\mathcal{L}}$.
- To allow for transfers between different modalities, we introduce station arcs, connecting each platform node (line-based or nonscheduled) to their respective station node. Furthermore, for each edge $e = (\nu_1, \nu_2) \in E_m$ of a nonscheduled modality $m \in \mathcal{M} \setminus \mathcal{M}^{\mathcal{L}}$, we introduce an arc in the MCGN connecting the two nonscheduled platform nodes. Define $\mathfrak{E} := (\bigcup_{m \in \mathcal{M}^{\mathcal{L}}} \mathfrak{E}_m) \cup \mathfrak{E}^N \cup (\bigcup_{m \in \mathcal{M} \setminus \mathcal{M}^{\mathcal{L}}} \mathfrak{E}_m)$ with
- station arcs $\mathfrak{E}^N := \{((\nu, 0, 0), (\nu, 0, m)), ((\nu, 0, m), (\nu, 0, 0)) : \nu \in N, m \in \mathcal{M}(\nu)\}$ and
 - nonscheduled arcs $\mathfrak{E}_m := \{((\nu_1, 0, m), (\nu_2, 0, m)) : (\nu_1, \nu_2) \in E_m\}$ for $m \in \mathcal{M} \setminus \mathcal{M}^{\mathcal{L}}$.



■ **Figure 2** Construction of a unimodal change&go network (left) and a multimodal change&go network (right). Each edge is directed and exists in both orientations.

Travel Times

In a unimodal change&go network, a passenger's route between two stops $\nu_1, \nu_2 \in V_m$ is represented by a path connecting the corresponding platform nodes $(\nu_1, 0, m), (\nu_2, 0, m) \in \mathfrak{N}_m$. It consists of both driving and platform edges, providing information not only about the route in the underlying road or track network, but also about which lines are used and where the passenger transfers between lines.

In a multimodal change&go network, passenger paths connect the corresponding station nodes $(\nu_1, 0, 0), (\nu_2, 0, 0) \in \mathfrak{N}^N$ and provide the additional information of transfers between different modalities. The travel times on the driving edges equal those on the corresponding PTN_m edge. The transfer times depend on the timetable, which, at the line planning stage of the planning process, is not known yet, hence, we instead put a fixed time penalty for each transfer within a modality or between different modalities.

► **Definition 3.7** (CGN travel times). For each driving arc $e \in \mathfrak{E}_m^d$ denote its corresponding edge in the underlying PTN _{m} by $E_m(e)$. Let t_m^{trans} be the time penalty for a transfer between two lines of modality $m \in \mathcal{M}^{\mathcal{L}}$ and t^{trans} be the time penalty for a transfer between two different modalities. Let $e \in \mathfrak{E}$ and define its travel time as

$$\tau_e := \begin{cases} t_{E_m(e)} & , \text{ if } e \in \mathfrak{E}_m^d \text{ for some } m \in \mathcal{M}^{\mathcal{L}}, \\ \frac{1}{2}t_m^{\text{trans}} & , \text{ if } e \in \mathfrak{E}_m^p \text{ for some } m \in \mathcal{M}^{\mathcal{L}}, \\ t_{E_m(e)} & , \text{ if } e \in \mathfrak{E}_m \text{ for some } m \in \mathcal{M} \setminus \mathcal{M}^{\mathcal{L}}, \\ \frac{1}{2}t^{\text{trans}} & , \text{ if } e \in \mathfrak{E}^N. \end{cases}$$

The travel time on a path p in a CGN _{m} of modality m or in the MCGN can be computed as the sum of the travel times of the arcs in the path $\tau(p) = \sum_{e \in p} \tau_e$. Note that, similar to the multimodal public transport network, the travel time of a passenger's path includes two "additional" platform and station edges for the initial boarding and the final alighting.

Line Planning in the Change&Go Network

The task in line planning is to choose lines from the line pool to be operated and determine their frequencies.

► **Definition 3.8** (Line Concept). Let $m \in \mathcal{M}^{\mathcal{L}}$ be a line-based modality, with line pool $\mathcal{L}_m$ and denote by $f_l \in \mathbb{N}$ for each line $l \in \mathcal{L}_m$ its *frequency*, i.e., the number of times the line is operated within a given planning period T . We call the tuple $(\mathcal{L}_m, f_m)$, with $f_m = (f_l)_{l \in \mathcal{L}_m} \in \mathbb{N}^{|\mathcal{L}_m|}$, a *line concept of modality m* .

There are many different models for line planning, with various objective functions and constraints for feasible solutions. In this work, we assume that the cost of operating a line $l \in \mathcal{L}_m$ with frequency $f_l \in \mathbb{N}$ is f_l times the cost of the line c_l , which is fixed and given. In the classic unimodal line planning problem, we need passenger demand for this modality, e.g., as an OD matrix $\mathcal{OD}^m \in \mathbb{N}^{|V_m| \times |V_m|}$ specifying the number of passengers traveling between two stops in PTN _{m} with the modality m . The passenger paths highly depend on the actual design of the system, which is not known in advance, hence the modal split can only be an estimation.

