Report from Dagstuhl Seminar 15261

# Logics for Dependence and Independence

Edited by

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#### Abstract

This report documents the programme and outcomes of Dagstuhl Seminar 15261 'Logics for Dependence and Independence'. This seminar served as a follow-up seminar to the highly successful seminar 'Dependence Logic: Theory and Applications' (Dagstuhl Seminar 13071). A key objective of the seminar was to bring together researchers working in dependence logic and in the application areas so that they can communicate state-of-the-art advances and embark on a systematic interaction. The goal was especially to reach those researchers who have recently started working in this thriving area.

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# Brief Introduction to the Topic

Dependence and independence are interdisciplinary notions that are pervasive in many areas of science. They appear in domains such as mathematics, computer science, statistics, quantum physics, and game theory. The development of logical and semantical structures for these notions provides an opportunity for a systematic approach, which can expose surprising connections between different areas, and may lead to useful general results.

Dependence Logic is a new tool for modeling dependencies and interaction in dynamical scenarios. Reflecting this, it has higher expressive power and complexity than classical logics used for these purposes previously. Algorithmically, first-order dependence logic corresponds exactly to the complexity class NP and to the so-called existential fragment of secondorder logic. Since the introduction of dependence logic in 2007, the framework has been



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generalized, e.g., to the contexts of modal, intuitionistic, and probabilistic logic. Moreover, interesting connections have been found to complexity theory, database theory, statistics, and dependence logic has been applied in areas such as linguistics, social choice theory, and physics. Although significant progress has been made in understanding the computational side of these formalisms, still many central questions remain unsolved so far.

The Dagstuhl seminar 'Dependence Logic: Theory and Applications' had a major impact to the field of dependence logic opening up connections to new application areas. The aim of this follow-up seminar was to gather together the people working in dependence logic and in the application areas, especially those researchers who have recently started working in this quickly developing area to communicate state-of-the-art advances and embark on a systematic interaction.

# Organization of the Seminar and Activities

The seminar brought together 38 researchers from mathematics, statistics, database theory, natural language semantics, and theoretical computer science. The participants consisted of both senior and junior researchers, including a number of postdocs and advanced graduate students.

Participants were invited to present their work and to communicate state-of-the-art advances. Over the five days of the seminar, 27 talks of various lengths took place. Introductory and tutorial talks of 90-60 minutes were scheduled prior to seminar. Most of the remaining slots were filled, mostly with shorter talks, as the seminar commenced. The organizers considered it important to leave ample free time for discussion.

The tutorial talks were scheduled during the beginning of the week in order to establish a common background for the different communities that came together for the seminar. The presenters and topics were:

- Jouko Väänänen and Juha Kontinen, Dependence Logic
- Bernhard Thalheim, Database Constraints A Survey
- Ilya Shpitser, Causal inference
- Lauri Hella, Modal dependence logic
- Ivanio Ciardelli, Dependency as Question Entailment
- Antti Hyttinen, Statistical Independence, Causality and Constraint Satisfaction

There were additionally two introductory talks with a more focused and technical topic:

- Alex Simpson, Sheaf semantics for independence logics
- Phokion Kolaitis, The Query Containment Problem: Set Semantics vs. Bag Semantics

Additionally, the following shorter presentations were given during the seminar:

- Asa Hirvonen, Model theoretic independence
- Kerkko Luosto, Dimensions for Modal Dependence Logic
- Gianluca Paolini, Measure teams
- Olaf Beyersdorff, Proof Complexity of Quantified Boolean Formulas
- Antti Kuusisto, Propositional dependence logic via Kripke semantics
- Johanna Stumpf, Characterisation of the expressive power of modal logic with inclusion atoms
- Sebastian Link, Dependence-driven, non-invasive cleaning of uncertain data
- Jonni Virtema, Complexity of Propositional Inclusion and Independence Logic
- Katsuhiko Sano, Characterizing Frame Definability in Team Semantics via The Universal Modality



**Figure 1** The blackboard after Jouko Väänänen's conclusion of the seminar.

- Raine Rönnholm, Expressing properties of teams in k-ary inclusion-exclusion logic
- Julian Bradfield, On the structure of events in Boolean games
- Fan Yang, Some proof theoretical results on propositional logics of dependence and independence
- Erich Grädel, Counting in Team Semantics
- Fredrik Engström, Generalized quantifiers and Dependence Logic
- Miika Hannula, Axiomatizing dependencies in team semantics
- Dietmar Berwanger, An NL-fragment of inclusion logic
- Nicolas de Rugy-Altherre, Tractability Frontier of Data Complexity in Team Semantics

For some of these, an abstract can be found below.

