

Passenger-Based Intermodal Connection Optimization with Elastic Demands

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Abstract

A railway network can attract additional passengers by improving the quality of service. One way to do that is to establish new train connections by slight shifts in arrival and departure times. We propose a connection optimization model to find a shifting of a given aperiodic timetable that maximizes the passenger volume, in which the demand is elastic and forecast by an integrated discrete choice logit model. The resulting large-scale mixed-integer non-linear non-convex programming problems can be solved by tailor-made heuristics for the entire Italian railway network with its 2,000 stations and 10,000 daily train trips, uncovering substantial potentials for connectivity improvements and boosts in demand.

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1 Introduction

The attractiveness of railway transportation to passengers, in comparison to other means of transport, depends on multiple factors such as availability, reliability, ticket price, travel duration, number of transfers, and transfer times. Railway companies are particularly interested in establishing opportunities for transfers, as they allow passengers to travel between origins and destinations that are not connected by direct lines; such connections generate additional business opportunities and generally improve the quality of service. It is therefore highly interesting to predict and ultimately optimize potential increases in passenger demand in dependence of the train connections, which is precisely the aim of connection optimization.

From an operational perspective, transfers are subject to a number of constraints. Indeed, a feasible transfer requires a minimum duration between the arrival of the first train and the departure of the second train, because otherwise it is not possible to change from the first train to the second. On the other hand, long transfer waiting times discourage passengers from traveling by train, such that an upper bound on the transfer time is also appropriate. There is hence a certain time window for a feasible transfer. This time window can in many cases be met by relatively small shifts in the arrival and departure times of the respective trains at the transfer station, and at essentially no additional operational cost to the service provider.

Giacco et al. [10] proposed a connection optimization approach in order to establish more and/or better transfer opportunities for a major Italian railway company. They provide a case study involving roughly 10,000 scheduled train trips between 2,225 stations for an entire day of operation on a nationwide scale. The increase in the number of feasible connections is significant: When allowing only small shifts in departure and arrival times of ± 1 minute, the number of connections can be increased by 5%. When allowing up to ± 8 minute shifts, the number of connections rises by up to 15%. However, a connection is only a transfer opportunity, and some connections are surely more valuable than others. Hence, the question arises of how much of this potential translates into an increase of railway passenger transportation volume, which is the overall goal of every railway operator. An assessment of this potential requires a prediction of the travel choices of potential passengers, which may or may not use the train depending on whether they have a (good) connection or not. Discrete choice logit models are well established to make such predictions. The aim of this paper is to integrate such a logit model into the connection optimization model of Giacco et al. in order to control the elasticity of the demand with respect to the connections, and to optimize the connections such that the number of railway passengers increases.

Previous work

Connection optimization changes the timetable by shifting arrival and departure times; the problem is therefore a timetabling problem and, in particular, a railway timetabling problem. Railway timetabling is one of the central planning problems in railway operations, as timetable quality directly affects infrastructure utilization, operational efficiency, and passenger satisfaction. It has been studied extensively in the Operations Research literature, starting with Szpigel's 1973 paper on railway timetabling on a single-track corridor [24]. Subsequent research extended these optimization approaches from single-track corridors to increasingly complex railway networks. As is highlighted in the survey articles by Caimi, Kroon & Liebchen [6] and Cacchiani & Toth [5], the two most common approaches to timetabling are periodic timetabling, i.e., creating timetables that repeat after a fixed time

interval, and aperiodic timetabling. Periodic timetabling allows for foreseeable planning under predictive circumstances for rolling stock deployment and maintenance scheduling, see, e.g., Peeters [19] or Liebchen [16]. Aperiodic timetables are mostly considered in the context of shorter time periods with unusual circumstances such as construction zones or holidays, or in connection with train routing, see e.g., the work of Caprara, Fischetti & Toth [8] on the train timetabling problem, of Borndörfer et al. [3] on railway track allocation, or the survey article by Lusby et al. [17].

Aperiodic timetables also arise from modifying a given (periodic or aperiodic) timetable by shifting departure and arrival times in order to improve some measure of connectivity; this idea has been explored repeatedly under terms such as timetable synchronization, timetable coordination, schedule optimization etc., sometimes in an intermodal context. For example, Ceder et al. [9], synchronize bus timetables to maximize the number of simultaneous arrivals, Wong et al. [27] synchronize the Hong Kong mass transit rail timetable to minimize the total transfer waiting time, Cao et al. [7] synchronize the Beijing subway timetable, Ke et al. [14, 15] coordinate the high-speed rail timetable on the corridor from Zhengzhou to Beijing to establish additional connections to and from flights, while Polinder, Schmidt and Huisman [20] show that slight deviations from periodic timetables can enable better passenger travel times.

Substantial progress has been made in the treatment of passenger behavior. Many timetabling models consider a fixed passenger routing and aim to optimize the timetable based on this travel behavior [6]. However, given the importance of the timetable for passenger travel decisions, researchers have started to investigate flexible passenger routing of a static demand in response to timetabling [4], [22]. Indeed, fixing passenger routes before timetable optimization may lead to arbitrarily bad results compared to approaches that integrate passenger routing into the timetable optimization [4]. Integrated design approaches for aperiodic timetabling were presented by Schmidt [21], and Schöbel and Scholl [23]. A notable approach is the one of Hartleb and Schmidt ([12]), who incorporated an advanced treatment of passenger routing that includes mode choice. Under the assumption that the travel choice depends on the number of available routes, they approximate a logit distribution of the passenger flow, uncovering improvement potentials over alternative timetabling methods. Indeed, logit models [18, 1, 25] are widely accepted as a reasonable way to anticipate travel choices, and timetabling models that integrate a logit routing can be considered as being particularly realistic.

The above approaches optimize the construction of a timetable while considering passenger behavior. Another line of research studies how already existing timetables can be modified to improve passenger service. This constitutes a switch from a purely operation-driven timetabling approach to passenger-centric scheduling without having to completely change the timetable and thus maintaining reliability for passengers in the process. We mention Binder et al. [2], who proposed a multi-objective algorithm for timetable rescheduling after a disruption, allowing for a consideration of both passenger and operational objectives.

This work

While the literature extensively covers timetabling and integrated models that include passenger response to the timetable, there is little research on refining existing timetables subject to a realistic demand response. Integrated models so far mostly consider changes in route choices subject to a fixed travel demand. The question of whether passengers want to travel at all has been rarely studied, and, as far as we know, not in the context of connection optimization.

We address the two points of connection optimization and demand elasticity by developing an integrated scheduling model that is based on an existing timetable, optimizes the connections, and predicts the associated demand elasticity. We employ a logit approach to model passenger behavior. Different from Hartleb and Schmidt [12], we do not discretize subject to the number of travel alternatives, but use an iterative flow distribution approach that alternates between connection optimization and passenger routing. This method works for large-scale scenarios and is exemplified in a case study for a European passenger railway network of 2,225 stations and a timetable for roughly 10,000 trips. The paper is organized as follows. Section 2 presents a mathematical definition of the problem as well as the development of the associated logit connection optimization model. An iterative flow distribution approach to tackle this problem is presented in Section 3. Section 4 reports and discusses computational results for a large-scale application. Section 5 concludes.