In the travel time model from [25], the passengers in $\mathcal{OD}^m$ form a multi-commodity flow in the CGN _{m} between the platform nodes. The frequencies have to be chosen in a way that accommodates the passenger flow on the driving arcs without exceeding a budget constraint on the overall cost of the line system, while the sum of passengers' travel times is minimized. In the multimodal line planning problem we want to determine line concepts for all line-based modalities simultaneously. The multimodal change&go network allows to apply the travel time model by routing the passengers of $\mathcal{OD}$ as a multi-commodity flow between the station nodes. Hence, the modal split is not fixed in advance but emerges endogenously from the line concepts determined by the optimization.

3.4 Event-Activity Network

In the timetabling step of the planning process, the task is to determine a time for each arrival and departure of a vehicle at a station. A vehicle's journey along a line can be represented by a sequence of departures and arrivals at the stops contained in the line.

► **Definition 3.9** (Event-Activity Network). For $m \in \mathcal{M}^{\mathcal{L}}$ with PTN _{m} = (V_m, E_m) and line concept $(\mathcal{L}_m, f_m)$, let

$$\mathcal{T}_m = \bigcup_{l \in \mathcal{L}_m} \{l^{(1)}, \dots, l^{(f_l)}\}$$

be the set of all trips of the line concept. A line l with frequency $f_l \in \mathbb{N}$ is operated f_l times within the planning period, and is therefore represented by f_l separate trips. Then, define the *event-activity network* as the directed simple graph $\text{EAN}_m = (\mathcal{E}_m, \mathcal{A}_m)$. The events $\mathcal{E}_m$ represent arrivals and departures of vehicles at stops or stations, i.e., $\mathcal{E}_m := \mathcal{E}_m^{\text{arr}} \cup \mathcal{E}_m^{\text{dep}}$ with

- $\mathcal{E}_m^{\text{arr}} := \{(v, t, \text{arr}, m) : t \in \mathcal{T}_m, v \in V_m \cap t\},$
- $\mathcal{E}_m^{\text{dep}} := \{(v, t, \text{dep}, m) : t \in \mathcal{T}_m, v \in V_m \cap t\}.$

The activities connect consecutive events and represent vehicles driving between or waiting at stations or possible transfers for passengers between trips at a station. Define $\mathcal{A}_m := \mathcal{A}_m^{\text{drive}} \cup \mathcal{A}_m^{\text{wait}} \cup \mathcal{A}_m^{\text{trans}}$, where

- $\mathcal{A}_m^{\text{drive}} := \{((v_1, t, \text{dep}, m), (v_2, t, \text{arr}, m)) : t \in \mathcal{T}_m, (v_1, v_2) \in E_m \cap t\},$
- $\mathcal{A}_m^{\text{wait}} := \{((v, t, \text{arr}, m), (v, t, \text{dep}, m)) : t \in \mathcal{T}_m, v \in V_m \cap t\},$
- $\mathcal{A}_m^{\text{trans}} := \{((v, t_1, \text{arr}, m), (v, t_2, \text{dep}, m)) : t_1, t_2 \in \mathcal{T}_m, v \in E_m \cap t_1 \cap t_2\}.$

Additionally, event-activity networks can be extended with further activity types beyond the basic ones presented here; we refer to [15, 18] for more advanced examples.

Multimodal Event-Activity Network

For the timetabling stage in the multimodal case, the set of events that need to be scheduled is the union of all sets of events from the unimodal line-based cases. But, additionally, we need to consider the possibility that passengers might walk between stops that are within acceptable distance of each other. Hence, we need to introduce additional activities that represent the use of nonscheduled modalities.

► **Definition 3.10** (Multimodal Event-Activity Network). Let $\mathcal{M}$ be a set of modalities with an event activity network $\text{EAN}_m = (\mathcal{E}_m, \mathcal{A}_m)$ for each line-based modality $m \in \mathcal{M}^{\mathcal{L}}$ and denote by $D_m > 0$ for each nonscheduled modality $m \in \mathcal{M} \setminus \mathcal{M}^{\mathcal{L}}$ the maximal duration of a leg of this modality in a passenger's journey. Define for a nonscheduled modality $m \in \mathcal{M} \setminus \mathcal{M}^{\mathcal{L}}$ its set of activities

$$\mathcal{A}_m := \left\{ \left((v_1, t_1, \text{arr}, m_1), (v_2, t_2, \text{dep}, m_2) \right) : \right. \\ \left. (v_1, t_1, \text{arr}, m_1) \in \mathcal{E}_{m_1}, (v_2, t_2, \text{dep}, m_2) \in \mathcal{E}_{m_2} \text{ with } \text{dist}_m(v_1, v_2) \leq D_m \right\},$$

where $\text{dist}_m(v_1, v_2)$ denotes the shortest path distance in PTN_m with respect to the edge travel times t_e . Define the *multimodal event-activity network* $\text{MEAN} = (\mathcal{E}, \mathcal{A})$, with $\mathcal{E} := \bigcup_{m \in \mathcal{M}^{\mathcal{L}}} \mathcal{E}_m$ and $\mathcal{A} := \bigcup_{m \in \mathcal{M}} \mathcal{A}_m$.

