

Fair Division

Edited by

Yonatan Aumann¹, Jérôme Lang², and Ariel D. Procaccia³

1 Bar-Ilan University – Ramat Gan, IL, yaumann@gmail.com

2 University Paris-Dauphine, FR, lang@lamsade.dauphine.fr

3 Carnegie Mellon University – Pittsburgh, US, arielpro@cs.cmu.edu

Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 16232 “Fair Division”. The seminar was composed of technical sessions with regular talks, and discussion sessions distributed over the full week.

Seminar June 5–10, 2016 – <http://www.dagstuhl.de/16232>

1998 ACM Subject Classification I.2.11 Distributed Artificial Intelligence

Keywords and phrases cake cutting, computational social choice, envy-freeness, fair division

Digital Object Identifier 10.4230/DagRep.6.6.10

Edited in cooperation with Nhan-Tam Nguyen

1 Executive Summary

Yonatan Aumann

Steven J. Brams

Jérôme Lang

Ariel D. Procaccia

License  Creative Commons BY 3.0 Unported license
© Yonatan Aumann, Steven J. Brams, Jérôme Lang, and Ariel D. Procaccia

Fair division has been an active field of research in economics and mathematics for decades. More recently, the topic has attracted the attention of computer scientists, due to its algorithmic nature and its real-world applications. There had been a first Dagstuhl Seminar on fair division, in 2007, and none since. The aim of the 2016 Dagstuhl seminar on fair division was to bring together top researchers in the field, from among the multiple disparate disciplines where it is studied, both within computer science and from economics and mathematics, to share knowledge and advance the state of the art.

The seminar covered fair division of both divisible and indivisible goods, with a good mix between economics and computer science (with a significant number of talks being about economics *and* computer science). Topics included algorithms, lower bounds, approximations, strategic behavior, tradeoffs between fairness and efficiency, partial divisions, alternative definitions of fairness, and practical applications of fair division. The ratio between the number of participants with a main background in computer science and in economics was about 3–1, with a couple of participants with another main background (mathematics or political science). This ratio is similar to the corresponding ratios for Dagstuhl seminars on computational social choice (2007, 2010, 2012, 2015).

The seminar started by a short presentation of the participants (3 minutes per attendee). The rest of the seminar was composed of technical sessions with regular talks, and discussion sessions distributed over the full week (Tuesday morning, Tuesday afternoon, Wednesday



Except where otherwise noted, content of this report is licensed under a Creative Commons BY 3.0 Unported license

Fair Division, *Dagstuhl Reports*, Vol. 6, Issue 6, pp. 10–25

Editors: Yonatan Aumann, Jérôme Lang, and Ariel D. Procaccia



DAGSTUHL
REPORTS

Dagstuhl Reports
Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

morning, Friday morning). One of these discussion sessions was specifically about *Fair division in the real world*, two were about open problems, and one was about high-level thoughts about the topic and its future. Moreover, there was a significant amount of time left for participants to interact in small groups.

2 Table of Contents

Executive Summary

<i>Yonatan Aumann, Steven J. Brams, Jérôme Lang, and Ariel D. Procaccia</i>	10
---------------------------------------------------------------------------------------	----

Overview of Talks

A discrete and bounded envy-free cake cutting protocol for any number of agents <i>Haris Aziz and Simon William Mackenzie</i>	14
Complexity of Manipulating Sequential Allocation <i>Haris Aziz, Sylvain Bouveret, Jérôme Lang, and Simon William Mackenzie</i>	14
Nash Social Welfare Approximation for Strategic Agents <i>Simina Brânzei, Vasilis Gkatzelis, and Ruta Mehta</i>	15
Equitable cake cutting <i>Katarina Cechlarova</i>	15
The Power of Swap Deals in Distributed Resource Allocation <i>Yann Chevaleyre</i>	16
Dividing homogeneous divisible goods among three players <i>Marco Dall’Aglío</i>	16
Price of Pareto Optimality in Hedonic Games <i>Edith Elkind</i>	16
Approximating the Nash Social Welfare <i>Vasilis Gkatzelis, Simina Brânzei, Richard Cole, Gagan Goel, and Ruta Mehta</i>	17
Matroids and Allocation of Indivisible Goods <i>Laurent Gourves</i>	18
The redesign of the Israeli medical internship lottery <i>Avinatan Hassidim</i>	18
Procedural Justice in Simple Bargaining Games <i>Dorothea Herreiner</i>	18
Inheritance Game <i>Mehmet Ismail</i>	19
Making the Rules of Sports Fairer <i>Mehmet Ismail and Steven J. Brams</i>	19
Open Problems: Rules That Make Service Sports More Competitive <i>Mehmet Ismail, Steven J. Brams, D. Marc Kilgour, and Walter Stromquist</i>	20
Direct algorithms for balanced two-person fair division of indivisible items: A computational study <i>Marc Kilgour and Rudolf Vetschera</i>	20
Maximin Envy-Free Division of Indivisible Items <i>Christian Klamler, Steven J. Brams, and Marc Kilgour</i>	20
What is the highest guaranteed maximin approximation? <i>David Kurokawa</i>	21
Fair Division under Additive Utilities: good and bad news <i>Hervé J. Moulin</i>	21