The seminar achieved its aim of bringing together researchers from various related communities to share state-of-the-art research. The organizers left ample time outside of this schedule of talks and many fruitful discussions between participants took place throughout the afternoons and evenings.

# **Concluding Remarks and Future Plans**

The organizers regard the seminar as a great success. Bringing together researchers from different areas fostered valuable interactions and led to fruitful discussions. Feedback from the participants was very positive as well.

Finally, the organizers wish to express their gratitude toward the Scientific Directorate of the Center for its support of this seminar.

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# **3** Overview of Talks

# 3.1 Proof Complexity of Quantified Boolean Formulas

Olaf Beyersdorff (University of Leeds, GB)

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 Main reference O. Beyersdorff, L. Chew, M. Janota, "Proof Complexity of Resolution-based QBF Calculi," in Proc. of the 32nd Int'l Symp. on Theoretical Aspects of Computer Science (STACS'15), LIPIcs, Vol. 30, pp. 76–89, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2015.
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The main aim in proof complexity is to understand the complexity of theorem proving. Arguably, what is even more important is to establish techniques for lower bounds, and the recent history of computational complexity speaks volumes on how difficult it is to develop general lower bound techniques. Understanding the size of proofs is important for at least two reasons. The first is its tight relation to the separation of complexity classes: NP vs. coNP for propositional proofs, and NP vs. PSPACE in the case of proof systems for quantified boolean formulas (QBF). The second reason to study lower bounds for proofs is the analysis of SAT and QBF solvers: powerful algorithms that efficiently solve the classically hard problems of SAT and QBF for large classes of practically relevant formulas.

In this talk we give an overview of the relatively young field of QBF proof complexity. We explain the main resolution-based proof systems for QBF, modelling CDCL and expansionbased solving. In the main part of the talk we will give an overview of current lower bound techniques (and their limitations) for QBF systems. In particular, we exhibit a new and elegant proof technique for showing lower bounds in QBF proof systems based on strategy extraction. This technique provides a direct transfer of circuit lower bounds to lengths of proofs lower bounds.

Potential connections to dependence logic arise through dependencies between quantified variables. These are used frequently in QBF and QBF solving, and are systematically studied in the NEXPTIME-complete logic of dependency QBFs (DQBF).

#### 3.2 On the structure of events in Boolean games

Julian Bradfield (University of Edinburgh, GB)

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 Joint work of Bradfield, Julian; Gutierrez, Julian; Wooldridge, Michael
 Main reference J. Bradfield, J. Gutierrez, M. Wooldridge, "On the structure of events in boolean games,"

Presentation at the 11th Conf. on Logic and the Foundations of Game and Decision Theory

(LOFT'14), University of Bergen, Norway, July 27–30, 2014.

As conventionally formulated, Boolean games assume that all players make choices simultaneously, and act in complete ignorance of the choices being made by other players. For many settings, these assumptions represent gross over simplifications. In this paper, we show how Boolean games can be enriched by *dependency graphs* which explicitly represent the dependencies between choices in a game. These dependency graphs allow us to directly specify what a player knows about other choices when that player makes a choice. In addition, they capture a richer and more plausible model of concurrency than the simultaneous action model implicit in conventional Boolean games. We refer to games played with dependency graphs

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as *partial order Boolean games*. After motivating and presenting the partial order Boolean games model, we explore its properties. We show that while some problems associated with our new games have the same complexity as in conventional Boolean games, for others the complexity blows up dramatically. We also show that the concurrent behaviour of partial order Boolean games can be represented using a closure operator semantics, and conclude by considering the relationship of our model to IF logic.

### 3.3 Generalized quantifiers and Dependence Logic

Fredrik Engström (University of Göteborg, SE)

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Dependence logic, proposed by Väänänen [2], is an elegant way of introducing dependencies between variables into the object language. The framework of dependence logic, so-called team semantics, has turned out to be very flexible and allows for interesting generalizations. Instead of considering satisfaction with respect to a single assignment s, team semantics considers sets of assignments X, called teams.

