

# Bidimensional Structures: Algorithms, Combinatorics and Logic

Edited by

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## Abstract

We provide a report on the Dagstuhl Seminar 13121: *Bidimensional Structures: Algorithms, Combinatorics and Logic* held at Schloss Dagstuhl in Wadern, Germany between Monday 18 and Friday 22 of March 2013. The report contains the motivation of the seminar, the abstracts of the talks given during the seminar, and the list of open problems.

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## 1 Executive Summary

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The monumental Graph Minors project developed by Robertson and Seymour in the 1980s is one of the most fundamental achievements of Combinatorics. The project had several groundbreaking consequences for Theoretical Computer Science. However, the wide spread opinion in the algorithmic community, expressed by David S. Johnson in his NP-Completeness Column (J. Algorithms 1987), was that it is mainly of theoretical importance. It took some time to realize that the techniques developed in Graph Minors can be used in the design of efficient and generic algorithms. One of the main techniques extracted from Graph Minors is based on the structural results explaining the existence (or the absence) of certain grid-like or bidimensional structures in graphs. The usage of bidimensional structures and the related width parameters in many areas of Computer Science and Combinatorics makes such techniques ubiquitous.

Historically, the first applications of bidimensional structures are originated in Graph Minors of Robertson and Seymour, because of the structure of the graphs excluding some fixed some graph as a minor. There is still an on-going work in Combinatorics on obtaining new



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structural theorems. There are much more examples in Combinatorics, where bidimensional structures and width parameters play a crucial role like in obtaining Erdős-Pósa type of results. Reed used bidimensional structures to settle Erdős-Hajnal conjecture on near-bipartite graphs. Kawarabayashi and Reed used bidimensional structures to bound the size of a minimal counterexample to Hadwiger's conjecture. Demaine and Hajiaghayi optimized the original grid-exclusion theorem on  $H$ -minor free graphs.

The usage of bidimensional structures and width parameters in Algorithms goes back to the parameter of treewidth, introduced in the Graph Minors series. Treewidth is now ubiquitous in algorithm design and expresses the degree of topological resemblance of a graph to the structure of a tree. Its algorithmic importance dates back in the early 90's to the powerful meta-algorithmic result of Courcelle asserting that all graph problems expressible in Monadic Second Order Logic can be solved in linear time on graphs of bounded treewidth. Bounded treewidth can be guaranteed by the exclusion of certain bidimensional structures. Intuitively, this exclusion is what enables the application of a series of classic algorithmic techniques (divide-and-conquer, dynamic programming, finite automata) for problems of certain descriptive complexity. This phenomenon was perhaps the first strong indication of the deep interleave between graph structure and logic in graph algorithms. However, a deeper understanding of it became more evident during the last decade and produced powerful meta-algorithmic techniques.

Apparently, graph-theoretic fundamentals emerging from the Graph Minors project developed by Robertson and Seymour, are used currently in several areas of Computer Science and Discrete Mathematics. Algorithmic fertilization of these ideas occurred mostly in the context of parameterized complexity and its foundational links to logic. The course of developing a structural algorithmic graph theory revealed strong connections between Graph Theory, Algorithms, Logic, and Computational Complexity and joined a rapidly developing community of researchers from Theoretical Computer Science and Discrete Mathematics.

Dagstuhl seminar 13121 brought together some of the most active researchers on this growing field.

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### 3 Overview of Talks

#### 3.1 A welcome to treewidth

*Dimitrios M. Thilikos (University of Athens, GR)*

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This talk is a short introduction to the various fields where the graph invariant of treewidth has been important. This includes: (i) the deep combinatorial results of the Graph Minors Series of Robertson and Seymour, (ii) the meta-algorithmic framework initiated by Courcelle's theorem, (iii) the derivation of dynamic programming algorithms and its multiple applications in parameterized algorithms and complexity, and (iv) the design of FPTAS for NP-hard problems. It is stressed that, in all above fields, combinatorial results concerning *bidimensional structures* in graphs, such as the *Grid Exclusion Theorem*, have played an important role.

#### 3.2 Bidimensionality and its applications I: Bidimensionality: Yesterday, Today and Tomorrow

*Saket Saurabh (University of Bergen, NO)*

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In this talk we give the chronological survey of the development of Bidimensionality as a field. This talk will set up all the necessary definitions for talks to come and explain the key developments in the area in details.

#### 3.3 Bidimensionality and its applications II: Graph Surgery and Kernelization


*Daniel Lokshantov (University of Bergen, NO)*

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Over the last few years there has been quite a few results in parameterized algorithms and kernelization that are based on cutting away a piece of the input instance, analyzing it, and replacing it by a smaller, equivalent piece. In this talk we outline a language in which it is natural to talk about such operations when the considered instances are graphs. In particular we define bounded graphs together with some operations on them, and show how these operations have some nice algebraic properties. We then proceed to use this language to show that a large class of parameterized problems admit linear kernels on any class of graphs excluding a fixed  $H$  as a minor.

### 3.4 Treewidth and dimension

*Gwenaël Joret (Université Libre de Bruxelles, BE)*

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Over the last 30 years, researchers have investigated connections between dimension for posets and planarity for graphs. Here we extend this line of research to the structural graph theory parameter tree-width by proving that the dimension of a finite poset is bounded in terms of its height and the tree-width of its cover graph.

### 3.5 Canonical tree-decompositions, $k$ -blocks and tangles

*Reinhard Diestel (University of Hamburg, DE)*

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A  $k$ -block in a graph  $G$  is a set  $X$  of at least  $k$  vertices no two of which are separated in  $G$  by fewer than  $k$  other vertices (which may or may not lie in  $X$ ). In joint work with Carmesin, Hundertmark and Stein I recently proved that, for each  $k$ , every graph has a ‘canonical’ tree-decomposition whose adhesion sets have order  $< k$  and separate any two  $k$ -blocks, which thus come to lie in different parts of the decomposition. The decompositions are *canonical* in that the automorphisms of the graph act on their sets of parts inducing automorphisms of the decomposition trees. The following algorithmic problems may be of interest, given an integer  $k$  and a graph  $G$ : to find all the  $k$ -blocks in  $G$ ; to construct a canonical tree-decomposition as above; to use these decompositions to solve Graph Isomorphism in polynomial time for suitable classes of graphs.