To respect shared infrastructure capacities in railway applications, additional headway activities derived from the infrastructure network can be included.

Timetabling in the Event-Activity Network

Consecutive departures and arrivals cannot be scheduled arbitrarily, there are restrictions on the amount of time that needs to pass between them: vehicles need to wait for some time at each stop such that passengers can board and alight, and there is an amount of time needed for driving from one station to the next. These constraints need to be taken into account when determining a timetable. In this work we focus on periodic timetabling, but the following definitions can be easily extended to the aperiodic case.

► **Definition 3.11** (Timetable). Given an event-activity network $\text{EAN}_m = (\mathcal{E}_m, \mathcal{A}_m)$ of a line-based modality $m \in \mathcal{M}^{\mathcal{L}}$ and a planning period $T \in \mathbb{N}$, a *periodic timetable* $\pi^m = (\pi_e^m)_{e \in \mathcal{E}_m} \in \mathbb{N}^{|\mathcal{E}_m|}$ fixes a point in time $\pi_e^m \in \{0, \dots, T-1\}$ for each event $e \in \mathcal{E}_m$ which

occurs in every period repetition throughout the day.

Suppose that for each activity $a \in \mathcal{A}_m$ we have lower and upper bounds $L_m(a), U_m(a) \in \mathbb{N}$ on its duration. A periodic timetable π^m is called *feasible*, if for each activity $a = (e_1, e_2) \in \mathcal{A}_m$ the periodic difference in time between the two events is within the given bounds, i.e.

$$L_m(a) \leq ((\pi_{e_2}^m - \pi_{e_1}^m - L_m(a)) \bmod T) + L_m(a) \leq U_m(a).$$

The usual objective for the timetabling step is to minimize the overall passenger-weighted travel time. Since the passenger paths, and thus their travel times, depend on the timetable, which is not known beforehand, we need an estimate c_a of how many passengers will use activity $a \in \mathcal{A}_m$. To obtain these estimates, one can do a passenger prerouting in EAN_m with respect to some duration assumption $\tilde{t}_a$ for all $a \in \mathcal{A}_m$. Finding an optimal timetable is known as the *Periodic Event Scheduling Problem* (PESP) [26].

Timetabling in the Multimodal Event-Activity Network

For a nonscheduled activity $a = (e_1, e_2) \in \mathcal{A}_m, m \in \mathcal{M} \setminus \mathcal{M}^{\mathcal{L}}$ connecting two events at stops v_1 and v_2 , we define the time bounds $L_m(a) := \text{dist}_m(v_1, v_2), U_m(a) := T + \text{dist}_m(v_1, v_2) - 1$. The task in the multimodal timetabling step is to determine a periodic timetable $\pi = (\pi^m)_{m \in \mathcal{M}^{\mathcal{L}}} \in \mathbb{N}^{|\mathcal{E}|}$ respecting all activity bounds which minimizes the travel time for given passenger weights c_a on each activity $a \in \mathcal{A}$. Note that the bounds on the nonscheduled activities do not impose any restrictions on the times of the incident events, but they are needed to calculate the travel time of a passenger in the objective function correctly.

The resulting problem is still a PESP instance and all solution methods known for the unimodal case can be applied.

4 Applications

In the following we present multiple possible applications for our framework. To demonstrate its practical relevance, we show a case study with real-world data for the first application.

Multimodal Sequential Planning. As already mentioned above, the supplies of modalities mutually influence each other, as it determines the modal split. Therefore, it is reasonable to perform the sequential planning stages like line planning and timetabling not for a single line-based modality in isolation, but to consider all line-based modalities in an integrated manner. Furthermore, one can also fix individual modalities, plan the remaining modalities while using the fixed ones only for passenger routing, and measure its influence on the fixed modalities.

Multiple Vehicle Types. In most public transport systems there are different vehicle types operating, e.g., small and articulated buses or slower and faster trains. Especially in the line planning step, the passenger capacity of a vehicle and its speed are crucial. Furthermore, different vehicle types might not be allowed to operate on every PTN edge due to height limitations, narrow corners or weight limitations. One can model each different vehicle type as a single modality m having a specific passenger capacity A_m , a specific speed $\text{speed}(m)$ and an individual line pool $\mathcal{L}_m$. Multimodal planning methods can then be used to plan lines, timetables and vehicle schedules when vehicles of different types are present.

