

Integrated Location-Routing-Scheduling for Multi-Trip Mobile Parcel Lockers with Time Windows

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Abstract

Mobile parcel lockers can reposition delivery capacity during the day, but this flexibility requires decisions that are tightly linked: where lockers stop, which customers are assigned to each stop, how lockers move, and when service takes place. We introduce the multi-trip mobile parcel locker problem with time windows (MT-MPLP-TW), which integrates locker deployment, customer assignment, parking selection, routing, scheduling, parking capacity, walking limits, and depot replenishment. We present a compact mixed-integer programming formulation and a logic-based Benders decomposition (LBBD) that separates route-and-assignment decisions from detailed time-expanded certification. Computational results on generated instances and on the single-trip mobile parcel locker benchmark show that the compact formulation is effective on small and medium instances, while LBBD gives stronger performance on larger and more constrained cases.

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1 Introduction

Parcel lockers reduce failed deliveries and consolidate last-mile demand, but fixed lockers require permanent infrastructure and cannot easily adapt to demand that varies over space and time. This limitation has motivated mobile-locker concepts, in which locker capacity can be repositioned during the day [4]. Mobile parcel lockers (MPLs) can be temporarily parked close to customers, replenished at the depot, and reused over multiple trips.

This paper introduces the multi-trip mobile parcel locker problem with time windows (MT-MPLP-TW). The problem extends single-trip mobile parcel locker planning by allowing each deployed locker to perform several depot-to-depot trips during the planning horizon. It asks which MPLs should be deployed, which parking locations they should visit, which



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customers should be assigned to those visits, and how the resulting trips should be scheduled. These decisions must respect customer availability, parking availability, parking capacity, locker capacity, walking limits, travel times, and depot replenishment. The problem fills a modeling gap because it treats locker location as an operational routing and scheduling decision rather than as a fixed network-design decision.

We make three contributions. First, we propose a compact mixed-integer programming formulation for MT-MPLP-TW. Second, we develop an LBBD algorithm in which the master problem selects a route-and-assignment structure and a certification subproblem checks the detailed schedule. Third, we compare the compact formulation and LBBD on generated instances and on the public ST-MPLP benchmark, showing that the decomposition is more scalable when assignment, routing, and timing are strongly coupled.

2 Problem Description and Formulation

We consider customers C , parking locations P , and a fleet M of MPLs. Each locker may perform several trips $r \in R$ from a depot 0, where a trip is a depot-to-depot tour that can visit several parking locations before returning for possible replenishment. Thus, the index r denotes the trip position of locker m during the planning horizon; trips are not generated in advance. Let $N = \{0\} \cup P$ and $A = \{(i, j) \in N \times N : i \neq j\}$. Each customer c has demand η_c , a maximum walking distance e_c , possible pickup locations L_c , and service durations θ_{cl} . The service duration is the time during which the locker must remain available at the selected parking location so that customer c can complete pickup from location l . Customer availability windows are associated with customer locations, while parking windows describe when a parking location can host an MPL. For each feasible triple (c, l, p) , the set W_{clp} contains service windows obtained by intersecting customer and parking availability. Triples without a feasible service window, or violating the walking limit, are removed from F . Here, B is a sufficiently large constant used to deactivate timing constraints when the associated binary decisions are not selected.

The binary variables are x_{ijmr} for arc use, y_{pmr} for parking visits, z_{cmr} for assigning customer c to locker m and trip r , w_{clp} for customer-location-parking assignment, o_{clpw} for service-window selection, ψ_{pkmr} for parking-window selection, and u_{mr} for active trips. The variable u_{m0} indicates whether MPL m is deployed. Continuous variables are departure times t_{mr} , parking arrival times ϕ_{pmr} , stay durations s_{pmr} , and service start times γ_{clmr} .