### 3.6 Subexponential-time parameterized algorithm for Steiner Tree on planar graphs

*Erik Jan van Leeuwen (Max-Planck Institut für Informatik, Saarbrücken, DE)*

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Bidimensionality theory provides a method for designing fast, subexponential-time parameterized algorithms for a vast number of NP-hard problems on sparse graph classes such as planar graphs, bounded genus graphs, or, more generally, graphs with a fixed excluded minor. However, in order to apply the bidimensionality framework the considered problem needs to fulfill a special density property. Some well-known problems do not have this property, unfortunately, with probably the most prominent and important example being the Steiner Tree problem. Hence the question whether a subexponential-time parameterized algorithm for Steiner Tree on planar graphs exists has remained open. In this talk, we answer this question positively and develop an algorithm running in time subexponential in  $k$  and polynomial space, where  $k$  is the size of the Steiner tree. Our algorithm does not rely on tools from bidimensionality theory or graph minors theory, apart from Baker’s classical approach.

Instead, we introduce new tools and concepts to the study of the parameterized complexity of problems on sparse graphs.

### 3.7 Rank based algorithms for bounded treewidth graphs

*Marek Cygan (University of Warsaw, PL)*

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**Joint work of** Marek Cygan, Hans Bodlaender, Jesper Nederlof, Stefan Kratsch

During the talk we will present a new approach to algorithms in bounded treewidth graph, which relates a DP computation with a rank computation of an appropriately defined matrix. Using this tool we obtain deterministic  $c^{\text{tw}(G)}n^{O(1)}$  time algorithms in graphs of treewidth  $\text{tw}$  for problems such as Steiner Tree, Hamiltonicity or Feedback Vertex Set. Moreover we show how to solve weighted variants of those problems in the same running time and describe a different approach which allowed us to compute the number of Hamiltonian cycles in  $c^{\text{tw}(G)}n^{O(1)}$  time.

### 3.8 Decomposing quantified conjunctive (or disjunctive) formulas

*Victor Dalmau (Universitat Pompeu Fabra, ES)*

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Model checking-deciding if a logical sentence holds on a structure-is a basic computational task that is well-known to be intractable in general. For first-order logic on finite structures, it is PSPACE-complete, and the natural evaluation algorithm exhibits exponential dependence on the formula. We study model checking on the quantified conjunctive fragment of first-order logic, namely, prenex sentences having a purely conjunctive quantifier-free part. Following a number of works, we associate a graph to the quantifier-free part; each sentence then induces a prefixed graph, a quantifier prefix paired with a graph on its variables. We give a comprehensive classification of the sets of prefixed graphs on which model checking is tractable, based on a novel generalization of treewidth, that generalizes and places into a unified framework a number of existing results.

### 3.9 Branch Decompositions and Linear Matroids

*Illya V. Hicks (Rice University, US)*

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**Joint work of** Illya V. Hicks, Edray Goins, Jing Ma, Susan Margulies

This talk gives a general overview of practical computational methods for computing branch decompositions for linear matroids and their usage for solving integer programs. The concept of branch decompositions and its related invariant branch width were first introduced by Robertson and Seymour in their proof of the Graph Minors Theorem and can be easily generalized for any symmetric submodular set function.

### 3.10 A 5-approximation for treewidth using linear time, single exponential in the treewidth

*Hans L. Bodlaender (Utrecht University, NL)*

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
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**Joint work of** Pål Grønås Drange, Markus S. Dregi, Fedor Fomin, Daniel Lokshantov and Michał Pilipczuk

We give an algorithm that, given a graph  $G = (V, E)$  and an integer  $k$ , either finds a tree decomposition of  $G$  of width at most  $5k + 4$ , or decides that the treewidth of  $G$  is more than  $k$ . The algorithm uses  $O(c^k \cdot n)$  time on graphs with  $n$  vertices. This is the first algorithm of the type that is both single exponential in the treewidth and linear in the number of vertices. Earlier algorithms either use quadratic time, or use  $2^{\Omega(k \log k)}$  steps when time is measured as a function of  $k$ . The algorithm uses various techniques, including a data structure that allows several queries to be executed in  $O(\log n)$  time, a table lookup technique for large values of  $n$ . As a consequence of our result, many problems allow algorithms whose time is linear in the number of vertices and single exponential in the treewidth; the result removes the need of a tree decomposition given as part of the input.

### 3.11 Contraction Decomposition: a new technique for H-minor-free graphs

*Erik Demaine (MIT, US)*


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Many problems are closed under contractions but not deletions, suggesting that we develop a Graph Contraction Theory to parallel Graph Minor Theory. Alas, some theorems like well-quasi-ordering do not hold in this setting. Nonetheless, we have been able to develop many contraction analogs to minor results. One powerful such result is contraction decomposition, which splits the edges of any graph into  $k$  pieces such that contracting any piece results in a graph of bounded treewidth. This approach has led to the best approximation algorithms for Traveling Salesman Problem on graphs.