Strategic Ridepooling. Ridepooling systems are demand-responsive, i.e., they react spontaneously to the passenger demand that arises in each situation, while in line-based systems, the vehicles travel along fixed lines according to a pre-determined timetable. This makes line systems very good at transporting large numbers of passengers while ridepooling tends to be more efficient in low-demand situations. Hence, planning both modalities simultaneously leads to integrated solutions where each is utilized to make use of its strengths and advantages. First models for the strategic aspects of ridepooling, meaning the decisions of where to offer a ridepooling service and with which capacity, have been presented in [8]. Analogously to the line planning step, the task is to choose ridepooling areas and a number of ridepooling vehicles to be deployed in each area. The proposed framework allows to combine strategic ridepooling with line planning for the integrated planning of a multimodal system consisting of line-based modalities and ridepooling services, while nonscheduled modalities are also available for passengers.

Planning Infrastructure for Nonscheduled Modalities. Starting from an infrastructure network and the information for each infrastructure link, which modality can (in principle) be established on it and in which direction, our framework can be used to actually plan a PTN, i.e., to decide for stops to be opened and for each connection between them, which infrastructure path it uses. For nonscheduled modalities, this would mean to decide where bike lanes or footpaths should be built. Our framework allows to integrate the effect on other modalities into the decision or to plan infrastructure for multiple modalities at once.

Modelling Shared Infrastructure within one Modality. Especially in railway applications shared infrastructure capacities are an important problem, even within a single modality (see, e.g., [17]). If for example two trains leave a station in the same direction, a headway distance must be respected for security reasons, but if their paths split at some point before their next station, the headway is no longer needed on the remaining track segments. Therefore, the infrastructure network can be used to model and even plan track occupations.

Integration of Walking to Line-based Planning Problems. When planning a line-based modality in the classic unimodal framework, one usually considers an OD matrix, specifying the number of passengers that wish to travel between pairs of stops. This is done by assigning the actual demand points (homes, schools, workplaces etc.) to the closest stop. However, depending on the resulting system, it might shorten the travel time of passengers to walk to another stop to save travel time or transfers. Similarly, walking might also be reasonable between two trips with a line vehicle within a passenger's journey. Our framework allows to consistently include the possibility of walking into the planning of line-based systems.

4.1 Case study

We present a case study using real-world data of the city of Helsinki, Finland. We consider five line-based modalities and one nonscheduled modality and perform the sequential planning process starting with an infrastructure network and an OD matrix on infrastructure level. The goal is to plan line concepts and timetables for all line-based modalities by applying multimodal planning methods in the three steps of line planning, passenger-prerouting to get EAN weights, and timetabling.

All experiments are conducted within LinTim [23]; an open-source software toolbox for public transport planning that provides a collection of algorithms covering the full sequential planning pipeline. We have extended LinTim with a multimodal planning module, implementing all

graph structures described in Section 3 and making them available for use in each planning step. The calculations were performed on an Intel(R) Xeon(R) Gold 6240R CPU @ 2.40GHz with 380 GB of memory and we used Gurobi 13.0.1 [11] as IP solver.

4.1.1 Dataset

We use real-world data from the city of Helsinki, Finland, for this numerical study. The Helsinki dataset from the LinTim toolbox [23] was cropped to include the city center of Helsinki as shown in Figure 6 in Appendix A. The resulting infrastructure network consists of 195 nodes and 2996 links. The OD matrix is based on Helsinki Region Transport (HSL) open data of passenger boarding in November 2016 and contains 49,544 passengers. We consider the five line-based modalities *bus*, *tram*, *rail*, *subway*, and *ferry* as well as the nonscheduled modality *walk*. Line pools for all line-based modalities are generated by computing a shortest path for each station pair of the corresponding modality and eliminating subpaths. The costs c_l of a line l are computed as $c_l = \text{cost}_{\text{fix}} + \sum_{e \in l} \sum_{\lambda \in \text{infram}(e)} \text{length}(\lambda) \cdot \text{cost}_{\text{length}}$. Table 1 in Appendix A gives an overview of the modalities' characteristics.

4.1.2 Methodology

In a traditional unimodal planning procedure we would consider every line-based modality independently of the other modalities. To apply unimodal planning, we need a preprocessing step to split the OD matrix on infrastructure level $\mathcal{OD}$ in modality-specific OD matrices $\mathcal{OD}^m \in \mathbb{N}^{|V_m| \times |V_m|}$ specifying the number of passengers traveling between two stops in PTN_{*m*} with the modality *m*. To achieve an OD split, we route all passengers on shortest paths in the MPTN with respect to the edge travel times t_e and split up the resulting paths as described in Algorithm 1 in Appendix B. Note that the sum of passengers included in the OD matrices $\mathcal{OD}^m$ for all $m \in \mathcal{M}$ can be larger than the sum of passengers included in $\mathcal{OD}$, since a single passenger can appear in the OD matrix of multiple modalities or even multiple times within the same modality.

