

Computing the Cost of Stability in Periodic Microscopic Railway Timetabling via Logic-Based Benders Decomposition

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Abstract

Periodic railway timetables require stability in operation, but the cost of increasing stability in terms of total travel time remains unclear. We study this cost of stability in microscopic periodic timetabling using the realisable cycle time as a stability measure. To implement stability, we compress the timetable to a shorter realisable cycle time that remains feasible at the microscopic level, and then rescale it back to the nominal period used in regular operation. The difference between this nominal period and the realisable cycle time becomes the time supplement available for absorbing delays. We formulate an optimization problem that jointly considers realisable cycle time and total travel time, and solve it with Logic-Based Benders Decomposition. The master problem optimizes commercial timing variables, while the subproblem verifies microscopic feasibility with routing, periodicity, and resource-conflict constraints. Infeasible subproblems yield activity cuts that constrain the master. Experiments on two Swiss railway case studies show that the proposed decomposition substantially improves computational performance on the larger instance and reveals nonlinear cost-of-stability frontiers with topology-dependent infeasible cycle-time regions and exceptionally high marginal costs at certain stability levels.

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1 Introduction

Periodic timetables are a central planning instrument in passenger railways because they provide a regular and memorable public service offer and are widely used in European passenger railway planning [2]. Their operational value, however, depends not only on feasibility in the nominal plan but also on stability: the ability of delayed trains to return to their scheduled train paths by using the available time supplement [5].

A useful stability proxy for periodic timetables is the minimum realisable cycle time. Given a nominal period T , a timetable whose minimum realisable cycle time ω satisfies $\omega < T$ has positive cycle-time slack which implies finite delay settling under realisability



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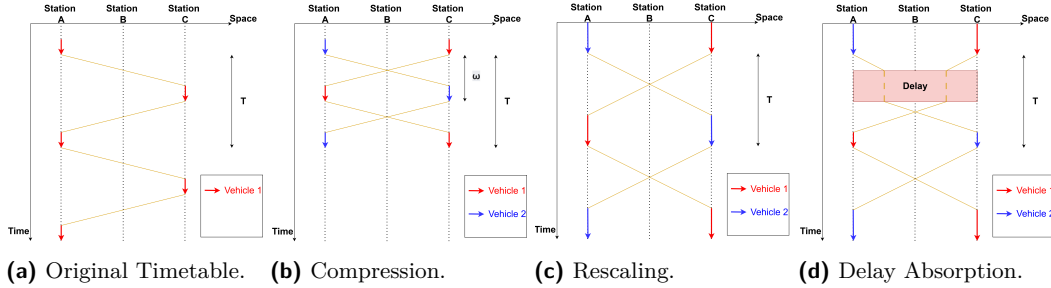


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assumptions [8]. A stable periodic timetable can be generated conceptually in two steps: compression and rescaling. In the compression step, one searches for a timetable structure that can be operated with a realisable cycle time $\omega < T$. In the rescaling step, this compressed timetable is rescaled back to the nominal period T for regular operation. The time difference $T - \omega$ is then available as time supplement [8]. Supplement could in principle be allocated in different ways, and this paper follows the established uniform rescaling construction, which guarantees the supplement in all the critical cycles after rescaling [8]. Fuchs et al. [4] extend the stability implementation method to single-track railway networks and prove the necessity of microscopic infrastructure modelling where resource conflicts dominate.

Figure 1 demonstrates an example of stability implementation and delay absorption. In an original timetable (Figure 1a) with no stability consideration, trains run at maximum speed between station A and C. In the compression step (Figure 1b), we alter the timetable structure by allowing trains to cross at B and optimize the realisable cycle time. The timetable is uniformly rescaled in Figure 1c. When a delay event occurs (Figure 1d), the timetable can adjust to absorb delay in the following train run and return to the normal schedule without intervention.



■ **Figure 1** An example of implementing timetable stability.