### 3.12 Topological problems in tournaments

*Michał Pilipczuk (University of Bergen, NO)*

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The containment theory for tournaments was developed recently by Chudnovsky, Fradkin, Kim, Scott, and Seymour. It appears that the natural containment notions in this setting form well-quasi-orderings, and correspond to two natural width measures, namely pathwidth and cutwidth. This creates possibilities for many algorithmic applications, including XP and FPT algorithms. During the talk, we will survey the status of algorithmic results on topological problems in tournaments, with particular focus on fixed-parameter tractability and similarities with bidimensionality



### 3.13 Kernelization using structural parameters on sparse graph classes

*Felix Reidl (RWTH Aachen, DE)*

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Meta-theorems for polynomial (linear) kernels have been the subject of intensive research in parameterized complexity. Heretofore, there were meta-theorems for linear kernels on graphs of bounded genus,  $H$ -minor-free graphs, and  $H$ -topological-minor-free graphs. To the best of our knowledge, there are no known meta-theorems for kernels for any of the larger sparse graph classes: graphs of bounded expansion, locally bounded expansion, and nowhere dense graphs. In this paper we prove meta-theorems for these three graph classes. More specifically, we show that graph problems that have finite integer index (FI) have linear kernels on graphs of bounded expansion when parameterized by the size of a modulator to constant-treewidth graphs. For graphs of locally bounded expansion, our result yields a quadratic kernel and for nowhere dense graphs, a polynomial kernel. While our parameter may seem rather strong, we show that a linear kernel result on graphs of bounded expansion with a weaker parameter will necessarily fail to include some of the problems included in our framework. Moreover, we only require problems to have FI on graphs of constant treewidth. This allows us to prove linear kernels for problems such as LONGEST PATH/CYCLE, EXACT  $(s, t)$ -PATH, TREewidth, and PATHwidth which do not have FI in general graphs.

### 3.14 A new proof for the weak-structure theorem with explicit constants

*Paul Wollan (University of Rome “La Sapienza”, IT)*

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The Weak Structure Theorem of Robertson and Seymour is the cornerstone of many of the algorithmic applications of graph minors techniques. The theorem states that any graph which has both large tree-width and excludes a fixed size clique minor contains a large, nearly planar subgraph. In this talk, we will discuss a new proof of this result which is significantly simpler than the original proof of Robertson and Seymour. As a testament to the simplicity of the proof, one can extract explicit constants to the bounds given in the theorem ensuring a linear relationship between the size of the grid minor and the size of the planar subgraph guaranteed by the theorem.

### 3.15 Approximability and fixed parameter algorithms: a new look

*Rajesh Chitnis (University of Maryland, US)*

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Traditionally fixed-parameter algorithms (FPT) and approximation algorithms have been considered as different approaches for dealing with NP-hard problem. The area of fixed-parameter approximation algorithms tries to tackle problems which are intractable to both

these techniques. In this talk we will start with the formal definitions of fixed-parameter approximation algorithms and give a brief survey of known positive and negative results. Then (under standard conjectures in computational complexity) we show the first fixed-parameter inapproximability results for Clique and Set Cover, which are two of the most famous fixed-parameter intractable problems. On the positive side we obtain polynomial time  $f(OPT)$ -approximation algorithms for a number of  $W[1]$ -hard problems such as Minimum Edge Cover, Directed Steiner Forest, Directed Steiner Network, etc. Finally we give a natural problem which is  $W[1]$ -hard, does not have a constant factor approximation in polynomial time, but admits a constant factor FPT-approximation.

### 3.16 Excluded vertex-minors for graphs of linear rank-width at most $k$

*Sang-il Oum (KAIST – Daejeon, KR)*

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Linear rank-width is a graph width parameter, which is a variation of rank-width by restricting its tree to a caterpillar. As a corollary of known theorems, for each  $k$ , there is a finite set  $\mathcal{O}_k$  of graphs such that a graph  $G$  has linear rank-width at most  $k$  if and only if no vertex-minor of  $G$  is isomorphic to a graph in  $\mathcal{O}_k$ . However, no attempts have been made to bound the number of graphs in  $\mathcal{O}_k$  for  $k \geq 2$ . We construct, for each  $k$ ,  $2^{\Omega(3^k)}$  pairwise locally non-equivalent graphs that are excluded vertex-minors for graphs of linear rank-width at most  $k$ . Therefore the number of graphs in  $\mathcal{O}_k$  is at least double exponential.

### 3.17 Definability of numerical graph parameters and various notions of width

*Johann A. Makowsky (Technion – Haifa, IL)*

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Graph parameters (and properties) definable in Monadic Second Order Logic (possibly with modular counting) are FPT computable for graphs of bounded width (where the width notion may vary depending on the logical presentation of the (hyper)-graph. It is therefore desirable to be able to show not only definability, but also non-definability. We present a method to show this for graph parameters which take values in a field which is even new for graph properties. For a graph parameter  $f$  and a binary operation on graphs  $\square$  the Hankel matrix  $H(f, \square)$  is the infinite matrix where rows and columns are labeled by graphs  $G_i$  and the entry  $H_{i,j}$  is given by  $f(G_i, G_j)$ . The methods are based in Hankel matrices (aka connection matrices) and the finite Rank Theorem (B. Godlin, T. Kotek and J.A. Makowsky, 2008) which states the MSOL-definable graph parameters have Hankel matrices of finite rank, provided  $\square$  behaves nicely with respect to the logic. We show that many examples of well studied graph parameters are not MSOL-definable even on ordered structures and with modular counting. A striking example is the chromatic number, for which non-definability was known before only using the complexity assumption ETH.

(Joint work with T. Kotek, published in the Proceedings of CSL'2012, LIPICS)

### 3.18 The $k$ -disjoint paths problem in directed planar graphs

*Dániel Marx (MTA – Budapest, HU)*

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**Joint work of** Marek Cygan, Marcin Pilipczuk, and Michał Pilipczuk

Given a graph  $G$  and  $k$  pairs of vertices  $(s_1, t_1), \dots, (s_k, t_k)$ , the  $k$ -vertex-disjoint paths problem asks for pairwise vertex disjoint paths  $P_1, \dots, P_k$  such that  $P_i$  goes from  $s_i$  to  $t_i$ . Schrijver proved that the  $k$ -vertex-disjoint paths problem on planar directed graphs can be solved in time  $O(n^k)$ . We give an algorithm with running time  $2^{2^{O(k^2)}} \cdot n^{O(1)}$  for the problem, that is, we show the fixed-parameter tractability of the problem.