Strategy-Proofness of Scoring Allocation Correspondences for Indivisible Goods <i>Nhan-Tam Nguyen</i>	22
The Single-Peaked Domain Revisited: A Simple Global Characterization <i>Clemens Puppe</i>	22
Preferences over Allocation Mechanisms and Recursive Utility <i>Uzi Segal and David Dillenberger</i>	23
Fair Division of Land <i>Erel Segal-Halevi</i>	23
Fairness and False-Name-Proofness in Randomized Allocation of a Divisible Good <i>Taiki Todo, Yuko Sakurai, and Makoto Yokoo</i>	24
Deceased Organ Matching in Australia and New Zealand. <i>Toby Walsh</i>	24
Participants	25

3 Overview of Talks

3.1 A discrete and bounded envy-free cake cutting protocol for any number of agents

Haris Aziz (Data61 / NICTA – Sydney, AU) and Simon William Mackenzie (UNSW – Sydney, AU)

License © Creative Commons BY 3.0 Unported license
© Haris Aziz and Simon William Mackenzie

Main reference H. Aziz, S. Mackenzie, “A Discrete and Bounded Envy-Free Cake Cutting Protocol for Any Number of Agents”, arXiv:1604.03655v10 [cs.DS], 2016.

URL <http://arxiv.org/abs/1604.03655v10>

We consider the well-studied cake cutting problem in which the goal is to find an envy-free allocation based on queries from n agents. The problem has received attention in computer science, mathematics, and economics. It has been a major open problem whether there exists a bounded and discrete envy-free protocol. We resolve the problem by proposing a discrete and bounded envy-free protocol for any number of agents. The maximum number of queries required by the protocol is a power tower of n of order six. We additionally show that even if we do not run our protocol to completion, it can find in at most n^{n+1} queries a partial allocation of the cake that achieves proportionality (each agent gets $\frac{1}{n}$ of the value of the whole cake) and envy-freeness. Finally we show that an envy-free partial allocation can be computed in n^{n+1} queries such that each agent gets a connected piece that gives the agent $\frac{1}{3n}$ of the value of the whole cake.

3.2 Complexity of Manipulating Sequential Allocation

Haris Aziz (Data61 / NICTA – Sydney, AU), Sylvain Bouveret (LIG – Grenoble, FR & Université Grenoble-Alpes, FR), Jérôme Lang (University Paris-Dauphine, FR), and Simon William Mackenzie (UNSW – Sydney, AU)

License © Creative Commons BY 3.0 Unported license
© Haris Aziz, Sylvain Bouveret, Jérôme Lang, and Simon William Mackenzie

Joint work of Haris Aziz, Sylvain Bouveret, Jérôme Lang, Simon William Mackenzie

Main reference H. Aziz, S. Bouveret, J. Lang, S. Mackenzie, “Complexity of Manipulating Sequential Allocation,” arXiv:1602.06940v1 [cs.GT], 2016.

URL <http://arxiv.org/abs/1602.06940v1>

Sequential allocation is a simple allocation mechanism in which agents are given pre-specified turns and each agent gets the most preferred item that is still available. It has long been known that sequential allocation is not strategyproof. This raises the question about the complexity of computing a preference report that yields more additive utility than the truthful preference. We show that is NP-complete. In doing so, we show that a previously presented polynomial-time algorithm for the problem is not correct. We complement the NP-completeness result by two algorithmic results. We first present a polynomial-time algorithm for optimal manipulation when the manipulator has Boolean utilities. We then consider stronger notions of manipulation whereby the untruthful outcome yields more utility than the truthful outcome for all utilities consistent with the ordinal preferences. For this notion of manipulation, we show that there exists a polynomial-time algorithm for computing a manipulation.