2 Connection Optimization

We discuss the connection optimization problem in three versions of increasing complexity that highlight the major characteristics. The first version, the connection optimization problem (COP), just counts connections; they are the decisive combinatorial part of the problem and decide which journeys are possible and which ones are not. The second version, the passenger connection optimization problem (PCOP), adds a passenger flow to maximize the volume of the satisfied demand; this model is more realistic and the workhorse of this paper. The third version, the logit(-based) connection optimization problem (LCOP), incorporates a discrete choice logit model to address the elasticity of the demand with respect to the service quality; it aims at attracting car drivers to use the train. The notation is summarized in Tab. 5 in the appendix.

2.1 The Connection Optimization Problem

Giacco et al. [10] considered the following basic version of the connection optimization problem. Let $G = (S, A)$ be a directed graph of railway stations S and (train) trips A . The trips can be partitioned into a set of directed paths I which represent trains. Each train $i \in I$ hence does a sequence of trips, and in doing so, visits a set of stations S_i , arriving at and departing from station $s \in S_i$ at default times $\bar{a}_i^s \in \mathbb{N}_0$ and $\bar{d}_i^s \in \mathbb{N}_0$, respectively. These times can be adjusted by timetable shifts $Inf \leq t_i \leq Sup$, resulting in shifted departure and arrival times $a_i^s = \bar{a}_i^s + t_i$ and $d_i^s = \bar{d}_i^s + t_i$, respectively; here $Inf, Sup \in \mathbb{Z}$ are global bounds for the minimum and maximum time shifts and typically $Inf = -Sup$. Let $C \subseteq S \times I \times I$ be a set of potential connections between the trains. A connection between trains i and j at station s is feasible iff

$$m_{ij}^s \leq \Delta_{ij}^s := d_j^s - a_i^s := \bar{d}_j^s - \bar{a}_i^s + t_j - t_i \leq m_{ij}^s + \beta$$

holds, where $m_{ij}^s \in \mathbb{N}_{\geq 0}$ is a minimum connection time between trains i and j at station s and $\beta \in \mathbb{N}_{\geq 0}$ is the width of the connection time window. The minimum connection time m_{ij}^s is mostly dependent on the station's topology, i.e., the distance of the involved platforms, while β is considered to be a parameter of the problem. The *Connection Optimization Problem* (COP) is to determine timetable shifts that maximize the number of feasible connections. This is important in sparse railway networks, where the quality of the connections decides on the passenger volume. If transfers are too short, the passengers cannot reach them, and if they are too long, then they effectively cease to exist, because people will not use them. The

combinatorial structure of high-quality connections therefore determines the connectivity of the network, and the connections can be influenced in a significant way by retiming the arrivals and departures.

The Connection Optimization Problem can be formulated as a mixed-integer linear programming problem in terms of timetable shift variables t_i , connection time variables Δ_{ij}^s , and binary connection variables x_{ij}^s as follows:

$$\max \sum x_{ij}^s, \quad (\text{COP})$$

$$\text{s.t. } \Delta_{ij}^s \geq \text{Inf}_{ij}^s - (\text{Inf}_{ij}^s - m_{ij}^s) x_{ij}^s \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (1)$$

$$\Delta_{ij}^s \leq \text{Sup}_{ij}^s - (\text{Sup}_{ij}^s - m_{ij}^s - \beta) x_{ij}^s \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (2)$$

$$\Delta_{ij}^s = \bar{d}_j^s + t_j - \bar{a}_i^s - t_i \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (3)$$

$$\text{Inf} \leq t_i \leq \text{Sup} \quad \forall i \in I, \quad (4)$$

$$x_{ij}^s \in \{0, 1\} \quad \forall i, j \in I, s \in S_i \cap S_j. \quad (5)$$

The objective of (COP) maximizes the number of feasible connections. Constraints (1) and (2) determine the feasibility of a connection; here, $\text{Inf}_{ij}^s := \bar{d}_j^s - \text{Inf} - \bar{a}_i^s + \text{Sup}$ is a lower and $\text{Sup}_{ij}^s := \bar{d}_j^s + \text{Sup} - \bar{a}_i^s - \text{Inf}$ an upper bound on the transfer time from train i to train j at station s . The equation (3) computes the transfer time, while (4) limits the time shifts. Constraint (5) sets the domain of the connection variables.

Note that the connection time variables can be eliminated. In fact, Giacco et al. [10] used the constraints

$$1 + \frac{\Delta_{ij}^s - m_{ij}^s}{1440} \geq x_{ij}^s \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (6)$$

$$1 + \frac{m_{ij}^s + \beta - \Delta_{ij}^s}{1440} \geq x_{ij}^s \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (7)$$

instead of COP (1-3), assuming a time resolution of one minute and a time horizon of one day or 1440 minutes; these are integer equivalent, but weaker from an LP relaxation point of view. There is furthermore no need to stipulate the integrality of the time shifts, because there is always an optimal integer solution if all input data is integer (as we required). We remark that one can replace the uniform shifts t_i by more flexible station-dependent shifts t_{is} , which can be increasing, or coupled, etc., but there is no space to discuss such versions here. We also do not consider headway times for pairs of preceding and succeeding trains sharing a track segment, but associated constraints can in principle be added to the model; for example, Ke et al. [14, 15] considered this degree of freedom.

The COP is theoretically difficult, but computationally tractable for the instances that we target.

► **Theorem 1.** *The Connection Optimization Problem is strongly NP-hard.*

Proof. A reduction of the Max-Cut Problem to the Passenger Connection Optimization Problem, in which all parameters are 0/1, can be found in [13]. ◀

2.2 The Passenger Connection Optimization Problem

A problem of the basic model is that it ignores the importance of the connections for the passengers, as one connection is as good as another. This issue could be mitigated by introducing connection weights, but a more satisfying solution is to compute the actual

passenger flow in order to maximize the satisfied demand, which is the overall purpose of improving the connections. In this vein, we assume that there is a maximal demand given by an od -cube $D \in \mathbb{Q}_{\geq 0}^{S \times S \times H}$ over a discrete (hourly) time horizon H (of one day). For each triple of origin and destination stations $o, d \in S$ and hour h we consider a set P_{odh}^r of railway paths that connect o and d along time-feasible trips and connections between these trips; the set $P^r = \bigcup P_{odh}^r$ of all these paths represents the services that the railway offers. A railway path p is feasible with respect to a timetable shifting if all of its connections are feasible. For a path $p \in P_{odh}^r$ let $y_p \in [0, 1]$ be the fraction of the demand D_{odh} routed on p , and let $\kappa_a \in \mathbb{Q}_{\geq 0}$ be the maximum capacity of trip $a \in A$. A passenger flow is feasible if it ships some or all of the demand on feasible railway paths and in such a way that the capacities on all trips are respected. The *Passenger Connection Optimization Problem* (PCOP) is to determine time shifts such that a weighted sum of the satisfied demand and the number of feasible connections is maximized.