$$\min \quad C_R \sum_{(i,j) \in A} \sum_{m \in M} \sum_{r \in R} d_{ij} x_{ijmr} + C_W \sum_{(c,l,p) \in F} e_{cl}^p w_{clp} + C_F \sum_{m \in M} u_{m0} \quad (1)$$

$$\text{s.t.} \quad \sum_{\substack{j \in N \\ j \neq p}} x_{pjmr} = y_{pmr} \quad \forall p \in P, m \in M, r \in R \quad (2)$$

$$\sum_{\substack{i \in N \\ i \neq p}} x_{ipmr} = y_{pmr} \quad \forall p \in P, m \in M, r \in R \quad (3)$$

$$\sum_{j \in P} x_{0jmr} = u_{mr} \quad \forall m \in M, r \in R \quad (4)$$

$$\sum_{i \in P} x_{i0mr} = u_{mr} \quad \forall m \in M, r \in R \quad (5)$$

$$\sum_{\substack{l \in L_c: \\ (c,l,p) \in F}} w_{clp} \leq \sum_{m \in M} \sum_{r \in R} y_{pmr} \quad \forall c \in C, p \in P \quad (6)$$

$$\sum_{(l,p):(c,l,p) \in F} w_{clp} = 1 \quad \forall c \in C \quad (7)$$

$$\sum_{m \in M} \sum_{r \in R} z_{cmr} = 1 \quad \forall c \in C \quad (8)$$

$$y_{pmr} \geq z_{cmr} + \sum_{\substack{l \in L_c: \\ (c,l,p) \in F}} w_{clp} - 1 \quad \forall c \in C, p \in P, m \in M, r \in R \quad (9)$$

$$z_{cmr} \leq u_{mr} \quad \forall c \in C, m \in M, r \in R \quad (10)$$

$$\sum_{w \in W_{clp}} o_{clpw} = w_{clp} \quad \forall (c,l,p) \in F \quad (11)$$

$$\gamma_{clmr} \geq a_{clp}^w - B(2 - o_{clpw} - z_{cmr}) \quad \forall (c,l,p) \in F, w \in W_{clp}, m \in M, r \in R \quad (12)$$

$$\gamma_{clmr} \leq b_{clp}^w - \theta_{cl} + B(2 - o_{clpw} - z_{cmr}) \quad \forall (c,l,p) \in F, w \in W_{clp}, m \in M, r \in R \quad (13)$$

$$\sum_{k \in K_p} \psi_{pkmr} = y_{pmr} \quad \forall p \in P, m \in M, r \in R \quad (14)$$

$$\phi_{pmr} \geq a_p^k - B(1 - \psi_{pkmr}) \quad \forall p \in P, k \in K_p, m \in M, r \in R \quad (15)$$

$$\phi_{pmr} + s_{pmr} \leq b_p^k + B(1 - \psi_{pkmr}) \quad \forall p \in P, k \in K_p, m \in M, r \in R \quad (16)$$

$$\phi_{jmr} \geq \phi_{imr} + s_{imr} + \frac{d_{ij}}{v} - B(1 - x_{ijmr}) \quad \forall i, j \in P, i \neq j, m \in M, r \in R \quad (17)$$

$$\gamma_{clmr} \geq \phi_{pmr} - B(2 - w_{clp} - z_{cmr}) \quad \forall (c,l,p) \in F, m \in M, r \in R \quad (18)$$

$$\gamma_{clmr} + \theta_{cl} \leq \phi_{pmr} + s_{pmr} + B(2 - w_{clp} - z_{cmr}) \quad \forall (c,l,p) \in F, m \in M, r \in R \quad (19)$$

$$u_{mr} \leq u_{m,r-1} \quad \forall m \in M, r = 2, \dots, |R| \quad (20)$$

$$t_{mr} \geq \phi_{p,m,r-1} + s_{p,m,r-1} + \frac{d_{p0}}{v} - B(1 - x_{p0,m,r-1}) \quad \forall p \in P, m \in M, r = 2, \dots, |R| \quad (21)$$

$$\sum_{(l,p):(c,l,p) \in F} e_{cl}^p w_{clp} \leq e_c \quad \forall c \in C \quad (22)$$

$$\sum_{c \in C} \eta_c z_{cmr} \leq q_m u_{mr} \quad \forall m \in M, r \in R \quad (23)$$

$$s_{pmr} \geq \theta_{cl}(w_{clp} + z_{cmr} - 1) \quad \forall (c,l,p) \in F, m \in M, r \in R \quad (24)$$

$$\sum_{m \in M} \sum_{r \in R} \psi_{pkmr} \leq b_p \quad \forall p \in P, k \in K_p \quad (25)$$

$$\phi_{pmr} \leq t_{mr} + \frac{d_{0p}}{v} + B(1 - x_{0pmr}) \quad \forall p \in P, m \in M, r \in R \quad (26)$$