### 3.19 Surface split decompositions: fast dynamic programming over branch decompositions for graphs of bounded genus

*Paul Bonsma (University of Twente, NL)*

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Surface split decompositions are a special kind of branch decomposition, for graphs embedded on a surface of bounded genus. They are a direct generalization of sphere cut decompositions for planar graphs. Surface split decompositions have been introduced in [P. Bonsma, Surface split decompositions and subgraph isomorphism in graphs on surfaces, STACS 2012], where also a surprisingly simple method is introduced to obtain improved complexity bounds for various dynamic programming algorithms, provided that the given branch decomposition is a surface split decomposition. (In the aforementioned paper, this is only applied to the Subgraph Isomorphism problem.) In this talk, I will first give an introduction to surface split decompositions, and discuss their algorithmic applications in general. Next, I will introduce the following conjecture: Given a branch decomposition of a graph embedded in a surface of bounded genus, in polynomial time it is possible to construct a surface split decomposition where the width is increased by at most an (additive) constant. A proof of this conjecture, combined with known bidimensionality techniques, would make it possible to easily prove strong (e.g. subexponential) complexity bounds for many problems on graphs of bounded genus. This would give simplified proofs for various best known complexity bounds of this kind, and also enable new results in this direction.

### 3.20 Tools for multicoloring with applications to planar graphs and bounded treewidth graphs

*Guy Kortsarz (Rutgers University-Camden, US)*

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In this paper, we study scheduling jobs with conflicts on two fundamental classes of graphs: planar graphs and bounded tree width graph. The problem is represented by a graph in which a vertex is a job and has a work demand  $p(v)$ , meaning it takes  $p(v)$  time units to finish

the job  $v$ . A solution for  $v$  is,  $p(v)$  integers, the least of which is 1 or larger, that represent the  $p(v)$  rounds in which  $v$  is active. And a general solution is an assignment for every  $v$ . The jobs (vertices) compete on resources and thus at every round only an independent set can be processed. The independent set has no cardinality bound meaning no bound on the number of processors (but the case of bounded number of processors can be handled easily). The colors of  $v$  can be non-preemptive  $i, i + 1, \dots, j - 1, j$  so that  $j - i + 1 = p(v)$ , or arbitrary (preemptive) Perhaps our main contribution is designing very *general tool* for multicoloring graphs that are of independent interest. These results *should be* by other papers (at least if they know our paper). The max measure calls for minimizing the maximum number of every vertex, hence the makespan. The sum version required to minimize the sum of largest numbers assigned to a vertex. Hence the sum of completing times of jobs. For the preemptive makespan multicoloring of bounded tree width graphs we give a PTAS with quite unique properties. The coloring depends on the independent sets chosen and the  $p(v)$  but never on the edges of  $G$ . Also there are always at most  $O(\log n)$  preemptions. For the non-preemptive minimum makespan of bounded tree width graphs we get a FPTAS that applies to a large collection of functions (not only max and sum). For sum multicoloring (both preemptive and non preemptive) of Planar graphs and bounded tree width graphs we provide a PTAS. Our algorithms are quite non-trivial.

### 3.21 What makes normalized weighted satisfiability tractable?

Iyad A. Kanj (DePaul University – Chicago, US)

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We consider the weighted monotone and antimonotone satisfiability problems on normalized circuits of depth at most  $t \geq 2$ , abbreviated  $\text{WSAT}^+[t]$  and  $\text{WSAT}^-[t]$ , respectively. These problems model the weighted satisfiability of monotone and antimonotone propositional formulas (including weighted monotone/antimonotone CNF-SAT in a natural way, and serve as the canonical problems in the definition of the parameterized complexity hierarchy. In particular,  $\text{WSAT}^+[t]$  ( $t \geq 2$ ) is  $W[t]$ -complete for even  $t$  and  $W[t - 1]$ -complete for odd  $t$ , and  $\text{WSAT}^-[t]$  ( $t \geq 2$ ) is  $W[t]$ -complete for odd  $t$  and  $W[t - 1]$ -complete for even  $t$ . Moreover, several well-studied problems, including important graph problems, can be modeled as  $\text{WSAT}^+[t]$  and  $\text{WSAT}^-[t]$  problems in a straightforward manner. We characterize the parameterized complexity of  $\text{WSAT}^+[t]$  and  $\text{WSAT}^-[t]$  with respect to the genus of the circuit. For  $\text{WSAT}^-[t]$ , we give a precise characterization:  $\text{WSAT}^-[t]$  is fixed-parameter tractable (FPT) on circuits whose genus is  $n^{o(1)}$ , where  $n$  is the number of the variables in the circuit, and it has the same  $W$ -hardness as the general  $\text{WSAT}^-[t]$  problem (i.e., with no restriction on the genus) on circuits whose genus is  $n^{\Omega(1)}$ . For  $\text{WSAT}^+[2]$  (i.e., weighted monotone cnf-sat and  $\text{WSAT}^+[2]$ , which are both  $W[2]$ -complete, the characterization is also precise:  $\text{WSAT}^+[2]$ , and  $\text{WSAT}^+[3]$ , are FPT if the genus is  $n^{o(1)}$  and  $W[2]$ -complete if the genus is  $n^{\Omega(1)}$ . For  $\text{WSAT}^+[t]$  where  $t > 3$ , we show that it is FPT if the genus is  $O(\sqrt{\log n})$ , and that it has the same  $W$ -hardness as the general  $\text{WSAT}^+[t]$  problem if the genus is  $n^{\Omega(1)}$ . The above results give, via standard parameterized reductions, precise characterizations of the parameterized complexity of several problems with respect to the genus of the underlying graph, as shown in the current paper.

### 3.22 An excluded grid theorem for digraphs with forbidden minors

*Stephan Kreutzer (TU Berlin, DE)*

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One of the fundamental results in graph structure theory is the excluded grid theorem, proved by Robertson and Seymour, which states that every graph of sufficiently high tree-width contains a large grid as a minor. This theorem provides the structural foundation for algorithmic techniques such as bidimensionality and is also the basis for more advanced structure theorems in the graph minor series. In 1997, Reed and later, in 2001, Johnson, Robertson, Seymour and Thomas conjectured an excluded grid theorem for directed graphs, i.e. the existence of a function  $f : \mathbb{N} \rightarrow \mathbb{N}$  such that every digraph of directed tree-width at least  $f(k)$  contains a directed grid of order  $k$ . Johnson et al. proved the conjecture for planar digraphs in 2001 but for all other cases the conjecture remained open. In this talk we present a proof of the conjecture for the case of digraphs excluding a fixed undirected graph as a minor. We present the main proof ideas and give examples of possible algorithmic applications.