Extending the basic IP model by passenger flow variables y_p , the PCOP can be formulated as a mixed-integer program as follows:

$$\begin{aligned}
\max \quad & \lambda \sum_{(i,j,s) \in C} x_{ij}^s + \mu \sum_{odh \in S^2 \times H} \sum_{p \in P_{odh}^r} D_{odh} y_p & (\text{PCOP}) \\
\text{s.t.} \quad & \Delta_{ij}^s + (\text{Inf}_{ij}^s - m_{ij}^s) x_{ij}^s \geq \text{Inf}_{ij}^s & \forall i, j \in I, s \in S_i \cap S_j, & (8) \\
& \Delta_{ij}^s + (\text{Sup}_{ij}^s - m_{ij}^s - \beta) x_{ij}^s \leq \text{Sup}_{ij}^s & \forall i, j \in I, s \in S_i \cap S_j, & (9) \\
& \Delta_{ij}^s = \bar{d}_j^s + t_j - \bar{a}_i^s - t_j & \forall i, j \in I, s \in S_i \cap S_j, & (10) \\
& \text{Inf} \leq t_i \leq \text{Sup} & \forall i \in I, & (11) \\
& x_{ij}^s \in \{0, 1\} & \forall i, j \in I, s \in S_i \cap S_j, & (12) \\
& \sum_{p \in P_{odh}^r} D_{odh} y_p \leq D_{odh} & \forall o, d \in Z, o \neq d, h \in H, & (13) \\
& \sum_{a \in p \in P^r} D_{odh} y_p \leq \kappa_a & \forall a \in A, & (14) \\
& 0 \leq y_p \leq x_{ij}^s & \forall p \in P^r, (i, j, s) \in p, & (15) \\
& 0 \leq y_p \leq 1 & \forall p \in P^r. & (16)
\end{aligned}$$

The objective of PCOP is a weighted sum of the number of connections and the satisfied demand subject to weights $\lambda, \mu \geq 0$. Constraints (8-12) are the same as in COP. Constraints (13-14) define the capacitated passenger flow, while 15 makes sure that only feasible paths are used. The demand constraints (13) could be normalized, and the upper bounds on the passenger flow (16) are redundant, but both constraints will play a role later with different right-hand sides. The PCOP is an integrated connection optimization and passenger routing problem, where the passengers are routed in a system-optimal way as a multicommodity flow over a given set of railway paths. It is clearly more difficult than the COP, but computationally tractable for large-scale instances and reasonable numbers of paths per odh -triple.

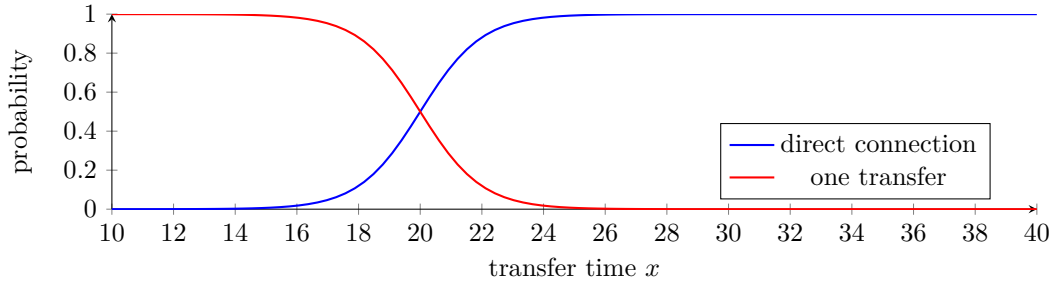
► **Theorem 2.** *The Passenger Connection Optimization Problem is strongly NP-hard.*

Proof. A reduction of the 3-SAT Problem to the Passenger Connection Optimization Problem, in which all parameters are polynomial, can be found in [13]. ◀

2.3 The Logit Connection Optimization Problem

The main drawback of the passenger connection optimization approach is that it computes the passenger flow as a system optimum, while travelers in reality make independent and selfish choices. They may, in particular, choose not to travel at all, or by another mode

such as the car; they may also switch from car to rail due to attractive connections. A widely accepted method to model this behavior in a more realistic way are discrete choice models [18, 1, 25]. These associate a utility with every travel alternative, which reflects its overall attractiveness, and stipulate that a high utility results in a high probability of choice, which is assumed to be feasible. This is a problem in the presence of constraints on the supply, such as capacity constraints. Various options are available to deal with this issue, such as modifying the utility by congestion terms, maximizing the feasible utility, or reallocating excess demand to secondary choices. We will use the latter approach by constructing a capacity-constrained passenger flow that is close to the passenger distribution arising from a logit model. This reflects the behavior of travelers who initially strive for the best trains, and resort to secondary alternatives when these are overcrowded. Logit models often consider zone-based demands, which make the selection of the start and end station a degree of freedom. This is important in regional rail traffic, and simultaneously improves and coarsens the demand data, so we do that, and we will consider the car as an additional mode and source of new railway customers.



■ **Figure 1** Probabilities of two alternative travel choices depending on a transfer time x according to a logit model in which the direct connection has observable utility -150 , the connection with one transfer has observable utility $-130 - x$, and the random variation is Gumbel $G(0, 1)$ distributed.

Let Z be a partition of the set of stations S into zones $z \in Z, z \subseteq S$, let $D \in \mathbb{Q}^{Z \times Z \times H}$ be the associated zone-based demand cube, and let P_{odh}^c and P_{odh}^r denote a set of car and railway paths, respectively, between origin zone o and destination zone d that start in hour h (between h and $h + 1$). Denote by $P_{odh} = P_{odh}^c \cup P_{odh}^r$ the set of all paths connecting the odh -triple odh , and by $P := P^c \cup P^r$ the set of all passenger paths. The paths are evaluated by a utility function $u : P \times \mathbb{Z}^I \rightarrow \mathbb{R}$ that associates with each path and time shift a value that represents the quality of this path for the passengers. A(n unnnested) logit model assumes that the percentage passenger flow on a path $p \in P_{odh}$ and a timetable shifting t , i.e., the fraction of the demand D_{odh} routed on p , is

$$\bar{y}_p = \frac{e^{u_p(t)}}{\sum_{p \in P_{odh}} e^{u_p(t)}}. \quad (17)$$

The logit(-induced passenger) flow $\bar{y}$ depends in a non-linear and non-convex way on the shifting of the timetable, see Fig. 1. However, this dependence is relatively small, and does by itself not lead to a substantial flow redistribution. The main impact of the shifting is still the effect on the feasibility of connections, as in the basic model, and this in turn redistributes the logit flow. The *Logit Connection Optimization Problem* (LCOP) is the task to find a shifting for the train timetable that maximizes the number of passengers taking the train