In the line planning step we use the travel time model as formulated by [25] and solve the IP formulation with Gurobi. This model minimizes the overall travel time with integrated passenger routing in the change&go network while respecting vehicle capacities and a total cost budget. To get a reasonable cost budget for each modality, we first solve the cost-model for line planning as introduced in [28] and increase its optimal objective value by 10%. Applying this procedure separately for each line-based modality *m* with CGN_{*m*} and $\mathcal{OD}^m$ as input gives a line concept $(\mathcal{L}_m, f_m)$ for each modality.

The travel time model can also be applied in a multimodal setting using the MCGN as graph structure. We use the sum of the unimodal cost budgets as budget for the multimodal model and the original OD matrix $\mathcal{OD}$ as input. To model shared infrastructure capacities correctly, we apply linear constraints of the form of Equation (1). Again, we end up with line concepts for all modalities.

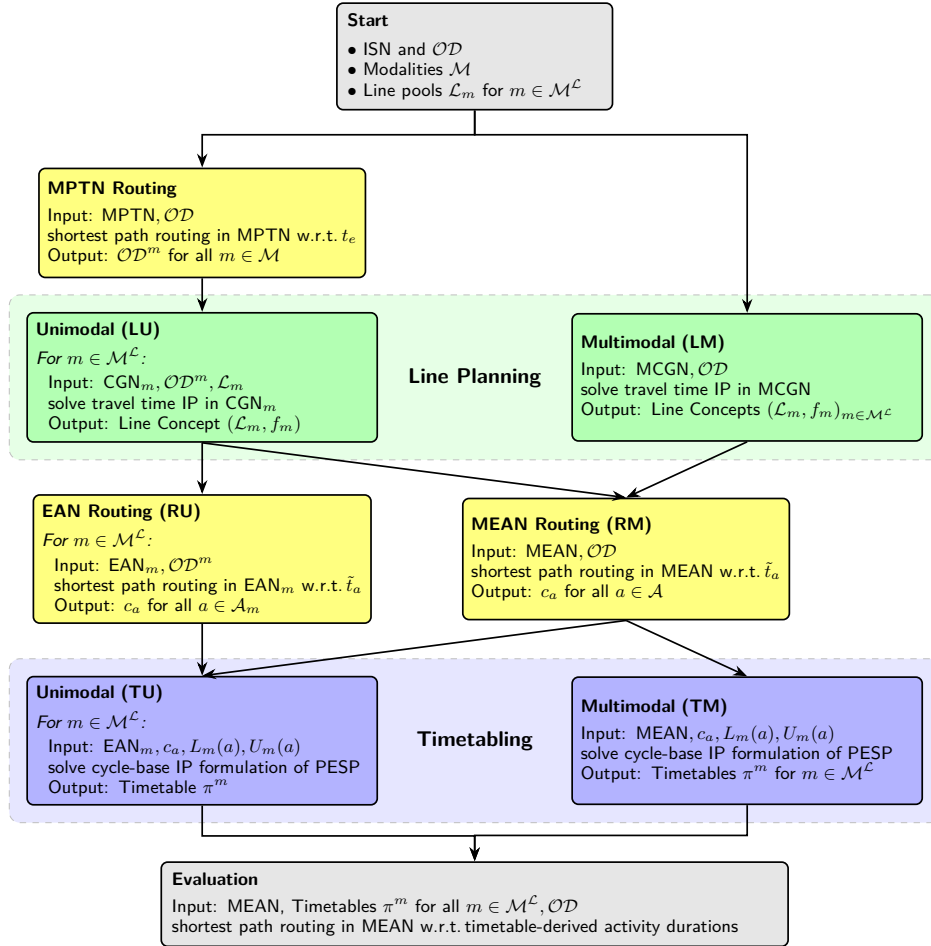
The timetabling step requires passenger weights for all activities and we consider two ways to construct them. In the traditional unimodal approach one takes the modality-specific OD matrix $\mathcal{OD}^m$ and routes all passengers in the EAN_{*m*} on shortest paths with respect to estimated activity durations $\hat{t}_a$. In a multimodal approach we build the MEAN and route all passengers in the OD matrix $\mathcal{OD}$ on shortest paths with respect to estimated activity durations. This allows passengers to use different modalities compared to the first prerouting in the MPTN and leads to a different modal split. Since transfer times can now be estimated more accurately from the line frequencies, this new routing is more realistic.