The semantics of Dependence logic is based on the principle that

a formula  $\varphi$  is satisfied by a team X if every assignment  $s \in X$  satisfies  $\varphi$ .

The compositional semantics of dependence logic, except for the case for the dependence atom, can be derived from this one principle.

In this talk we introduce a new semantics, which is better suited for non-monotone increasing generalized quantifiers, where the above is replaced by the principle that

a formula  $\varphi$  is satisfied by a team X if for every assignment  $s : \operatorname{dom}(X) \to M^k, s \in X$  iff s satisfies  $\varphi$ ,

replacing an implication by an equivalence. When only first-order logic is considered in this new setting nothing exciting happens. It is only when we introduce atoms, like dependence atoms, or new logical operations that things start to get more exciting.

This alternative semantics will allow us to extend the logic with any generalized quantifier, not only monotone increasing ones as in [1].

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# 3.4 Counting in Team Semantics

Erich Grädel (RWTH Aachen, DE)

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We explore different ways to add counting constructs to logics with team semantics, such as counting quantifiers and forking atoms. While on the level of existential second-order

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definability, counting in finite structures is available without additional constructs and without a separate numerical sort, the addition of counting quantifiers enhances the expressive power of weaker logics, such as inclusion logic.

In the context of descriptive complexity theory, fixed-point logic with counting (FPC) is of central importance and actually the logic of reference in the quest for a logic for polynomial time. We extend the equivalence of inclusion logic and positive greatest fixed-point logic, proved by Galliani and Hella on the way back from the last Dagstuhl seminar on dependence logic, to an equivalence between FPC and inclusion logic with counting. Our proof is based on a new class of games, called threshold safety games, an on interpretation arguments for such games.

This talk is partially based on joint work with Stefan Hegselmann and on discussions with Pietro Galliani and Lauri Hella.

# 3.5 Axiomatizing dependencies in team semantics

Miika Hannula (University of Helsinki, FI)

We consider implication problems of different database dependencies. Simulating the chase algorithm at the logical level we show how different classes of dependencies can be axiomatised in the team semantics framework. In the associated proof systems, intermediate steps of deductions are inclusion dependencies that are implicitly existentially quantified as in lax semantics.

#### 3.6 The expressive power of modal inclusion logic

Lauri Hella (University of Tampere, FI)

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Modal inclusion logic is the extension of basic modal logic with inclusion atoms, and its semantics is defined on Kripke models with teams. A team of a Kripke model is just a subset of its domain. In [1] we give a complete characterisation for the expressive power of modal inclusion logic: a class of Kripke models with teams is definable in modal inclusion logic if and only if it is closed under k-bisimulation for some integer k, it is closed under unions, and it has the empty team property. We also prove that the same expressive power can be obtained by adding a single unary nonemptiness operator to modal logic. Furthermore, we establish an exponential lower bound for the size of the translation from modal inclusion logic to modal logic with the nonemptiness operator.

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#### 3.7 Model theoretic independence

Åsa Hirvonen (University of Helsinki, FI)

Independence is a crucial notion in the branch of model theory called classification theory. Classification theory aims at classifying collections of models: when are they well behaved, meaning that all models can be characterized up to isomorphism with a relatively small set of invariants, and when are they ill-behaved and exhibit the maximum number of models in each cardinality, with the models hard to distinguish from one another.

The most studied notion of independence in model theory is that of forking independence, developed by Shelah. It is designed to generalize linear independence in vector spaces and algebraic independence in algebraically closed fields. However, it gives relevant information also on more complex models. A classifiable model need not have just one dimension that characterizes it, but may have a collection of different dimensions (hence the set of invariants and not just one invariant).

With a more general independence notion, the connection between 'dependent' and 'independent' changes. The natural notion of dependence is no longer 'not independent of' but one needs a notion of generation. This is handled with various notions of primeness.

The use of model theoretic independence has extended to various contexts outside the original scope, and the properties studied vary depending on what can be achieved in different contexts as well as on different authors' different viewpoints. However, in the cases where one can define a well-behaved independence notion, it tends to be unique.

I will present the background to and some features of model theoretic independence, giving a short overview of the roles of dependence and independence in classification theory.