subject to a passenger flow that differs as little as possible from the logit flow. One way to make this goal precise is to formulate the LCOP as a mixed-integer non-linear program adding variables $\bar{y}_p$ for the logit flow and s_p for a path-wise excess over the logit flow:

$$\max \lambda \sum_{(i,j,s) \in C} x_{ij}^s + \mu \sum_{odh \in Z^2 \times H} \sum_{p \in P_{odh}^r} D_{odh} y_p - \nu \sum_{p \in P^r} s_p \quad (\text{LCOP})$$

$$\text{s.t. } \Delta_{ij}^s + (Inf_{ij}^s - m_{ij}^s) x_{ij}^s \geq Inf_{ij}^s \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (18)$$

$$\Delta_{ij}^s + (Sup_{ij}^s - m_{ij}^s - \beta) x_{ij}^s \leq Sup_{ij}^s \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (19)$$

$$\Delta_{ij}^s = \bar{d}_j^s + t_j - \bar{a}_i^s - t_j \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (20)$$

$$Inf \leq t_i \leq Sup \quad \forall i \in I, \quad (21)$$

$$x_{ij}^s \in \{0, 1\} \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (22)$$

$$\sum_{\substack{p \in P_{odh}^r \\ u_p(t) \leq u(q,t)}} D_{odh} y_p \leq \frac{\sum_{\substack{p \in P_{odh}^r \\ u_p(t) \leq u(q,t)}} e^{u_p(t)}}{\sum_{p \in P_{odh}^r} e^{u_p(t)}} D_{odh} \quad \forall o \neq d \in Z, h \in H, q \in P_{odh}^r, \quad (23)$$

$$\sum_{a \in p \in P^r} D_{odh} y_p \leq \kappa_a \quad \forall a \in A, \quad (24)$$

$$0 \leq y_p \leq x_{ij}^s \quad \forall p \in P^r, (i, j, s) \in p, \quad (25)$$

$$0 \leq y_p \leq \bar{y}_p + s_p \quad \forall p \in P^r, \quad (26)$$

$$\bar{y}_p = \frac{e^{u_p(t)}}{\sum_{p \in P_{odh}^r} e^{u_p(t)}} \quad \forall p \in P^r, \quad (27)$$

$$0 \leq s_p \quad \forall p \in P^r. \quad (28)$$

The first two terms of the objective and the constraints (18-22) and (24-25) are as in the PCOP. The new demand constraints (23) limit the railway passenger flow for each odh -triple to the railway share of the modal split according to the logit model; this share is computed for all levels of utilities in order to enforce a similarity to the logit flow. Constraint (27) computes the logit flow, while (26) and (28) capture the individual deviations from the logit flow for each railway path; here, the constraints (26) replace the upper bounds on the passenger flow in the PCOP. The sum of the deviations is penalized in the new third term of the objective, which is weighed by a factor $\nu \geq 0$. We remark that the LCOP is certainly more complex than its predecessors, but that a formal analysis would require a definition that avoids the computation of irrational numbers.

It is known that a logit flow can also be computed by maximizing the associated entropy or, alternatively, minimizing the perceived travel time, which are concave resp. convex functions, see, e.g., van Lieshout & Dalmeijer [26]. One could therefore alternatively replace the demand constraints (23) and (27) by such an objective, resulting in a MIP with a concave objective. Such a model is usually treated by outer approximation, which in this case will produce cutting planes with the same purpose as our demand constraints. This approach is technically more demanding and not the subject of this paper.

3 Solving the Logit Connection Optimization Problem

LCOP is a mixed-integer non-linear non-convex programming problem of high algebraic complexity. For the scenarios with 2,000 stations and 10,000 trains that we have in mind, it is also very large. As an exact solution appears extremely difficult, if not out of reach, we

propose two heuristic approaches that approximate the LCOP in terms of PCOP problems. The first, which we call mode-based flow distribution, solves a single PCOP to distribute an “idealized logit flow”. The second tries to get closer to the logit distribution by additionally minimizing the total path deviation.

3.1 Mode-Based Flow Distribution

The mode-based flow distribution method attempts to solve the relaxation of the LCOP that arises from deleting the path-based deviation part in the objective as well as the associated constraints (26-28), and from replacing the right-hand sides of the demand constraints (23) by best possible shares. These shares are computed in terms of best possible utilities for the railway paths, which overestimates the demand for rail transport. This produces a high railway passenger volume, which is useful in order to identify timetable optimization potentials. Let $\bar{u}_p$ and $\underline{u}_p$ be the maximal and minimal utility of a passenger path p , i.e., the utility for a (hypothetical) timetable shifting that always produces minimal and maximal connection times, respectively, and let

$$\bar{U}_q := \frac{\sum_{p \in P_{odh}^r : \underline{u}_p \leq \bar{u}_q} e^{\bar{u}_p}}{\sum_{p \in P_{odh}} e^{\underline{u}_p}} \quad (29)$$

be an idealized maximal railway share of the modal-split for the best possible utility level of path q . We remark that in our application $\underline{u}$ and $\bar{u}$ are not as far apart as one might fear. The mode-based approach tries to distribute a flow with these idealized shares according to the following mixed-integer program:

$$\max \lambda \sum_{(i,j,s) \in C} x_{ij}^s + \mu \sum_{odh \in Z^2 \times H} \sum_{p \in P_{odh}^r} D_{odh} y_p \quad (\text{MCOP})$$

$$\text{s.t. } \Delta_{ij}^s + (\text{Inf}_{ij}^s - m_{ij}^s) x_{ij}^s \geq \text{Inf}_{ij}^s \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (30)$$

$$\Delta_{ij}^s + (\text{Sup}_{ij}^s - m_{ij}^s - \beta) x_{ij}^s \leq \text{Sup}_{ij}^s \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (31)$$

$$\Delta_{ij}^s = \bar{d}_j^s + t_j - \bar{a}_i^s - t_j \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (32)$$

$$\text{Inf} \leq t_i \leq \text{Sup} \quad \forall i \in I, \quad (33)$$

$$x_{ij}^s \in \{0, 1\} \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (34)$$

$$\sum_{\substack{p \in P_{odh}^r \\ \underline{u}_p(t) \leq \bar{u}_q(t)}} D_{odh} y_p \leq \bar{U}_q D_{odh} \quad \forall o \neq d \in Z, h \in H, q \in P_{odh}^r, \quad (35)$$

$$\sum_{a \in p \in P^r} D_{odh} y_p \leq \kappa_a \quad \forall a \in A, \quad (36)$$

$$0 \leq y_p \leq x_{ij}^s \quad \forall p \in P^r, (i, j, s) \in p. \quad (37)$$

This MIP is a PCOP with a lot of demand constraints, namely, for say k paths per odh -triple, k such constraints instead of one in the standard PCOP. For reasonable values of k , this is still tractable.