3.3 Nash Social Welfare Approximation for Strategic Agents

Simina Brânzei (The Hebrew University of Jerusalem, IL), Vasilis Gkatzelis (Stanford University, US), and Ruta Mehta

License © Creative Commons BY 3.0 Unported license
 © Simina Brânzei, Vasilis Gkatzelis, and Ruta Mehta
Main reference S. Brânzei, V. Gkatzelis, R. Mehta, “Nash Social Welfare Approximation for Strategic Agents,”
 arXiv:1607.01569v1 [cs.GT], 2016.
URL <http://arxiv.org/abs/1607.01569v1>

The fair division of resources among strategic agents is an important age-old problem that has led to a rich body of literature. At the center of this literature lies the question of whether there exist mechanisms that can implement fair outcomes, despite the agents’ strategic behavior. A fundamental objective function used for measuring fair outcomes is the *Nash social welfare* (NSW), mathematically defined as the geometric mean of the agents’ values in a given allocation. This objective function is maximized by widely known solution concepts such as Nash bargaining and the competitive equilibrium with equal incomes.

In this work we focus on the question of (approximately) implementing this objective. The starting point of our analysis is the Fisher market, a fundamental model of an economy, whose benchmark is precisely the (weighted) Nash social welfare. We study two extreme classes of valuations functions, namely perfect substitutes and perfect complements, and find that for perfect substitutes, the Fisher market mechanism has a constant price of anarchy (PoA): at most 2 and at least $e^{\frac{1}{e}}$ (≈ 1.44). However, for perfect complements, the Fisher market mechanism has an arbitrarily bad performance, its bound degrading linearly with the number of players.

Strikingly, the Trading Post mechanism – an indirect market mechanism also known as the Shapley-Shubik game – has significantly better performance than the Fisher market on its own benchmark. Not only does Trading Post attain a bound of 2 for perfect substitutes, but it also implements almost perfectly the NSW objective for perfect complements, where it achieves a price of anarchy of $(1 + \epsilon)$ for every $\epsilon > 0$. Moreover, we show that all the equilibria of the Trading Post mechanism are pure (so these bounds extend beyond the pure PoA), and satisfy an important notion of individual fairness known as proportionality.

3.4 Equitable cake cutting

Katarina Cechlarova (Pavol Jozef Safarik University – Kosice, SK)

License © Creative Commons BY 3.0 Unported license
 © Katarina Cechlarova
Joint work of Katarina Cechlarova, J. Dobos, E. Pillarova

The cake is represented by real interval $[0,1]$ and each of n players has her valuation of the cake in the form of a nonatomic probability measure. We look for equitable divisions, i.e. such that the values received by players by their own measures are equal, and everybody gets one contiguous piece. We show that such divisions always exist but they cannot be computed by a finite algorithm. Therefore we propose a simple algorithm to find approximately equitable divisions.

3.5 The Power of Swap Deals in Distributed Resource Allocation

Yann Chevaleyre (University of Paris North, FR)

License  Creative Commons BY 3.0 Unported license
© Yann Chevaleyre

Joint work of Y. Chevaleyre, A. Damamme, A. Beynier, N. Maudet

In the simple resource allocation setting consisting in assigning exactly one resource per agent, the top trading cycle procedure stands out as being the undisputed method of choice. It remains however a centralized procedure which may not well suited in the context of multiagent systems, where distributed coordination may be problematic. In this paper, we investigate the power of dynamics based on rational bilateral deals (swaps) in such settings. While they may induce a high efficiency loss, we provide several new elements that temper this fact: (i) we identify a natural domain where convergence to a Pareto-optimal allocation can be guaranteed, (ii) we show that the worst-case loss of welfare is as good as it can be under the assumption of individual rationality, (iii) we provide a number of experimental results, showing that such dynamics often provide good outcomes, especially in light of their simplicity, and (iv) we prove the NP-hardness of deciding whether an allocation maximizing utilitarian or egalitarian welfare is reachable.