This is joint work with Ken-ichi Kawarabayashi.

### 3.23 Exclusion theorems for immersions on surface embedded graphs

*Archontia Giannopoulou (University of Bergen, NO)*

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In this talk, we provide a structural characterization of graphs that forbid a grid as an immersion and can be embedded in a surface of Euler genus  $g$ . In particular, we prove that a graph  $G$  that excludes some grid as an immersion and is embedded in a surface of Euler genus  $g$  has either “small” treewidth (bounded by a function of  $H$  and  $g$ ) or “small” edge connectivity (bounded by 3). By generalizing these techniques we also provide a structural characterization for the case where the excluded graph is any graph  $H$ .

### 3.24 Packing edge-disjoint odd S-cycles in 4-edge-connected graphs

*Yusuke Kobayashi (University of Tokyo, JP)*

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We develop theory for 4-edge-connected graphs. It has been known that for a 4-edge-connected graph  $G$ ,

1. there is a much simpler polynomial-time algorithm for the  $k$  edge-disjoint paths problem for fixed  $k$  (compared to a graph minor algorithm), and
2. the Erdős-Pósa property holds for edge-disjoint odd cycles, and there is a simple polynomial-time algorithm to test whether or not  $G$  has  $k$  edge-disjoint odd cycles for fixed  $k$ .


Note that the Erdős-Pósa property does not hold for edge-disjoint odd cycles in general. In this paper, we generalize the above results as follows:

1. there is a simple polynomial-time algorithm for the PARITY  $k$  EDGE-DISJOINT PATHS problem for fixed  $k$  (i.e., the length of each path is of a specified parity), and
2. the Erdős-Pósa property holds for edge-disjoint odd  $S$ -cycles (i.e., odd cycles through a vertex in a specified vertex set  $S$ ), and there is a simple polynomial-time algorithm to test whether or not  $G$  has  $k$  edge-disjoint odd  $S$ -cycles for fixed  $k$ .

This is joint work with Naonori Kakimura and Ken-ichi Kawarabayashi.

### 3.25 Tree-width of hypergraphs and surface duality

Frédéric Mazoit (Université Bordeaux, FR)

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In Graph minors III, Robertson and Seymour write: “It seems that the tree-width of a planar graph and the tree-width of its geometric dual are approximately equal? indeed, we have convinced ourselves that they differ by at most one”. They never gave a proof of this. We recently published a generalisation of this statement to embedding of hypergraphs on general surfaces, and we prove that our bound is tight. Although the result is purely graph theoretical, the proof uses a “surface-cut” decomposition which may find algorithmic applications for graphs on surfaces.

## 4 Open problems

We give a list of the problems presented on Monday, March 18, 2013 and Thursday, March 21, 2013 at the open-problem session of the Seminar on *Bidimensional Structures: Algorithms, Combinatorics and Logic*, held at Schloss Dagstuhl in Wadern, Germany.

### Canonical tree-decompositions, $k$ -blocks and tangles

Reinhard Diestel, University of Hamburg, Hamburg, R.Diestel@math.uni-hamburg.de

A  $k$ -block in a graph  $G$  is a set  $X$  of at least  $k$  vertices no two of which are separated in  $G$  by fewer than  $k$  other vertices (which may or may not be in  $X$ ). Such  $k$ -blocks might be of interest in applications; in a computer network, for example, they might specify the nodes at which one would place those servers that should be able to communicate with each other even when some  $\ell < k$  other nodes have failed. We thus pose the following algorithmic problem:

**Algorithmic Problem 1.** Given an integer  $k$  and a graph  $G$ , find all the  $k$ -blocks in  $G$ . A simple ad-hoc algorithm can do this in  $O(kn^4)$  time [1], but maybe this can be improved.

An important feature of  $k$ -blocks is that their connectivity need not lie in the subgraph they induce, but will more typically be provided by the ambient graph. Thus, while  $k$ -connected subgraphs are obvious candidates for  $k$ -blocks, other  $k$ -blocks might induce no edge at all. See [1] for examples of quite different types.

Although  $k$ -blocks were already considered by Mader [5], they came to the fore more recently as a key ingredient of a solution offered in [3] to an old problem in graph connectivity:

the problem of how to decompose a  $(k - 1)$ -connected graph into its ‘ $k$ -connected pieces’ in some structured way. The main results of [3] can be summarized as follows.

**Theorem 1.** For every integer  $k$ , every graph has a canonical tree-decomposition that distinguishes all its  $k$ -blocks efficiently.

(A tree-decomposition is *canonical* if it is invariant under the automorphisms of the graph. It *distinguishes* two  $k$ -blocks  $X_1, X_2$  if it has an adhesion set  $S$  of order  $< k$  that separates them, and it does so *efficiently* if  $S$  is no larger than the smallest  $X_1$ – $X_2$  separator in the graph (which will always have order  $< k$ ). Thus, every  $k$ -block lies in some part of the tree-decomposition, and distinct  $k$ -blocks lie in different parts.)

**Algorithmic Problem 2.** Given  $k$  and a graph  $G$ , find in  $G$  a tree-decomposition such as in Theorem 1.

We believe we can do this in  $O(k^3 n^4)$  time, but have not pursued the matter.

One may wonder whether every graph even has one unified tree-decomposition that distinguishes all its  $k$ -blocks, for all  $k$  simultaneously. This is not true in this generality, but it is almost true:

**Theorem 2.** Every graph has a canonical tree-decomposition that distinguishes all its robust blocks efficiently.

(*Robust*  $k$ -blocks are defined in [3], and the definition is technical. But most  $k$ -blocks are robust; for example, all  $k$ -blocks of size at least  $\frac{3}{2}k$  are. Loosely speaking, non-robust blocks are a rare technical phenomenon that can occur but usually does not. A *block* is a  $k$ -block for some  $k$ .)