► **Theorem 3.** *MCOP is a relaxation of LCOP and hence provides an upper bound.*

Proof.

$$\bar{U}_q = \frac{\sum_{p \in P_{odh}^r : \underline{u}_p \leq \bar{u}_q} e^{\bar{u}_p}}{\sum_{p \in P_{odh}} e^{\underline{u}_p}} \geq \frac{\sum_{p \in P_{odh}^r : \underline{u}_p(t) \leq \bar{u}_q(t)} e^{\underline{u}_p(t)}}{\sum_{p \in P_{odh}} e^{\underline{u}_p(t)}} \quad \forall p \in P^r, t \in [\text{Inf}, \text{Sup}]^I. \quad (38)$$

◀

3.2 Iterative Flow Distribution

One might expect that the many demand constraints of the mode-based approach guarantee a good fit with the logit flow not only on a utility-level basis, but also for each individual path. This is, however, not the case, because the logit flow needs to be substantially rerouted in order to avoid congested trains, and this destroys the similarity. The iterative flow distribution method attempts to improve the fit on a path basis by restricting the deviation. We do that by means of upper bounds $\bar{y}_p$ on the path flow, which are initialized with the logit flow y_p . Recall that the logit model assigns too much flow to attractive trains, more than these trains can transport, such that the logit upper bounds for such paths will be inactive for a capacitated passenger flow. The excess logit flow can only be rerouted along other paths, which then have to transport more than the logit flow, but they can't, because their upper bound constraints coincide with the logit flow and are hence tight. We must therefore raise the active upper bound constraints in order to provide capacity for the excess flow, and do so in a uniform way. This allows to reroute some excess flow, but normally not all of it. We therefore raise the active bounds again and repeat until either all excess flow is routed, or all upper bounds become inactive. Denoting the upper bounds by $\bar{y}_p$, the following mixed-integer programming problem is solved in every iteration:

$$\max \lambda \sum_{(i,j,s) \in C} x_{ij}^s + \mu \sum_{odh \in Z^2 \times H} \sum_{p \in P_{odh}^r} D_{odh} y_p \quad (\text{ICOP}(\bar{y}))$$

$$\text{s.t. } \Delta_{ij}^s + (\text{Inf}_{ij}^s - m_{ij}^s) x_{ij}^s \geq \text{Inf}_{ij}^s \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (39)$$

$$\Delta_{ij}^s + (\text{Sup}_{ij}^s - m_{ij}^s - \beta) x_{ij}^s \leq \text{Sup}_{ij}^s \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (40)$$

$$\Delta_{ij}^s = \bar{d}_j^s + t_j - \bar{a}_i^s - t_j \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (41)$$

$$\text{Inf} \leq t_i \leq \text{Sup} \quad \forall i \in I, \quad (42)$$

$$x_{ij}^s \in \{0, 1\} \quad \forall i, j \in I, s \in S_i \cap S_j, \quad (43)$$

$$\sum_{\substack{p \in P_{odh}^r \\ u_p(t) \leq u(q,t)}} D_{odh} y_p \leq \bar{U}_q D_{odh} \quad \forall o \neq d \in Z, h \in H, q \in P_{odh}^r \quad (44)$$

$$\sum_{a \in p \in P^r} D_{odh} y_p \leq \kappa_a \quad \forall a \in A, \quad (45)$$

$$0 \leq y_p \leq x_{ij}^s \quad \forall p \in P^r, (i, j, s) \in p, \quad (46)$$

$$0 \leq y_p \leq \bar{y}_p \quad \forall p \in P^r. \quad (47)$$

The update of the bounds is documented in Alg. 1, which raises the bounds uniformly and only so much as to produce a theoretical chance to reroute all of the excess flow. As the bounds only increase, and the number of paths is finite, this method stops after a finite number of iterations. It aims at distributing all the logit flow, keeping the deviation as uniform as possible. The method is also satisfactory from a behavioral point of view, as it mimics the experiences of travellers who run into problems trying to buy tickets for overcrowded trains, or trying to board them in case of time-card owners, and then must change to secondary alternatives, which might again be overcrowded, and so on. ICOP($\bar{y}$) adds the upper bound constraints (47) to the MCOP and is also a PCOP problem. As such, it is computationally tractable for our application.

Algorithm 1 Iterative Flow Distribution.

Input: (LCOP)**Output:** feasible timetable shifting t , connections x , passenger flow y

```

1:  $(t, x, y) \leftarrow \text{argmin (MCOP)}$   $\triangleright$  compute initial timetable shifting and passenger flow
2:  $\bar{y} \leftarrow \bar{y}(t)$   $\triangleright$  compute logit flow associated with this shifting
3:  $\epsilon = \sum_{p \in P^r} \bar{y}_p - \sum_{p \in P^r} y_p$   $\triangleright$  compute total excess flow
4:  $Q \leftarrow \{p \in P^r : y_p = \bar{y}_p\}$   $\triangleright$  get paths with tight upper bounds
5: if  $\epsilon = 0$  or  $Q = \emptyset$  then  $\triangleright$  everything routed?
6:   return  $(t, x, y)$ 
7: end if
8:  $\bar{y}_p \leftarrow \bar{y}_p + \epsilon/|Q|$  for all  $p \in Q$   $\triangleright$  raise active bounds uniformly to route excess flow
9:  $(t, x, y) \leftarrow \text{argmin (ICOP)}$   $\triangleright$  resolve
10: goto 3  $\triangleright$  iterate

```

4 Computational Experiments

We evaluated our solution approach for the LCOP on a real-world timetable of Italy. It contains roughly 10,000 trips, performing 115,000 stops at 2,225 stations. The demand data originates from mobile phone data on (long-distance) public transport, car, and flight movements. It was then aggregated and assigned to origin zones, destination zones, and starting hours. In this way, 800 zones with an hourly granularity over a single day come up. We decided to neglect the air demand as the overall number of travelers was very small, and air travel mostly connected islands with each other and the mainland of Italy. For each of the remaining *odh*-triples with a demand of at least one passenger and each possible shifting interval, a set of at most 5 paths was computed in the railway graph that contains all connections that are potentially feasible with respect to the chosen shifting interval. The six resulting path sets were merged into a single one, such that each *odh*-triple had at most 30 paths to choose from. This set was considered in every run. The path set was constructed in such a way that it is possible to route the railway demand given from the mobile phone data in the unshifted static case, i.e., without additional demand drawn from other modes of transport. This path choice plays a crucial role for the optimization process. Tab. 1 reports the instance characteristics in dependence of a shifting interval. The row headlined *Given paths* shows the total number of paths in the scenario, i.e., ≈ 2.7 million paths. These paths were filtered by checking whether the required connections can be feasible within the shifting window of the instance. This leads to ≈ 1.56 million *Considered paths* for a shifting window of $[0, 0]$ minutes up to ≈ 2.735 million paths for a shifting window of $[-5, 5]$ minutes. The last row shows the number of *odh-triples with demand* larger than a given threshold, namely, in all our instances, a value of 1, which leads to 376,000 considered *odh-triples*. In all our computations, a connection time window of $\beta = 30$ minutes was considered in all runs, and we changed the objective function slightly by scaling the demand induced by an *odh*-triple by the distance in kilometers between the two zones. This makes longer train connections more important than short ones.