$$\phi_{pmr} \geq t_{mr} + \frac{d_{0p}}{v} - B(1 - x_{0pmr}) \quad \forall p \in P, m \in M, r \in R \quad (27)$$

$$\phi_{pmr} \geq t_{mr} + \frac{d_{0p}}{v} - B(1 - y_{pmr}) \quad \forall p \in P, m \in M, r \in R \quad (28)$$

$$x_{ijmr}, y_{pmr}, u_{mr}, z_{cmr}, w_{clp}, o_{clpw}, \psi_{pkmr} \in \{0, 1\} \quad \forall (i,j) \in A, p \in P, c \in C, m \in M, r \in R \quad (29)$$

$$t_{mr}, \gamma_{clmr}, \phi_{pmr}, s_{pmr} \geq 0 \quad \forall (c,l,p) \in F, m \in M, r \in R \quad (30)$$

The objective balances locker travel, customer walking, and deployment costs. Constraints (2)–(5) define the trip routes. Constraints (6)–(10) assign customers, link them to visited parking locations, and activate trips. Constraints (11)–(16) select compatible customer and parking windows. Constraints (17)–(19) propagate route timing and synchronize customer service with locker presence. Constraints (20) and (21) impose the multi-trip structure. The remaining constraints enforce walking limits, locker capacity, parking capacity, stay duration, first-arrival timing, and variable domains. In the compact formulation, trips are optimized through the arc variables x_{ijmr} ; the index r only limits the maximum number of depot-to-depot tours each MPL may perform.

The formulation is strengthened with standard inequalities that remove weak or symmetric solutions: active trips must serve at least one customer, service-window variables can be selected only when the corresponding assignment is selected, homogeneous MPLs are ordered, trips of the same MPL are ordered, and the first departure is fixed to break symmetry. The model specializes to the ST-MPLP of [4] by imposing a single trip, full-day parking availability, and the corresponding benchmark cost structure.

3 Logic-Based Benders Decomposition

Benders decomposition separates a problem into a master problem and a subproblem [1]. Logic-based Benders decomposition extends this idea to settings where the subproblem has discrete or logical structure and cuts are derived from inference rather than linear duality [3, 2]. We use this principle to separate route-and-assignment decisions from detailed time-expanded scheduling.

The compact formulation becomes difficult because routing, assignment, and continuous timing are optimized together. The LBB master problem keeps the structural part of the model: active trips, deployed lockers, parking visits, routing arcs, customer-to-trip assignments, customer-location-parking assignments, and coarse window choices. It contains the flow, assignment, linking, walking, locker-capacity, parking-capacity, and symmetry-breaking constraints of the compact formulation, together with static compatibility inequalities. The detailed continuous timing constraints are not fully enforced in the master. They are checked only after the master proposes an integer route-and-assignment structure.

For a master solution $\bar{\Omega}$, the certification subproblem fixes the selected discrete structure. It extracts the selected trips, the ordered parking visits of each trip, the customers assigned to each trip, the selected customer-location-parking choices, and the parking and service windows selected by the master. The subproblem then searches for feasible departure times, parking arrival times, stay durations, customer service times, and trip-to-locker assignments. It verifies travel-time propagation along each trip, parking-window feasibility, customer-window feasibility, synchronization between customer service and locker presence, depot return, and replenishment between consecutive trips of the same MPL.

If the subproblem is feasible, the candidate solution is certified. The resulting objective value is compared with the incumbent, and the incumbent is updated if the certified solution is better. If the subproblem is infeasible, the selected structure cannot appear in any feasible solution of the original compact model, and a lazy feasibility cut is added to the master.

Let ξ be the vector of binary master decisions defining a candidate structure. For an infeasible structure $\bar{\xi}$, define $S^1(\bar{\xi}) = \{a : \bar{\xi}_a = 1\}$ and $S^0(\bar{\xi}) = \{a : \bar{\xi}_a = 0\}$. The fallback no-good cut is

$$\sum_{a \in S^1(\bar{\xi})} (1 - \xi_a) + \sum_{a \in S^0(\bar{\xi})} \xi_a \geq 1. \quad (31)$$

This cut removes the exact infeasible master structure. It is always valid, but it is weak because it forbids only one binary pattern.