Increasing stability does not come for free. Existing approaches show that obtaining stability may require relaxing service frequencies, regularity requirements, or running-time bounds [1]. Here, we focus on the travel-time cost induced by the compression and rescaling. We define travel time as the sum of time durations of travel activities, including running between stations and dwelling at stations. When the compressed timetable is uniformly rescaled from cycle time ω to the nominal period T , each duration Δt is multiplied by the factor T/ω . With A^{com} denoting the set of commercial activities a in the timetable that contribute to the travel time, the total travel time of the rescaled timetable TT^r is

$$TT^r = \frac{T}{\omega} \cdot \sum_{a \in A^{\text{com}}} \Delta t_a, \quad (1)$$

The realisable cycle time ω can affect the total travel time in two aspects: Reducing ω increases the rescaling factor T/ω , which increases the rescaled travel time if the sum of Δt_a is the same. The effect can be observed between Figure 1c and Figure 1a where trains have longer running times after rescaling. At the same time, the combination and duration of Δt_a depend on the timetable structure solved under the chosen cycle time ω , including crossings, waiting-time allocations, route choices, and resource-conflict resolutions. The second effect can only be observed in a more complex network. The net effect of ω on travel time is therefore not evident a priori. This motivates the research question: For a periodic timetable solved with the stable timetable generation process, including compression and uniform rescaling, how much more travel time must be paid as costs for additional stability?

The paper makes three contributions. First, we propose a microscopic periodic timetabling model that jointly optimizes the stability and total travel time and quantifies the trade-off. Second, we extend an LBB approach with activity cuts that exploit the timetable's periodic structure and accelerate the solution process in the larger case study. Third, we conduct experiments on Swiss railway networks and reveal nontrivial properties of the trade-off frontier between stability and travel time.

2 Problem Setting and Stability Measure

We consider a given line plan, fixed train frequencies, and a microscopic infrastructure model. The underlying feasibility model follows the periodic microscopic timetabling framework of Fuchs et al. [4]. It combines a train-flow representation for route choices with an event-activity network for timing decisions. Events $e \in E$ have continuous times t_e , activities $a = (i, j) \in A$ impose lower and upper duration bounds (adjusted consistently for the compressed and rescaled timetables), and selected train movements occupy microscopic resource paths. Resource conflicts require a precedence decision and a minimum separation time. Periodic constraints impose that each event repeats after a cycle time ω .

We quantify the trade-off between two objectives: stability and travel time. Stability is represented by the realisable cycle time ω . Lower ω indicates more time supplement.

The total travel time is defined as the sum of durations of all commercial activities including running between stations and dwelling at stations. Let $A^{\text{com}} \subseteq A$ be the set of commercial activities in the timetable, whose durations contribute to the total travel time. Following the generation steps of timetable compression with realisable cycle time ω and rescaling, the total travel time of the generated stable timetable is

$$TT^r = \frac{T}{\omega} \cdot \sum_{a=(i,j) \in A^{\text{com}}} (t_j - t_i) \quad (2)$$

For design purposes, the two objectives can be combined in the weighted scalarization

$$\min \quad \lambda \cdot c_\omega \cdot \omega + (1 - \lambda) \cdot c_{TT} \cdot TT^r \quad (3)$$

where $\lambda \in [0, 1]$ is the weight, and $c_\omega, c_{TT} > 0$ normalize the two terms. In the computational evaluation below, we use an alternative fixed-stability formulation. For each sampled cycle time $\bar{\omega}$, we fix $\omega = \bar{\omega}$ and solve $f(\bar{\omega}) = \min TT^r$ subject to the same microscopic feasibility constraints. This measures the travel-time cost required for a prescribed stability level.

3 Logic-Based Benders Decomposition with Activity Cuts

3.1 Decomposition Structure

The microscopic stability model is a mixed-integer programming (MIP) model with continuous event times and binary decisions for routing, resource activation, and train precedence on conflicting resource paths [4]. Solving the full MIP directly is computationally challenging because microscopic feasibility is governed by a large set of routing and resource-conflict decisions, while the objective depends only on a smaller set of commercial timing variables. This structure motivates a decomposition.

We extend the microscopic-timetabling LBB framework of Leutwiler and Corman [7] with the activity cuts introduced below. The original problem is decomposed into a master problem, which contains only commercial event times, cycle time ω and accumulated cuts, and a subproblem, which verifies routing and resource-conflict feasibility.