In the logit based models, we assume that the observable utility ν_p of a railway path depends affinely on the travel time in high-speed (ht) and low-speed travel time (lt), the number of connections (nc), the waiting time between two trips (ct), access (a) and egress (e) times, the ticket price (tp), and the usage of high-speed trains (hu); these were the attributes for which data was available. Let $b_a(d(o, d))$ be coefficients for each attribute from

■ **Table 1** Demand and path characteristics for all considered experiments.

	[-0,0]	[-1,1]	[-2,2]	[-3,3]	[-4,4]	[-5,5]
Given paths ($\cdot 10^6$)	2.735	2.735	2.735	2.735	2.735	2.735
Considered paths ($\cdot 10^6$)	1.556	1.828	2.125	2.360	2.567	2.735
<i>odh</i> -triples with demand ($\cdot 10^6$)	0.376	0.376	0.376	0.376	0.376	0.376

$\{a, ht, lt, nc, ct, tp, hu, e\}$, depending on the distance $d(o, d)$ of the origin and destination zones o and d . Further, let $v(p, a)$ be the value of attribute a associated with path p , i.e., in case of for example ht the time spent on board of high-speed trains along p . The observable utility of a railway path p is then computed as

$$\nu_p := \sum_{a \in \{a, ht, lt, nc, ct, tp, hu, e\}} b_a(d(o_p, d_p)) v(p, a)$$

and the overall utility as $u_p = \nu_p + \epsilon_p$, where the non-observable utility ϵ_p is Gumbel distributed. In the case of a car path, the observable utility depends on the total travel time and the fuel consumption, which are both independent of the timetable shifting. As we did not have any further information, we consider the observable car utility as constant for any *od*-pair; this can be interpreted as considering exactly one car path for every *odh*-triple. All coefficients associated with the utility functions are industrial data that were calibrated to match the passenger flows of the unshifted timetable. Of course, the observable rail utility increases when the connection time decreases.

All computations were performed on an Intel(R) Core(TM) i7-9700K CPU @ 3.60GHz with 64 GB of RAM using Gurobi 10.0.1 [11] with an LP-IP gap tolerance of 0.01, a run time limit of 3,600 seconds (per iteration), and a maximum of eight threads in parallel.

4.1 Mode-Based Flow Distribution Results

Tab. 2 reports statistics of the solutions computed by the mode-based flow distribution approach. They provide upper bounds on the actual optimization potentials. The first row of the table contains the run time in seconds, and the second the integrality gap. The approach produces nearly optimal solutions within one hour of computation time. Only the two runs for a shifting interval of $[-4, 4]$ and $[-5, 5]$ minutes terminate at the runtime limit, but with reasonably small optimality gaps of 1.6% and 2.5%, respectively. The next row headlined *Aggregated routed pkm* shows the total passenger kilometers, i.e., the routed demand of an *odh*-triple times the distance in kilometers, added over all *odh*-triples. One can see that even the slight shift of plus-minus one minute bears the potential of increasing the aggregated routed passenger kilometers by almost 2.5%, and up to a $\approx 6.7\%$ increase in case of the largest shifting interval $[-5, 5]$; these are significant improvement potentials. Row *Aggregated expected pkm* reports the sum of all idealized maximum demand values $\bar{U}_q D_{odh}$ over all *odh*-triples such that q is feasible. These numbers can be interpreted as the maximum routeable passenger kilometers over all *odh*-triples if there were no train capacities. Comparing the different values for the different shifting intervals reveals that there is also an increase, though a slightly smaller one, ranging from 1.8% for no shifts vs. plus-minus one minute shifts to $\approx 4.9\%$ increase for $[-5, 5]$ minute shifts. Comparing with the values of the flow that gets routed and the expected flow without capacities, it becomes clear that there is a lost potential between 10.3% for $[-5, 5]$ minute shifts up to 11.8% for $[0, 0]$ shifts. The next two rows of the table labeled *Aggregated routed demand* and *Aggregated expected demand* show the numbers for the aggregated routed demand of the computed solutions,

■ **Table 2** Results for the mode-based flow distribution approach.

	[-0,0]	[-1,1]	[-2,2]	[-3,3]	[-4,4]	[-5,5]
Runtime(s)	8.47	26.02	96.79	724.94	3600	3600
Gap	0.0	0.007	0.009	0.009	0.016	0.025
Aggregated routed pkm($\cdot 10^6$)	104.33	106.89	108.93	110.47	111.02	111.34
Aggregated expected pkm($\cdot 10^6$)	118.35	120.51	121.93	123.20	123.77	124.18
Aggregated routed demand	531573	550468	563670	574419	579490	581223
Aggregated expected demand	592074	611250	622890	632757	638196	641971
Average deviation	8.80	7.70	6.90	6.50	6.30	6.20

■ **Table 3** Results for the iterative flow distribution approach after one iteration.

	[-0,0]	[-1,1]	[-2,2]	[-3,3]	[-4,4]	[-5,5]
Total time(s)	70.14	98.83	170.39	732.73	3703.20	3707.14
Aggregated routed pkm ($\cdot 10^6$)	97.54	99.74	101.36	102.59	103.37	103.50
Aggregated expected pkm ($\cdot 10^6$)	118.35	120.52	121.91	123.19	124.05	124.18
Aggregated routed demand	502470	518964	530252	538093	545226	546659
Aggregated expected demand	592074	611389	622726	632491	640047	642400
Number routed paths	371354	405574	425314	437012	456732	449807
Average deviation	0.70	0.80	0.80	0.80	0.80	0.80

and the aggregated values of the theoretically routable demand for the final shifting. Not surprisingly, these numbers follow a pattern that is similar to the travel kilometer values. In a nutshell, there is an even slightly larger potential to increase the demand when increasing the shifting interval, ranging from a $\approx 3.5\%$ to a $\approx 9.3\%$ increase for $[-1, 1]$ and $[-5, 5]$ shifts, and likewise for the demand that could not be routed ranging between $\approx 10.2\%$ and $\approx 9.5\%$. The values in the last row of the table report the *Average deviation* of the routed flow from the logit distribution of the flow over all feasible paths; it is unfortunately quite large.