# 3.8 Statistical Independence, Causality and Constraint Satisfaction

Antti Hyttinen (University of Helsinki, FI)

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A causal model and the accompanied causal graph explain how statistically modeled system behaves when interventions are applied to it. I will explain the connection between statistical dependence/independence and reachability/separation in the causal graphs, the so-called dseparation. Using this, I will present the idea of constraint-based causal discovery, in which one can deduce properties of the causal graph structure from independencies and dependencies in passively observed data. I will also go through further concepts of independence used in causality and machine learning research. I will outline connections to the implication problem of conditional independence and its possible generalizations.

# 3.9 Dependence logic

Juha Kontinen (University of Helsinki, FI)

We survey some recent work in the area of dependence logic and team semantics. We focus on results on the computational aspects of various logics in the team semantics framework. We also discuss axiomatizability of certain fragments and variants of dependence logic.

#### 3.10 Propositional dependence logic via Kripke semantics

Antti Kuusisto (Stockholm University, SE)

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We propose an alternative semantics for propositional dependence and independence logics. The semantics gives a classical interpretation for the Boolean connectives and is ultimately based on treating dependence and independence atoms as generalized modalities in a framework with a Kripke- style semantics. We argue for the naturality of the novel semantics from the point of view of natural language. We also give sound and complete axiomatizations for both propositional dependence and independence logics in the case of a global accessibility relation. Furthermore, we show that propositional dependence and independence logics in the new framework are equiexpressive with modal logic with the global modality, and thus the corresponding logics based on team semantics are strictly weaker in expressivity than the novel systems. Interestingly, however, there exists no compositional translation from standard dependence or independence logic into the novel logics.

#### 3.11 Dependence-driven, non-invasive cleaning of uncertain data

Sebastian Link (University of Auckland, NZ)

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One classical approach to cleaning relational databases is to remove tuples which violate some given data dependency. To identify which of the conflicting tuples are dirty, one tries to find a minimal set of tuples to remove. For several popular classes of data dependencies, such as keys and differential dependencies, this minimization problem turns out to be equivalent to vertex cover. The classical approach ignores the uncertainty with which tuples occur and dependencies hold on the data. The classical approach is also invasive in the sense that tuples are removed from the database. In practice this is often unacceptable as some deleted tuples represent invaluable information. We propose an entirely new view on data cleaning in which both shortcomings are overcome. We depart from the classical view in which the data is considered to be dirty, and instead, view the degree of uncertainty attributed to the data as dirty. The talk will present a well-founded framework for this new view, a fixed-parameter tractable algorithmic solution using a generalization of vertex cover, and some experimental results. Fraud detection is identified as a new application of data cleaning, and several open problems are outlined for joint future research.

# 3.12 Dimensions for Modal Dependence Logic

Kerkko Luosto (University of Tampere, FI)

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 Joint work of Hella, Lauri; Luosto, Kerkko; Sano, Katsuhiko; Virtema, Jonni
 Main reference L. Hella, K. Luosto, K. Sano, J. Virtema, "The Expressive Power of Modal Dependence Logic," arXiv:1406.6266v1 [cs.LO], 2014.
 URL http://arxiv.org/abs/1406.6266v1

Instead of the ordinary semantics for modal logics, we use throughout this work team semantics in connection with Kripke structures. Several extensions of the basic modal logic are considered. These include the modal dependence logic and extended modal dependence logic, resp., with the added dependence atoms (dependencies between propositional symbols or basic modal formulas, resp.), and the modal logic with intuitionistic disjunction.

In the case of team semantics, the teams satisfying a given formula in a fixed Kripke structure is a family of sets whose combinatorial properties reflect the properties of the formula. It is easy to see that this family is downwards closed, implying that it is generated by its maximal elements which form a so-called Sperner family. Two dimension concepts, upper and lower one, may now be introduced to study the expressive power of formulas of these logics. The upper dimension is simply related to the number of maximal set in the family, whereas the lower dimension is the size of the largest minimal set that avoids the family, when we run over all Kripke structures.

It is proved, among other results, that even if the extended modal dependence logic and modal logic with intuitionistic disjunction have the same expressive power, the translation form the former to latter involves an exponential blow-up.