**Algorithmic Problem 3.** Find a tree-decomposition such as in Theorem 2 for a given input graph.

An analysis of the proofs of Theorems 1 and 2 due to Hundertmark [4] shows that the only information about  $k$ -blocks we really use is how they relate to the  $(< k)$ -separations of the given graph  $G$ . Every  $k$ -block *orients* every  $(< k)$ -separation of  $G$  towards the side that contains it. For each  $k$ -block, these orientations are consistent in two ways. Given two nested separations  $(A, B)$  and  $(C, D)$  such that  $B \supseteq D$  and  $(C, D)$  is oriented towards  $D$ , then  $(A, B)$  is oriented towards  $B$ . Given two crossing separations  $(A, B)$  and  $(C, D)$  such that  $(A, B)$  is oriented towards  $B$  and  $(C, D)$  is oriented towards  $D$ , the ‘corner separation’  $(A \cup C, B \cap D)$  will be oriented towards  $B \cap D$  if it is oriented at all, i.e., if it has order  $< k$ . (If  $G$  is  $(k - 1)$ -connected, which is an important special case when we consider  $k$ -blocks, that will in fact be so.)

Call a set of orientations, one of each  $(< k)$ -separation of  $G$ , a *k-profile* if it satisfies these two consistency requirements. Thus, every  $k$ -block defines a *k-profile*. Hundertmark [1] showed that we can adapt the proofs of Theorems 1 and 2 so as to establish the existence of canonical tree-decompositions that distinguish all the *k-profiles* for any given  $k$  (Theorem 1), or all the ‘robust’ *k-profiles* for all  $k$  simultaneously. (A tree-decomposition *distinguishes* two *k-profiles* if it induces a separation of order  $< k - 1$  corresponding to an edge of the decomposition tree – that is oriented differently by the two profiles.)

The advantage of this more abstract approach is that there are *k-profiles* that do not come from  $k$ -blocks; so Theorems 1 and 2 become more comprehensive. For example, it is easy to see that every tangle of order  $k$  is a *k-profile*, and so we have canonical tree-decompositions that also distinguish all the maximal tangles in a graph (as well as their robust blocks). This strengthens a theorem of Robertson and Seymour [6], who proved the existence of such a

tree-decomposition which, however, is not canonical. (In order to select the required nested subset of  $(< k)$ -separations from the set of all these separations, they need a ‘tie-breaker’ that depends on a chosen vertex enumeration.)

All we need in order to define a  $k$ -profile on a set is to have a notion of separations of this set, and a notion of the order of such separations. These things are also given for matroids, and hence all our results generalize to matroids too [4].

Profiles not only generalize tangles, they are also special cases of preferences [4], which in turn are the prime examples of brambles. They are the weakest known way of consistently orienting all the  $(< k)$ -separations of a graph that still allows some tree-decomposition (canonical or not) to distinguish them all: there is in general no tree-decomposition that does this for preferences or brambles, since a graph of order  $n$  may have more than  $n$  preferences and brambles (but any minimal tree-decomposition separating them would have at most  $n$  parts).

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## Hyperbolicity

Blair D. Sullivan, Oak Ridge National Lab, [sullivanb@ornl.gov](mailto:sullivanb@ornl.gov)

An unweighted undirected graph  $G$  is  $\delta$ -hyperbolic if, for any four vertices  $u, v, w, x$  ordered so that  $d(u, v) + d(w, x) \geq d(u, w) + d(v, x) \geq d(u, x) + d(v, w)$ , we have  $[d(u, v) + d(w, x)] - [d(u, w) + d(v, x)] \leq \delta$ . Define the *hyperbolicity*  $\delta(G)$  to be the minimum such  $\delta$ . Define  $\nu(G)$  to be the maximum hyperbolicity over all metric cycles of  $G$ . (A cycle in  $G$  is *metric* if the distance between any two of its vertices is the same in  $G$  and in the cycle. So  $\nu(G) \approx |V(G)|/4$ .)

1. Is  $\delta(G) < \text{tw}(G) + \nu(G) - 1$ ? Or more generally, is  $\delta = O(\text{tw}(G) + \nu(G) + 1)$ ?
2. Can treewidth and treelength be simultaneously approximated when the hyperbolicity  $\delta$  is small? That is, is there a tree decomposition whose width is within a constant factor of the treewidth, and whose length is within a constant factor of the treelength? The *length* of a tree decomposition is the largest diameter of a bag (whereas width measures the maximum size of a bag). With large hyperbolicity, no simultaneous approximation is possible [1].

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## Spanning Trees Respecting Faces

Illya V. Hicks, Rice University, [ivhicks@rice.edu](mailto:ivhicks@rice.edu)

Given an embedded planar graph, what is the complexity of finding a spanning tree  $T$  that minimizes  $\max\{d_T(u, v) : u, v \text{ on a common face}\}$ ? If it is easier, you can assume that the graph is bipartite.

Blair Sullivan asks whether there is a polynomial-time  $O(1)$ -approximation.

The problem is motivated by a possible relation to computing planar branchwidth.

## Odd Immersion Hadwiger Conjecture

Bojan Mohar, Simon Fraser University, [mohar@sfu.ca](mailto:mohar@sfu.ca)

A graph  $H$  *immerses* in  $G$  if there are distinct vertices  $v_1, v_2, \dots, v_h$  in  $G$  corresponding to the  $h$  vertices  $\{1, 2, \dots, h\}$  in  $H$ , and there are edge-disjoint paths  $p_{ij}$  joining  $v_i$  to  $v_j$  in  $G$  for every edge  $(i, j)$  in  $H$ . Such an immersion is *odd* if all the paths  $p_{ij}$  have odd length.

**Conjecture:** If  $G$  has chromatic number at least  $k$ , then  $K_k$  oddly immerses into  $G$ .

This conjecture is a stronger (odd) form of the immersion Hadwiger conjecture [1, 2], which in turn is a variation of the classic Hadwiger's Conjecture: if  $G$  has chromatic number at least  $k$ , then  $K_k$  is a minor of  $G$ . The immersion Hadwiger conjecture, for example, is proved for  $k \leq 7$  [3].