4.2 Iterative Flow Distribution Results

The characteristics of the solutions computed by the iterative flow distribution approach can be found in Tab. 3 and 4 for one iteration and five iterations. The first row of both tables, headlined *Total time*, shows the total computation time in seconds, i.e., the time that it takes from computing the initial shifting and the associated flow to the final distributed flow. Most of the time is needed in the first iteration; indeed, the four additional iterations require in the worst case approximately 4 minutes of additional computation time.

Tab. 3 reports, as before, the numbers for the *Aggregated routed pkm*, *Aggregated expected pkm*, *Aggregated routed demand*, and *Aggregated expected demand* for each instance. Comparing the numbers for the aggregated routed pkm and the demand with those of the mode-based approach reveals a notable dip that results from enforcing the logit distribution of the flow over all feasible paths of each *odh*-triple with demand. The decrease ranges between 6.5% and 7.1% for the aggregated routed pkm and 5.4% and 6.3% for the routed demand. The values for the aggregated expected pkm and the aggregated expected demand, however, do not decrease at all. This implies that the timetable shifting does not differ too much between the solutions of the mode distribution approach and the iterative distribution approach, such that the passenger flow is simply more realistic. Investigating the numbers for the average deviation of the routed flow from the logit distribution reveals that the iterative approach achieves a significant improvement in that category, as the maximum average deviation is reduced to 0.8 over all instances in comparison to 8.80 for the mode-based approach.

■ **Table 4** Results for the iterative flow distribution approach after five iterations.

	[-0,0]	[-1,1]	[-2,2]	[-3,3]	[-4,4]	[-5,5]
Total time(s)	201.28	260.28	354.65	952.22	3923.97	3936.02
Aggregated routed pkm($\cdot 10^6$)	100.44	102.99	104.68	106.19	107.04	107.19
Aggregated expected pkm($\cdot 10^6$)	118.35	120.52	121.91	123.19	124.05	124.18
Aggregated routed demand	517335	536208	548000	557654	565443	567190
Aggregated expected demand	592074	611389	622726	632491	640047	642400
Number routed paths	362917	388528	405733	415298	432269	425393
Average deviation	0.90	0.90	1.00	1.00	1.00	1.00

Tab. 4 reveals at first glance that the additional iterations to redistribute the unrouted excess flow require only a marginal increase in computation time, while the numbers for aggregated routed pkm and routed demand show a significant increase in comparison to the numbers for the solutions after one iteration of the iterative flow distribution method. The values for aggregated expected pkm and demand do not differ at all from the values after one iteration and only slightly from the numbers computed by the mode-based flow distribution. Comparing the numbers in Tab. 4 for aggregated routed pkm and demand to the other two tables shows that the additional iterations of flow redistribution make up for 3.6% to 3.9% of the aggregated routed pkm, and 2.6% to 2.9% of the aggregated routed demand. Thus the gap between the mode-based upper bound and the result of the iterative flow distribution is closed by at least 52% for the aggregated routed pkm and by 40% for the aggregated routed demand. This is quite a lot.

As more flow is routed on secondary paths, the values for the average deviation from the logit flow on feasible paths increase, but rather marginally, from at most 0.8 for the solutions found after the first iteration to a deviation of at most 1.0 for solutions found after five iterations. can be observed for the number of routed paths in Tab. 3 and 4. Here each path is counted if it bears at least some flow. One can see that the iterative flow distribution approach routes the flow on way more paths, but that this number decreases with an increasing number of iterations, which is a direct consequence of the rerouting procedure and a tradeoff with respect to routing more demand. that there is a steeper improvement for the shifting intervals of $[-1, 1]$, $[-2, 2]$, $[-3, 3]$ minutes, while the additional improvement flattens a bit for the larger intervals. This shows that most of the potential can be harvested with relatively small shiftings.

To provide some insight into the transfer volumes at stations, Fig. 2 plots the aggregated numbers of passengers using newly established connections per station. In each of the four subfigures, there is a circle for each station with newly established connections, centered at the station's location, with a radius corresponding to the number of transferring passengers, i.e., a larger radius means more transferring passengers. Subfigure 2a shows the results for the solution of the mode-based approach and shifting interval $[-5, 5]$. There are only green circles, which correspond to positive numbers of passengers using newly established connections. In contrast to that, the other three subfigures 2b, 2c, and 2d, show differences in numbers of passengers using newly established connections between each combination of the solution of the iterative distribution approach with one (I1) and five (I5) rounds and the mode-based approach (MB), e.g., subfig. 2b (I1-MB) plots in green/red circles how many more/less passengers use newly established connections as a result of I1 in comparison to MB, and so on for the other combinations. All figures show that newly established connections are used to route passengers all over the country and not only at regional clusters. The two subfigures comparing the solutions of the iteratively distributed flow to the mode-based approach solution(2b and 2c) show more green circles, i.e., more stations with