#### 3.13 A Team Based Variant of CTL

Arne Meier (Leibniz Universität Hannover, DE)

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 $\bar{\mathbb{O}}$  Arne Meier Joint work of Meier, Arne; Krebs, Andreas; Virtema, Jonni

Main reference A. Krebs, A. Meier, J. Virtema, "A Team Based Variant of CTL," in Proc. of the 22nd Int'l Symp. on Temporal Representation and Reasoning (TIME'15), to appear; pre-print available as

arXiv:1505.01964v2 [cs.LO], 2015. URL http://arxiv.org/abs/1505.01964v2

We introduce two variants of computation tree logic CTL based on team semantics: an asynchronous one and a synchronous one. For both variants we investigate the computational complexity of the satisfiability as well as the model checking problem. The satisfiability problem is shown to be EXPTIME-complete. Here it does not matter which of the two semantics are considered. For model checking we prove a PSPACE-completeness for the synchronous case, and show P-completeness for the asynchronous case. Furthermore we prove several interesting fundamental properties of both semantics.

#### 3.14 Measure Teams

Gianluca Paolini (University of Helsinki, FI)

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We give an overview of recent probabilistic developments in team semantics centered around the notion of measure team.

#### 3.15 Expressing properties of teams in k-ary inclusion-exclusion logic

Raine Rönnholm (University of Tampere, FI)

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The k-ary inclusion-exclusion logic, INEX[k], is obtained by extending first order logic with k-ary inclusion and exclusion atoms. The truth conditions for these atoms correspond to inclusion and exclusion dependencies in the database theory. We will examine the expressive power of INEX[k] on the level of formulas, by analyzing what kind properties of teams can be defined with it. By our earlier results, INEX[k] is closely related with k-ary existential second order logic, ESO[k]. We know that all ESO[k]-definable properties of k-ary relations of teams can be defined in INEX[k] and that all INEX[k]-definable properties are ESO[k]-definable.

However, when the arity of relations becomes higher than the arity of atoms, things get more exotic. We will show that for any k there are some very simple FO-definable properties of (k+1)-ary relations that cannot be defined in INEX[k]. For example, by using only unary inclusion and exclusion atoms, we cannot define the symmetry of a binary relation. But interestingly in INEX[1] we can define many properties of binary relations that are not FO-definable, such as disconnectivity of a graph.

To prove our undefinability results we will introduce a new method: Suppose that phi is an INEX[k]-formula and X is a team. Suppose also that phi is true in X, i.e. verifier has a winning strategy in the corresponding semantic game. We will then consider the reducts of this strategy for semantic games of subteams Y. We can show that if X is large enough compared to the size of phi, then the reduct strategy corresponding the team  $X \$  is a winning strategy for any assignment s in X.

# 3.16 Causal Inference and Logics of Dependence and Independence

Ilya Shpitser (University of Southampton, GB)

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In this talk, I introduce modern causal inference, an interdisciplinary area spanning statistics, philosophy, computer science, and the empirical sciences. Causal inference is based on the notion of counterfactual responses to *interventions*, which mathematical idealizations of randomized treatment assignment. Interventions allow clean reasoning about confounding, a phenomenon responsible for the adage 'correlation does not imply causation.'

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In the first part of my talk, I describe some recent work on techniques that allow inferring cause effect relationships from observational data, by by appropriately handling confounding [6, 7, 4]. In the second, I discuss an encoding of interventionist causality via dynamic modal logic due to [1]. Finally, I motivate a generalization of the graphoid axioms for reasoning about conditional independence to the case of generalized independence that arises in the theory of hidden variable models. [8, 5, 2, 3].

I conclude by proposing avenues for future collaboration between the causal inference community, and the community that works on logic and model theory. These included introducing quantification and other first order logic features to Halpern's logic, combining logic and probability for reasoning about uncertainty and causality together, finding connections with dependence logics, and appropriately generalizing the graphoid axioms to the hidden variable case.

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#### 3.17 Sheaf semantics for independence logics

Alex Simpson (University of Ljubljana, SI)

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Hodges' team/trump semantics provides a general semantic framework for logics based on dependence and independence, which has proven to be flexible in adapting to several varieties of such logics. However, it also gives rise to quirks such as an uneven treatment of negation (which is often restricted to atomic formulas), the failure of classical logic, and a potential proliferation of logical connectives.