In the timetabling step we minimize the overall travel time with respect to the fixed precomputed passenger routing while satisfying all activity time bounds. We use the cycle-base IP formulation of the resulting PESP (from [19], see also [14]). The traditional unimodal approach solves an IP for each line-based modality independently using the modality's EAN_m , the lower and upper activity time bounds $L_m(a), U_m(a)$ and the passenger weights c_a from the previous routing step as input. The multimodal approach solves one single IP with the MEAN as underlying graph structure.

This gives five possible ways to get from the infrastructure network and the OD matrix to timetables for all modalities. Figure 3 visualizes our procedure and the five described paths where we combine unimodal and multimodal planning methods in different steps.

To evaluate a solution, we do a final routing in the MEAN with respect to the actual timetable-derived activity durations, delivering final passenger paths and travel times.

Note that we use warm starts for the IP solver whenever possible, e.g., the solution of the line planning cost-model is used as starting solution for the line planning travel time model and already computed timetables are used as initial solutions in the timetabling step, whenever they are based on the same line concept.



■ **Figure 3** Schematic overview of the sequential planning framework for the case study. Starting from an infrastructure network and OD matrix, five combinations of unimodal and multimodal planning steps in line planning, passenger prerouting, and timetabling yield timetables for all line-based modalities, evaluated via a final routing in the MEAN.

4.1.3 Results

An overview of all numerical results can be found in Table 2 in Appendix C.

The left part of Figure 4 shows the total costs of the line concepts resulting from unimodal and multimodal planning and how much each modality contributes. The costs for the line concepts resulting from multimodal planning are 5.2% higher, since the travel time model for line planning allows to increase the costs up to the given budget when decreasing the (estimated) travel time. This is possible in the multimodal model since passengers can be re-assigned to use different modalities. Within the line concept resulting from multimodal planning, the cost shares of bus and tram increase relative to the unimodal baseline, while those of subway and ferry decrease. This redistribution of resources reflects the jointly optimized passenger flows across all modalities.

Regarding the average travel time of a passenger depicted in the right part of Figure 4, we can see a clear monotone trend: the overall travel time decreases from left to right, i.e., the more multimodal aspects are involved during the planning process, the better the average travel time gets in the end. Most strikingly, the time passengers spend on intermodal transfers drops by more than 30% as soon as multimodal line planning is applied, indicating that the integrated MCGN formulation produces line concepts that are inherently better coordinated across modalities.

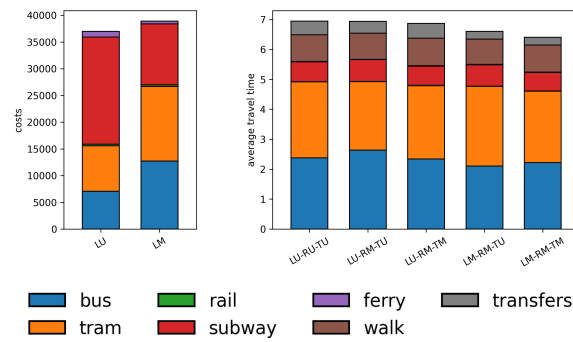


Figure 4 Modal split for costs (left) and average travel time (right).

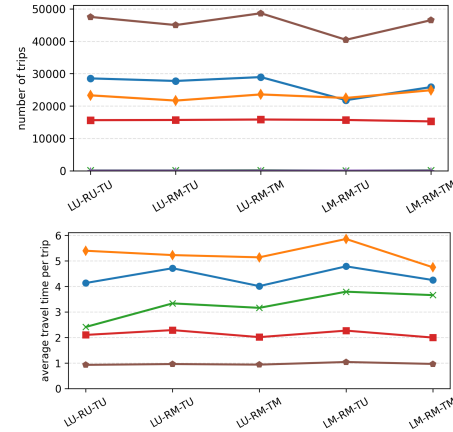


Figure 5 Trips per modality (top) and average travel time per trip (bottom).

However, the modality shares in the right part of Figure 4 show the time an average passenger spends within one modality during his entire journey. If this share decreases, it may either come from an improvement of service for this modality or reflect that less passengers use this modality and switched to another. To disentangle these effects, Figure 5 reports the number of trips per modality recorded in the final MEAN routing. Note that the sum of trips across all modalities exceeds the total number of passengers, since a single passenger journey may involve multiple modalities and is therefore counted once per leg. The upper part of Figure 5 reveals substantial shifts in the distribution of trips across experimental configurations: in particular, the number of trips for bus, tram and walk varies considerably, indicating that multimodal planning redistributes passengers across modalities rather than merely accelerating existing routes. Turning to the lower part of Figure 5, which shows the average travel time per trip and modality, the best results are achieved under multimodal timetabling, except for the rail modality.