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## Grid Minors

MohammadTaghi Hajiaghayi, University of Maryland, College Park, [hajiaghayi@gmail.com](mailto:hajiaghayi@gmail.com)

What is the smallest integer function  $f$  such that, if  $G$  is a graph of treewidth  $f(r)$ , then it has an  $r \times r$  grid minor? In particular, is  $f(r)$  polynomial?

**Conjecture:**  $f(r) = O(r^3)$  [2].

Robertson, Seymour, and Thomas [4] proved that  $f(r) \leq 2^{O(r^5)}$  and  $f(r) = \Omega(r^2 \log r)$ , and conjectured that the latter bound may be closer to the truth. Kawarabayashi and Kobayashi [3] improved the upper bound to  $f(r) \leq 2^{O(r^2 \log r)}$ . For  $H$ -minor-free graphs,  $f(r) = O(r)$  [1]. For map graphs,  $f(r) = O(r^3)$  [2]. Constant powers of such graphs also have polynomial bounds on  $f(r)$  [2].

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### Subexponential FPT Algorithms on Planar Graphs

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Is there a fixed-parameter algorithm with running time  $2^{\tilde{O}(\sqrt{k})}n^{O(1)}$  for the following problems?

1.  $k$  disjoint paths in directed planar graphs.
2. Steiner tree in planar graphs, parameterized by the size of the tree, or even by the number of terminals.
3. Subgraph isomorphism in planar graphs, parameterized by the size of the small graph.
4. Weighted versions of e.g. independent set or vertex cover.
5. Tree spanner: find a spanning tree of distortion at most  $k$ , parameterized by  $k$ . [Posed by Fedor Fomin.]

### Disjoint Paths in Planar Graphs

Daniel Lokshtanov, University of Bergen, daniello@ii.uib.no

For the  $k$  disjoint paths problem in planar graphs, is there a fixed-parameter algorithm with running time  $2^{k^{O(1)}}n^{O(1)}$ ?

We have a  $k^{2^{O(k)}}n^{O(1)}$ -time algorithm [1].

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### Spaghetti Treewidth

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A tree decomposition is *spaghetti* if every vertex appears in a path of bags (instead of a general subtree). *Spaghetti treewidth* is the minimum width of a spaghetti tree decomposition.

Is the class of graphs of spaghetti treewidth 2 closed under minors? If so, can we find the excluded minors?

A related, previously studied notion is “special treewidth” [1]. A rooted tree decomposition is *special* if every vertex appears in a rooted path of bags. The resulting parameter *special treewidth* falls somewhere between treewidth and pathwidth.

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## Bounded Rank

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The *bounded rank* approach leads to several singly exponential dynamic-programming algorithms [1].

Extend the applicability of this approach to the widest possible variety of problems.

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## Fast Treewidth

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Is there a fixed-parameter algorithm with running time  $2^{o(k^3)}n$  for computing the treewidth  $k$  in a general graph? Or is there some sort of lower bound? (There is an algorithm with running time  $2^{O(k^3)}n$  [1].)

Also recall the famous open problem: is treewidth NP-hard for planar graphs? Only a 1.5-approximation is known [2].

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## Disjoint Paths in Tournaments

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What is the parameterized complexity of the  $k$  disjoint paths problem in tournaments? In particular, is it fixed-parameter tractable? Chudnovsky, Scott, and Seymour [1] proved that it is in XP (there is an algorithm of running time  $n^{f(k)}$ ).

The  $k$  disjoint paths problem in general directed graphs is known to be NP-hard even for  $k = 2$  [2]. For directed acyclic graphs, it is known to be in XP but  $W[1]$ -hard [3].

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## Grid Theorem for Immersions

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Wollan [1] proved that, if  $G$  is 3-connected and does not contain a “large” wall as an immersion, then it should have large degree and large treewidth.

Is the class of graphs with small treewidth and small degree algorithmically interesting?

Are there parameterized problems that, while hard in general, are fixed-parameter tractable when restricted to this class?

Are there problems that, when parameterized with respect to treewidth or maximum degree are  $W[1]$ -hard, while they admit a fixed-parameter algorithm when both maximum degree and treewidth are bounded?

Is it possible to develop a bidimensionality theory based on immersions instead of minors?

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## Local Search Variant of Vertex Cover

Fedor Fomin, University of Bergen, fomin@ii.uib.no

In the  $k$ -LOCAL SEARCH FOR VERTEX COVER, we are given a graph  $G$  and a vertex cover  $S$  of  $G$ , the question is if  $k$ -exchange neighborhood of  $S$  contains a smaller vertex cover. In other words, is it possible to remove  $k$  vertices from  $S$  and add at most  $k - 1$  such that the resulting set is still a vertex cover? On general graphs the problem is known to be  $W[1]$ -hard parameterized by  $k$ . However, for planar graphs, it admits a singly exponential parameterized algorithm, i.e., of running time  $2^{O(k)}n^{O(1)}$  [1]. Is there a subexponential parameterized algorithm for this problem, i.e., of running time  $2^{o(k)}n^{O(1)}$ ? Similar questions can be asked for local search variants of Dominating Set and Odd Cycle Transversal. The other question is if the “local search” variants of other “connectivity” problems such as PLANAR TSP, LONGEST PATH, and PLANAR FEEDBACK VERTEX SET are FPT on planar graphs? See [2, 3] for  $W[1]$ -hardness of TSP on non-planar graphs.

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## Upper and Lower Bounds for Algorithms on Parameterized Problems

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**Problem 1:** Is there a  $3^k n^{O(1)}$  algorithm for FEEDBACK VERTEX SET?

**Problem 2:** It is known that there is a  $c^{\text{tw}(G)} n^{O(1)}$  algorithm for HAMILTONIAN CYCLE. Let  $c$  be the smallest (if exists) constant for which such an algorithm exists. Current lower and upper bounds establish that  $2 + \sqrt{2} \leq c \leq 4$  (the lower bound is up to the Strong Exponential Time Hypothesis). Find better estimations of the constant  $c$ .