I present instead an alternative semantic framework based on sheaves. In one direction, this framework generalises team semantics, in that teams arise as just one kind of possible structure for interpreting variables. In another direction, it departs from team semantics, in that the interpretation of logical connectives and quantifiers is different. In combination, these changes result in an embedding of independence primitives in ordinary classical logic, augmented by a layer of modalities, which reflect the nature of the notion of independence under consideration.

While the proposed framework arises abstractly from the interpretation of logic in certain atomic Grothendieck toposes, I instead introduce it directly from first principles. No knowledge of sheaf theory is assumed The presentation is aimed at logicians.

The material divides naturally into two parts.

- **Part 1: Sheaf semantics for logical independence.** I introduce the framework of sheaf semantics in the case of 'logical independence', which is the form of independence usually considered in team semantics. This gives rise to classical logic augmented with independence primitives, together with two modalities (necessity and possibility).
- **Part 2: Sheaf semantics for probabilistic independence.** I make use of the generality of the framework by giving a sheaf semantics in terms of random variables. This again gives rise to classical logic augmented with independence primitives. But now the independence primitives express probabilistic (conditional) independence, and there is a whole family of modalities capturing probability thresholds.

# 3.18 Characterisation of the expressive power of modal logic with inclusion atoms

Johanna Stumpf (TU Darmstadt, DE)

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The syntax of modal inclusion logic and team semantics are presented. Additionally, modal logic with nonempty disjunction is introduced. It is shown that exactly those properties of teams which are closed under unions and closed under team k-bisimulation are definable in modal logic with nonempty disjunction. Then modal logic with inclusion atoms is introduced and it is proven that the same result holds there. Furthermore, a lower bound for the size of the translation from modal logic with inclusion atoms to modal logic with nonempty disjunction is established.

# 3.19 Complexity of Propositional Inclusion and Independence Logic

Jonni Virtema (University of Tampere, FI)

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 Joint work of Hannula, Miika; Kontinen, Juha; Virtema, Jonni; Vollmer, Heribert
 Main reference M. Hannula, J. Kontinen, J. Virtema, H. Vollmer, "Complexity of Propositional Inclusion and Independence Logic," in Proc. of 40th Int'l Symp. on Mathematical Foundations of Computer Science (MFCS'15) – Part I, LNCS, Vol. 9234, pp. 269–280, Springer, 2015.
 URL http://dx.doi.org/10.1007/978-3-662-48057-1\_21

We classify the computational complexity of the satisfiability, validity and model-checking problems for propositional independence and inclusion logic and their extensions by the classical negation. Our main result shows that the satisfiability and validity problems of the extensions of propositional independence and inclusion logic by the classical negation are complete for alternating exponential time with polynomially many alternations.

#### 15261 – Logics for Dependence and Independence

# 3.20 Some proof theoretical results on propositional logics of dependence

Fan Yang (Utrecht University, NL)

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Fan Yang
Joint work of Yang, Fan; Iemhoff, Rosalie

In this talk, we present some proof theoretical results on propositional dependence logic and its variants, including propositional inquisitive logic. We prove the interpolation theorem for these logics. We also prove that propositional logics of dependence are structurally complete with respect to flat substitutions, that is, all admissible rules (with respect to flat substitutions) of these logics are derivable in their deductive systems.

#### 3.21 Tractability Frontier of Data Complexity in Team Semantics

Nicolas de Rugy-Altherre (University Paris-Diderot, FR)

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 Joint work of de Rugy-Altherre, Nicolas; Kontinen, Juha; Durand, Arnaud; Väänänen, Jouko
 Main reference A. Durand, J. Kontinen, N. de Rugy-Altherre, J. Väänänen, "Tractability Frontier of Data Complexity in Team Semantics," arXiv:1503.01144v2 [cs.LO], 2015.
 URL http://arxiv.org/abs/1503.01144v2

We study the data complexity of model-checking for logics with team semantics. For dependence and independence logic, we completely characterize the tractability/intractability frontier of data complexity of both quantifier-free and quantified formulas. For inclusion logic formulas, we reduce the model-checking problem to the satisfiability problem of so-called Dual-Horn propositional formulas. While interesting in its own right, this also provides an alternative proof for the recent result of P. Galliani and L. Hella in 2013 showing that the data complexity of inclusion logic is in PTIME. In the last section we consider the data complexity of inclusion logic under so-called strict semantics.

#### Erich Grädel, Juha Kontinen, Jouko Väänänen, and Heribert Vollmer



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