Table 3 in Appendix C reports the optimization runtimes for line planning and timetabling, where entries without a gap value indicate that the solver reached optimality within the time limit of 6 hours. We can observe that the unimodal models are solved to optimality in negligible time (below 0.02 s) for the three “small” modalities rail, subway and ferry, while especially unimodal line planning was challenging for the modalities bus and tram, reaching only an optimality gap of 11.45% and 10.93%, respectively.

Notably, the multimodal models achieve optimality gaps comparable to their unimodal counterparts. We attribute this to the systematic use of warm starts throughout all experiments, which proves effective in practice, yielding substantially shorter runtimes for the multimodal models than a cold-start baseline would suggest. The increased runtime for unimodal timetabling after unimodal line planning in the bus modality reflects two factors: the resulting event-activity network contains twice as many events and five times as many activities as the tram event-activity network, and since the full unimodal planning served as the baseline, no warm starts could be exploited.

5 Conclusion and Further Research

We presented a general framework for multimodal public transport planning. We introduced the three categories line-based, ridepooling, and nonscheduled for transport modalities and proposed a hierarchy of graph structures that extend their well-known unimodal counterparts to the multimodal setting. These structures explicitly model interactions between modalities, including passenger transfers and shared infrastructure capacity. We showed that established optimization models for unimodal line planning and periodic timetabling extend naturally to the multimodal setting using these graph structures.

Our computational study on the Helsinki dataset demonstrates that incorporating multimodal aspects into the planning process leads to measurable improvements in passenger travel time. In particular, the results show that multimodal line planning significantly reduces the time passengers spend on intermodal transfers, and that a fully integrated multimodal planning pipeline consistently outperforms unimodal approaches. At the same time, the multimodal timetabling step can be solved as a single PESp instance.

Several directions for future research remain open. First, the integration of ridepooling into the timetabling stage is a natural next step: while strategic ridepooling has already been incorporated into line planning, the operative aspects of ridepooling, such as vehicle routing and assignment, are demand-responsive and do not fit directly into the periodic timetabling framework. Developing suitable models that bridge the strategic and operative levels of ridepooling within the sequential planning pipeline is therefore an important open problem.

Second, the multimodal IP formulations are substantially larger than their unimodal counterparts, and dedicated solution methods are needed to make them tractable at scale. One promising approach is to solve the multimodal problem iteratively by fixing the line concepts or timetables of individual modalities in turn and re-optimizing the remaining ones, exploiting the decomposable structure of the multimodal graphs.

Third, the planning of transport infrastructure has been studied for multiple line-based modalities in [20]. Our framework allows to extend the planning of infrastructure to other modality categories. In particular the planning problem for nonscheduled modalities, i.e., determining where bike lanes or footpaths should be built, has so far only been sketched as a potential application of the framework. Formulating this as a concrete optimization problem and studying its interaction with the planning of line-based and ridepooling modalities constitutes a promising research direction.

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A Helsinki Dataset

■ **Table 1** Overview of the modalities’ characteristics within the Helsinki dataset.

	$ V_m $	$ E_m $	$ \mathcal{L}_m $	vehicle capacity	cost _{fix}	cost _{length}
bus	147	347	1399	80	68	0.6
tram	93	207	436	150	160	4.5
rail	3	2	1	300	220	10
subway	9	14	3	200	300	14
ferry	4	4	4	225	250	6
walk	195	4842	-	-	-	-