For a candidate master solution (ω^*, t^*) , the subproblem fixes the variables from the master and verifies all the routing, periodicity, and resource-separation constraints. The subproblem is a pure feasibility problem that can be solved with high efficiency [7]. If the subproblem is feasible, the candidate solution is optimal for the original problem. If the subproblem is infeasible, a cut is generated and added to the master for the next iteration.

3.2 Positive-Cycle Certificates and Activity Cuts

The subproblem can be represented as a graph of difference constraints. A difference constraint $t_j - t_i \geq \delta_{ij}$ is represented by a directed arc from i to j with a weight δ_{ij} . A set of selected routing and precedence decisions therefore induces a weighted directed graph. The subproblem is infeasible if the corresponding graph contains a positive cycle. Let a cycle be decomposed into master-induced arcs S_m^D and activated subproblem arcs S_s^D , where D denotes a routing and precedence decision pattern that induces the cycle. Summing the difference constraints around the cycle cancels the event times and requires

$$0 \geq \sum_{(i,j) \in S_m^D} \delta_{ij} + \sum_{(k,l) \in S_s^D} \delta_{kl}. \quad (4)$$

A positive right-hand side violates inequality (4) and therefore certifies infeasibility. To avoid this infeasibility, the following activity cut derived from inequality (4) is added to the master:

$$\bigvee_{D \in \mathcal{C}} \left(\sum_{(i,j) \in S_m^D} (t_j - t_i) \leq -\Delta_D \right) \quad (5)$$

where $\Delta_D = \sum_{(k,l) \in S_s^D} \delta_{kl}$ is the constant aggregate contribution of the subproblem arcs for a fixed decision pattern D and $\mathcal{C}$ is the set of cycle-inducing decision patterns.

The activity cut is particularly effective in periodic timetabling, where infeasibility often arises from aggregation of multiple periodicity links. If the master-induced arcs in a certificate contain only periodicity links, each such link can be written in the form of $t_j - t_i = k_{ij}\omega$, $k_{ij} \in \mathbb{Z}$. The aggregate master term in the cut (5) then collapses to a direct period cut. With $K_D = \sum_{(i,j) \in S_m^D} k_{ij}$, the cut becomes

$$K_D \omega \leq -\Delta_D \quad (6)$$

The cut allows the LBBD scheme to use periodic infeasibility certificates to constrain ω directly, rather than only excluding local timing patterns.

3.3 Algorithmic Outline

The LBBD scheme applies to both formulations. For the weighted formulation in Equation (3), the master jointly minimizes ω and TT^r . For the fixed-stability instances used in the experiments, the same scheme is applied with the additional master constraint $\omega = \bar{\omega}$ and the single objective $\min TT^r$.

1. Solve the master problem with commercial constraints, ω , and accumulated cuts.
2. Fix the resulting commercial event times and ω in the microscopic subproblem.
3. If the subproblem is feasible, store (ω, TT^r) and the corresponding timetable.
4. If the subproblem is infeasible, extract a positive-cycle certificate and add the corresponding activity cut to the master.
5. Repeat until the current instance is solved or proved infeasible.

4 Computational Experiments

4.1 Case Studies

We study two cases on the Swiss Rhaetian Railway: CHUR-AROSA, a single-track corridor with passing loops, and CHUR-CORE, a mostly double-track network. For each sampled cycle time $\bar{\omega}$, we solve a fixed-stability instance with $\omega = \bar{\omega}$, nominal cycle time $T = 3600$ s, and the objective of minimizing TT^r . We compare the benchmark MIP and the LBB method on these instances; Table 1 reports the instance characteristics and runtime statistics. All experiments ran on an AMD Ryzen 7 PRO 8840U with 32 GB RAM, using OpenBus [3] in Python and `gurobipy` 13.0.1 [6], and terminated at a 0.5% optimality gap or after 1800 s.

Table 1 Case-study statistics.