The implementation therefore tries to identify smaller infeasible explanations. When the certification model proves that only a subset of selected decisions is responsible for infeasibility, the cut is written on that subset instead of on the full master solution. For example, if a subset $C' \subseteq C$ of customers cannot be scheduled together on trip (m, r) , the following customer-core cut is added:

$$\sum_{c \in C'} z_{cmr} \leq |C'| - 1. \quad (32)$$

Similarly, if a subset $P' \subseteq P$ of parking visits cannot be jointly scheduled on trip (m, r) , we add

$$\sum_{p \in P'} y_{pmr} \leq |P'| - 1. \quad (33)$$

If infeasibility is caused by a selected route order, represented by a set of arcs $A' \subseteq A$, the route-core cut is

$$\sum_{(i,j) \in A'} x_{ijmr} \leq |A'| - 1. \quad (34)$$

These cuts are generated from the infeasibility information returned by the certification subproblem. When an irreducible infeasible subsystem is available, its constraints are mapped back to the master decisions that activated them. Otherwise, a deletion-filtering procedure removes selected customers, parking visits, or arcs one at a time and keeps only the decisions that are necessary to preserve infeasibility.

The algorithm is implemented in a branch-and-cut framework. Integer master solutions are intercepted through callbacks, certified by the subproblem, and either accepted as incumbents or cut off. A cache stores previously evaluated route-and-assignment structures, avoiding repeated certification of identical candidates. This is important because the master may rediscover the same structure through different fractional paths in the branch-and-bound tree. The overall scheme remains exact because every accepted incumbent is certified by the time-expanded subproblem, and every infeasible cut excludes only structures that cannot be scheduled.

4 Computational Results

All experiments were conducted on a workstation equipped with an Intel Core Ultra 9 285K processor with 24 cores and 62 GB of RAM, running Ubuntu 24.04.3 LTS. Unless stated otherwise, each run used five solver threads. The time limits were 3600 seconds for generated MT-MPLP-TW instances and 1800 seconds for the ST-MPLP benchmark.

Generated instances. The generated set contains small, medium, and large instances with different operational structures. Table 1 focuses on the large instances, where scalability differences are clearest. LBBD improves the average gap in all three families and is especially effective on the multi-trip-tight family.

■ **Table 1** Average performance on large generated MT-MPLP-TW instances.

Family	Method	Incumbent	Lower bound	Gap (%)	Time (s)
Homogeneous	Compact	6069.57	5816.04	4.69	2955.79
Homogeneous	LBBD	6011.80	5952.00	0.94	1952.74
Capacity buffer	Compact	5846.67	5725.35	1.91	2007.94
Capacity buffer	LBBD	5824.32	5824.25	0.00	855.38
Multi-trip tight	Compact	7444.87	6761.89	9.66	3282.85
Multi-trip tight	LBBD	7144.27	7010.33	1.99	2796.34

ST-MPLP benchmark. For the ST-MPLP benchmark, we use the public benchmark instances of [4] and implement the formulation described in their paper. We compare this implementation with our compact formulation and LBBD under the same computational

setting. Table 2 shows that the compact model is competitive on small instances, but its gap grows with the number of customers. LBBD gives the best average gaps on the two largest groups.

■ **Table 2** Average results for ST-MPLP instance groups.

Group	ST-MPLP		Compact MIP		LBBD	
	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)
5 customers	0.000	9.44	0.000	0.83	0.000	0.36
15 customers	4.803	467.80	4.808	405.28	0.000	17.21
30 customers	43.992	1801.20	39.487	1800.28	5.284	1139.05
50 customers	11.382	1800.57	27.200	1800.41	1.508	1650.77

5 Conclusion

We introduced the MT-MPLP-TW, an integrated model for multi-trip mobile parcel locker planning with time windows. The compact formulation captures locker deployment, assignment, parking selection, routing, scheduling, parking capacity, walking limits, and replenishment. The same formulation can be specialized to the single-trip mobile parcel locker benchmark by limiting each deployed locker to one trip.

The experiments show that the compact formulation is effective for small and medium instances, while LBBD improves performance on large and more constrained cases. The main computational difficulty is the coupling between customer assignment, route structure, and time feasibility. Future work should strengthen the master problem with sharper incompatibility cuts and extend the model to stochastic or dynamic demand.

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