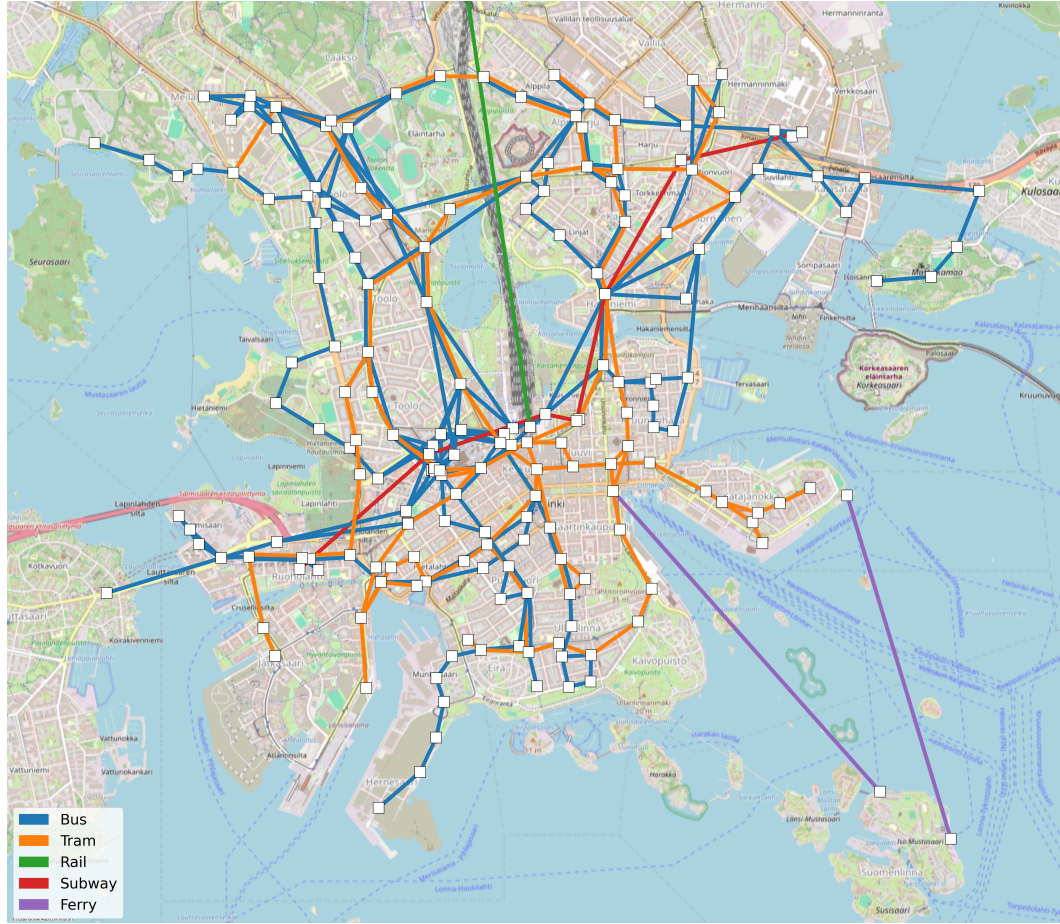


Figure 6 Visualization of the line-based modalities in the Helsinki dataset.

B OD Splitting

Algorithm 1 Splitting the OD matrix on infrastructure level into modality-specific OD matrices based on an MPTN routing.

Input: $\mathcal{OD}$, MPTN

Output: $\mathcal{OD}^m$ for all $m \in \mathcal{M}$

- 1: Initialize an empty OD matrix $\mathcal{OD}^m \leftarrow 0 \in \mathbb{N}^{|V_m| \times |V_m|}$ for all $m \in \mathcal{M}$
- 2: **for** all non-zero entries (ν_1, ν_2) in $\mathcal{OD}$ **do**
- 3: $p \leftarrow$ shortest path from ν_1 to ν_2 in MPTN with respect to t_e
- 4: split p in subpaths $p_1, \dots, p_k$ by removing transfer edges E^{trans}
- 5: **for** $i = 1, \dots, k$ **do**
- 6: Let m be the modality of subpath p_i starting at ν^s and ending at ν^t
- 7: $\mathcal{OD}_{\nu^s, \nu^t}^m \leftarrow \mathcal{OD}_{\nu^s, \nu^t}^m + \mathcal{OD}_{\nu_1, \nu_2}$
- 8: **return** $\mathcal{OD}^m$ for all $m \in \mathcal{M}$

C Numerical results

Table 2 Computational results - solution characteristics.

Costs	bus	7,054.4329			12,722.4196	
	tram	8,528.4775			13,974.6190	
	rail	280.0000			280.0000	
	subway	20,048.8140			11,400.3060	
	ferry	1,059.3400			530.4200	
	total	36,971.0644			38,907.7646	
Average Travel Time	bus	2.3807	2.6389	2.3434	2.1065	2.2171
	tram	2.5375	2.2873	2.4460	2.6589	2.3862
	rail	0.0051	0.0070	0.0105	0.0022	0.0102
	subway	0.6627	0.7246	0.6431	0.7184	0.6142
	ferry	0.0116	0.0116	0.0116	0.0116	0.0116
	walk	0.8883	0.8708	0.9188	0.8484	0.9047
	intermodal transfers	0.4598	0.3932	0.4975	0.2564	0.2610
	overall	6.9458	6.9334	6.8709	6.6024	6.4051
Number of journeys	bus	28,534	27,745	28,941	21,808	25,851
	tram	23,297	21,681	23,581	22,488	24,892
	rail	105	104	164	29	138
	subway	15,634	15,688	15,827	15,694	15,246
	ferry	48	48	48	48	48
	walk	47,527	45,021	48,666	40,465	46,536
	total passengers	49,544	49,544	49,544	49,544	49,544

Table 3 Computational results - runtime.

Runtime (gap)	Line planning	bus	6 h (11.45%)			6 h (0.02%)	
		tram	6 h (10.93%)				
		rail	0.01 s				
		subway	0.02 s				
		ferry	0.01 s				
	Timetabling	bus	6 h (2.44%)	9.64 s	6 h (1.13%)	0.01 s	1672.77 s
		tram	549.96 s	0.55 s		0.02 s	
		rail	0.01 s	0.01 s		0.01 s	
		subway	0.01 s	0.01 s		0.01 s	
		ferry	0.01 s	0.01 s		0.01 s	