Case	Topology	# of Nodes	# of Links	# of Conflicts	Avg. # of cuts	Avg. MIP runtime	Avg. LBB runtime
CHUR_AROSA	Single-track	3 169	5 032	21 640	10.79	1.45 s	2.93 s
CHUR_CORE	Double-track	58 945	83 635	607 338	3.6	*	148.36 s

* The MIP did not converge within the time limit.

4.2 Trade-Off Frontiers

Figure 2 shows the cost-of-stability frontier and the marginal costs of stability for both case studies. The plotted frontier uses cycle-time intervals of 36 seconds, equal to 1% of the nominal 3600-second period. Infeasible grid points are shown as dotted segments. For a solution at cycle time ω , the marginal cost is computed as the increase in rescaled total travel time when moving from the neighboring sampled solution at $\omega + 36$ s to ω .

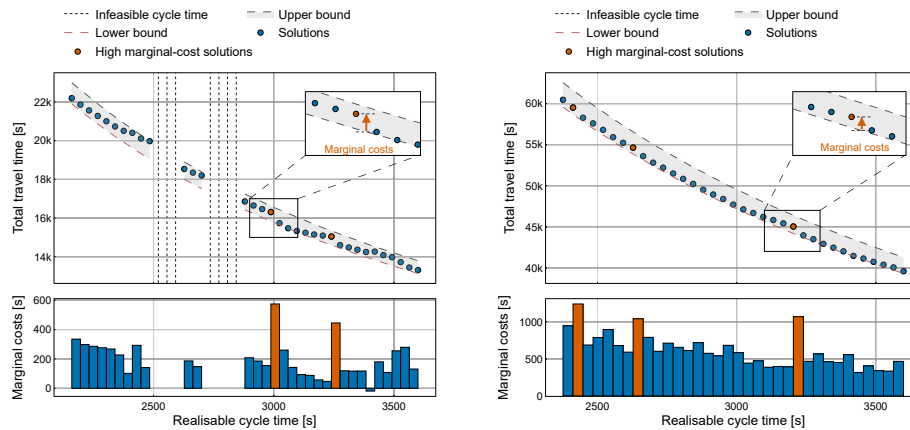


Figure 2 Trade-off between realisable cycle time ω and rescaled total travel time for CHUR-AROSA (left) and CHUR-CORE (right). Solutions with high marginal costs are highlighted.

The frontier shows that within some cycle-time ranges, the realisable cycle time can be reduced with moderate increases in rescaled total travel time, indicating moderate marginal costs. The costs reflect the combined effect of reducing cycle time: the rescaling increases travel time, while travel-time optimization can alleviate the marginal costs. Marginal costs at certain cycle times are exceptionally high, e.g., at 3024 s in CHUR-AROSA and 3204 s in CHUR-CORE.

in CHUR-CORE. This shows that stability should not be optimized only toward a single prescribed level: for operational planning, selecting a stability level without inspecting the frontier may therefore lead to unnecessarily high travel-time costs.

The two cases differ mainly in the infeasible cycle-time regions. The single-track infrastructure in CHUR-AROSA excludes certain sampled cycle-time levels and creates discontinuous feasible regions, whereas the mostly double-track infrastructure in CHUR-CORE allows a smoother adjustment of the timetable structure. The benefit of LBBD is most visible in the larger CHUR-CORE instance: the benchmark MIP does not converge within the 1800 s time limit, whereas LBBD solves the instances in 148.36 s on average.

5 Conclusion

We quantified the travel-time cost of periodic stability in microscopic railway timetabling through the trade-off between realisable cycle time and total travel time. We used Logic-Based Benders Decomposition, where commercial timing decisions are handled in the master problem and detailed routing, periodicity, and resource conflicts are checked in the subproblem. Infeasible subproblems with positive-cycle certificates yield activity cuts for the master. Experiments on two Swiss networks show that the frontier is topology-dependent: moderate stability gains can often be obtained at low travel-time cost, while certain stability requirements can be increasingly expensive or infeasible. Further work should extend the analysis to other networks, interpret infeasibility certificates in physical infrastructure terms, and develop stronger cut aggregation for LBBD.

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