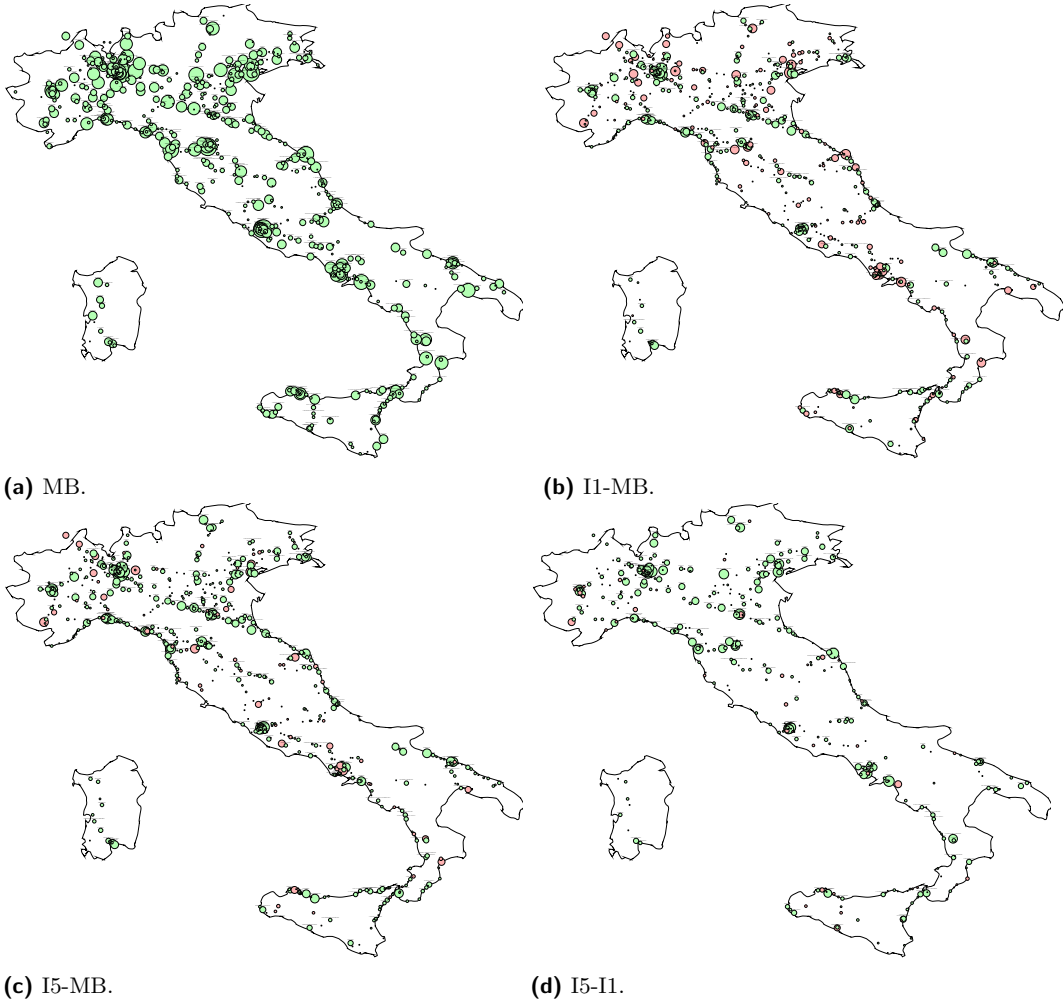


Figure 2 Illustration of the aggregated transfer volumes for newly established connections at stations for solutions of the different approaches considering a shifting interval of $[-5, 5]$.

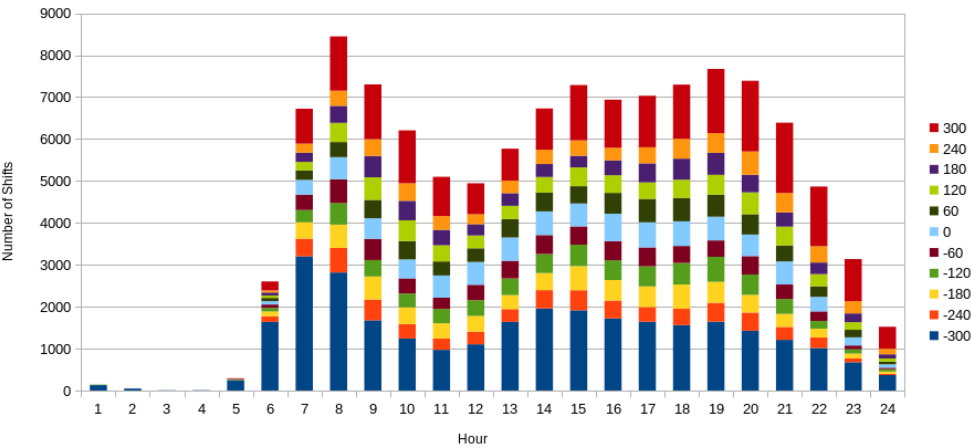


Figure 3 Hourly distribution of time shifts of trains for the solution computed with five iterations of flow distribution considering a shifting interval of $[-5, 5]$.

newly established transfers, which is a consequence of the larger number of paths over which the passenger flow becomes distributed. Looking at the red circles, i.e., stations with less transfers, we see more and larger red circles for Subfig. 2b as compared to Subfig. 2c. The reason for that is that the overall amount of flow is smaller for I1 compared to I5, as well as that I5 routes more flow on paths shared with MB than I1. Finally, Subfig. 2b compares the transfers for the solutions of I1 and I5. The larger number and radii of green circles here show that there is a significantly increased number of passengers using newly established connections for solutions of I5.

Fig. 3 shows the distribution of time shifts of the trains in the optimized timetable computed with five rounds of iterative flow distributions considering a shifting interval of $[-5, 5]$. It is representative for all our approaches: they all show a similar pattern in shifting the largest number of trains by the most negative value, followed by the largest positive value as a runner-up. For most hours, these two shifts comprise more than 50% of all time shifts. Especially in the morning, there is a large number of negative time shifts that decreases to the end of the day, while the number of positive shifts tends to increase toward the end of the day, i.e., the timetable is spread apart. Additionally, one can see that roughly 90% of all trains get shifted in some way. Thus, even if the individual shifts of the trains are small, the number of shifts and their impact on the timetable is large. The overall curve follows the classical steep-morning-wider-afternoon peak “camel distribution”.

5 Conclusion

In this paper, we study the complex task of connection optimization considering elastic demands. To define the problem, we iteratively transformed a basic formulation of the connections problem into a version that includes a logit model to predict passenger behavior. As the problem is non-linear and even non-convex, we presented two approximation heuristics, the mode-based distribution method and the iterative distribution method, based on a mixed-integer linear programming formulation that is used as the backbone of the heuristics. The heuristics are evaluated on real-world data of the Italian long-distance railway network. It turns out that it is possible to solve the model for nationwide scenarios in reasonable time, and that there is a significant potential to increase passenger demand by shifting the timetable by a small margin. In comparison to operating or buying more vehicles, not to speak of building more infrastructure, timetable shifting is a promising way to increase passenger satisfaction and generate higher revenues at a relatively low cost.

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A

 Appendix

Table 5 Definition of used variables and sets.

I	Set of trains.
S	Set of stations.
S_i	Set of stations at which train i stops.
A	Set of trips.
Z	Set of zones.
t_i^s	Integer variable deciding how the stop of train i at station s is shifted.
x_{ij}^s	Binary decision variable if connection between i and j at station s is feasible.
y_p	Rational variable measuring the fraction of demand of D_{od}^h routed on path p .
$u_p(t)$	Function computing the utility of path p for timetable shifting t .
$\bar{u}_p$	Upper bound on the utility on path p for all timetable shiftings.
$\underline{u}_p$	Lower bound on the utility on path p for all timetable shiftings.
$\bar{y}_p$	Rational variable measuring the logit passenger flow on path p .
s_p	Rational variable measuring the deviation from the logit flow on path p .
Inf_{ij}^s	Minimum time shift difference of stops at s of trips i and j .
Sup_{ij}^s	Maximum time shift difference of stops at s of trips i and j .
Inf_i^s	Minimum time shift difference of stop s of trip i .
Sup_i^s	Maximum time shift difference of stop s of trip i .
m_{ij}^s	Minimum transfer time between stops of i and j at s .
β	Maximum connection time.
P_{odh}^r	Set of railway paths between zones o and d departing between hour h and $h + 1$.
P_{odh}^c	Set of car paths between zones o and d departing between hour h and $h + 1$.
P_{odh}	Set of travel paths between zones o and d departing between hour h and $h + 1$.
P	Set of all travel paths.
κ_a	Capacity of trip a .
$\bar{U}_p$	Upper bound on the modal split share for railway paths with utility of p or less.
λ	Non-negative objective weight for connections.
μ	Non-negative objective weight for the railway passenger volume.
ν	Non-negative objective penalty for path-based deviations from the logit flow.