**Problem 3:** Can we count the number of vertex feedback sets of size at most  $k$  of a graph in  $c^{\text{tw}(G)} n^{O(1)}$  time?

**Problem 4:** Recent advances [1, 2] on dynamic programming on tree width reveal that problems like HAMILTONIAN CYCLE, STEINER TREE, and FEEDBACK VERTEX SET can be solved by singly exponential randomized parameterized algorithms when parameterized by the treewidth of the input graph. Specifically, for each of these problems  $\Pi$ , there is a constant  $c_\Pi$  such that  $\Pi$  can be solved by a randomized algorithm in  $c_\Pi^{\text{tw}(G)} \cdot n$  time. For the same parameterized problems, there are also deterministic singly exponential algorithms, but the constants are worse. Is it possible to improve the deterministic constant to match the randomized constant?

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- 2 Marek Cygan, Jesper Nederlof, Marcin Pilipczuk, Michał Pilipczuk, Johan van Rooij, and Jakub Onufry Wojtaszczyk. Solving connectivity problems parameterized by treewidth in single exponential time. arXiv:1103.0534.

## Planar Completion to Bounded Diameter

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Given a plane graph  $G$  and a nonnegative integer  $k$ , is it possible to add at most  $k$  edges such that the resulting graph remains plane and has diameter at most  $k$ ?

If  $(G, k)$  is a YES instance of the above problem, and  $G'$  is a minor of  $G$ , then  $(G', k)$  is also a YES instance of the same problem. From the meta-algorithmic consequence of the Graph Minors series, it follows that the above problem is fixed-parameter tractable, i.e., there exists an algorithm with running time  $f(k)n^{O(1)}$  (see e.g. [1]). However, so far no such algorithm has been constructed.

A possible approach is to use the fact that, if  $(G, k)$  is a YES instance, then  $G$  has treewidth bounded by some function of  $k$ . Then a dynamic programming algorithm for PLANAR COMPLETION TO BOUNDED DIAMETER on graphs of bound treewidth would result in the construction of the desired algorithm.

More generally, how can we design bounded-treewidth dynamic-programming algorithms for planar completion problems?

## References

- 1 Michael R. Fellows and Michael A. Langston. Nonconstructive tools for proving polynomial-time decidability. *Journal of the ACM* 35(3):727–739, 1988.

## 5 Program

### Monday, March 18

- [09:00–09:15] A welcome to Dagstuhl
- [09:15–09:30] **Dimitrios M. Thilikos:** *A welcome to treewidth*
- [09:30–09:45] **Saket Saurabh:** *Bidimensionality and its applications (part I)*
- [09:45–11:00] **Daniel Lokshtanov:** *Bidimensionality and its applications (part II)*
- [11:00–11:30] Break
- [11:30–12:00] **Gwenaël Joret:** *Treewidth and dimension*
- [12:15] Lunch
- [15:00–15:40] **Reinhard Diestel:** *Canonical tree-decompositions, k-blocks and tangles*
- [15:45–16:30] Cake
- [16:30–17:00] **Erik Jan van Leeuwen:** *Subexponential-time parameterized algorithm for Steiner Tree on planar graphs*
- [17:00–18:00] Open Problem Session

### Tuesday, March 19

- [09:00–10:00] **Marek Cygan:** *Rank based algorithms for bounded treewidth graphs*
- [10:00–10:30] Break
- [10:30–11:00] **Victor Dalmau** *Decomposing quantified conjunctive (or disjunctive) formulas*
- [11:00–11:30] **Illya V. Hicks** *Branch Decompositions and Linear Matroids*
- [11:30–12:00] **Hans L. Bodlaender** *A 5-approximation for treewidth using linear time, single exponential in the treewidth*
- [12:00] **Photo** *in front of the Chapel*
- [12:15] Lunch
- [15:00–15:45] **Erik Demaine:** *Contraction Decomposition: a new technique for H-minor-free graphs*
- [15:45–16:30] Cake
- [16:30–17:00] **Michał Pilipczuk:** *Topological problems in tournaments*
- [17:00–17:30] **Felix Reidl** *Kernelization using structural parameters on sparse graph classes*

### Wednesday, March 20

- [09:00–10:00] **Paul Wollan:** *A new proof for the weak-structure theorem with explicit constants*
- [10:00–10:30] Break
- [10:30–11:00] **Rajesh Chitnis** *Approximability and fixed parameter algorithms: a new look*
- [11:00–11:30] **Sang-Il Oum:** *Excluded vertex-minors for graphs of linear rank-width at most k*
- [11:30–12:00] **Janos Makowski:** *Definability of numerical graph parameters and various notions of width*
- [12:15] Lunch

**Thursday, March 21**

- [09:00–10:00] **Dániel Marx:** *The  $k$ -disjoint paths problem in directed planar graphs*  
[10:00–10:30] Break  
[10:30–11:00] **Paul Bonsma:** *Surface split decompositions: fast dynamic programming over branch decompositions for graphs of bounded genus*  
[11:00–11:30] **Guy Kortsarz:** *Tools for multicoloring with applications to planar graphs and bounded treewidth graphs*  
[11:30–12:00] **Iyad A. Kanj:** *What makes normalized weighted satisfiability tractable?*  
[12:15] Lunch  
[15:30–17:00] Cake  
[17:00–18:00] Open Problem Session

**Friday, March 22**

- [09:00–10:00] **Stephan Kreutzer:** *An excluded grid theorem for digraphs with forbidden minors*  
[10:00–10:30] Break  
[10:30–11:00] **Archontia C. Giannopoulou:** *Exclusion theorems for immersions on surface embedded graphs*  
[11:00–11:30] **Yusuke Kobayashi:** *Packing edge-disjoint odd  $S$ -cycles in 4-edge-connected graphs*  
[11:30–12:00] **Frédéric Mazoit:** *Tree-width of hypergraphs and surface duality*  
[12:15] Lunch



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