3.6 Dividing homogeneous divisible goods among three players

Marco Dall’Aglío (LUISS Guido Carli – Rome, IT)

License  Creative Commons BY 3.0 Unported license
© Marco Dall’Aglío

Joint work of Marco Dall’Aglío, Camilla Di Luca, Lucia Milone

Main reference M. Dall’Aglío, C. Di Luca, L. Milone, “Characterizing and Finding the Pareto Optimal Equitable Allocation of Homogeneous Divisible Goods Among Three Players,” arXiv:1606.01028v1 [math.OA], 2016.

URL <http://arxiv.org/abs/1606.01028v1>

We consider the division of a finite number of homogeneous divisible items among three players. Under the assumption that each player assigns a positive value to every item, we characterize the optimal allocations and we develop two exact algorithms for its search. Both the characterization and the algorithm are based on the tight relationship two geometric objects of fair division: the Individual Pieces Set (IPS) and the Radon-Nykodim Set (RNS).

3.7 Price of Pareto Optimality in Hedonic Games

Edith Elkind (University of Oxford, GB)

License  Creative Commons BY 3.0 Unported license
© Edith Elkind

Joint work of Edith Elkind, Angelo Fanelli, Michele Flammini

Price of Anarchy measures the welfare loss caused by selfish behavior: it is defined as the ratio of the social welfare in a socially optimal outcome and in a worst Nash equilibrium. A similar measure can be derived for other classes of stable outcomes. In this paper, we argue that Pareto optimality can be seen as a notion of stability, and introduce the concept of Price of Pareto Optimality: this is an analogue of the Price of Anarchy, where the maximum is computed over the class of Pareto optimal outcomes, i.e., outcomes that do not permit a

deviation by the grand coalition that makes all players weakly better off and some players strictly better off. As a case study, we focus on hedonic games, and provide lower and upper bounds of the Price of Pareto Optimality in three classes of hedonic games: additively separable hedonic games, fractional hedonic games, and modified fractional hedonic games; for fractional hedonic games on trees our bounds are tight.

3.8 Approximating the Nash Social Welfare

Vasilis Gkatzelis (Stanford University, US), Simina Brânzei, Richard Cole, Gagan Goel, and Ruta Mehta

License  Creative Commons BY 3.0 Unported license
© Vasilis Gkatzelis, Simina Brânzei, Richard Cole, Gagan Goel, and Ruta Mehta

We study the problem of allocating a collection of items among a set of agents with the goal of maximizing the geometric mean of their utilities, i.e., the Nash social welfare. We consider both the computational tractability of this problem as well as the issues that arise when the participating agents behave strategically, aiming to maximize their own utility.

When the items are divisible, the problem of maximizing the Nash social welfare is known to be computationally tractable, so we focus on the strategic interactions among the agents that arise when their preferences are private. We first analyze the efficiency of simple mechanisms in terms of their price of anarchy using the Nash social welfare measure. That is, we study the ratio of the optimal Nash social welfare for a given instance and the Nash social welfare at the worst Nash equilibrium, and we prove upper and lower bounds for this ratio [3]. Furthermore, we design novel mechanisms that achieve strategy-proofness by keeping some of the items unallocated. We show that these mechanisms combine strategy-proofness with a good approximation of the optimal Nash social welfare [2].

When the items are indivisible, the problem of maximizing the Nash social welfare becomes APX-hard, even when the valuations of the agents are additive. Our main result is the first efficient constant-factor approximation algorithm for this objective. We first observe that the integrality gap of the natural fractional relaxation is exponential, so we propose a different fractional allocation which implies a tighter upper bound and, after appropriate rounding, yields a good integral allocation [1].

References

- 1 Richard Cole and Vasilis Gkatzelis, Approximating the Nash Social Welfare with Indivisible Items. Proceedings of the Forty-Seventh Annual ACM on Symposium on Theory of Computing, STOC 2015, Portland, OR, USA, June 14-17, 2015
- 2 Richard Cole and Vasilis Gkatzelis and Gagan Goel, Mechanism design for fair division: allocating divisible items without payments. ACM Conference on Electronic Commerce, EC'13, Philadelphia, PA, USA, June 16-20, 2013
- 3 Simina Brânzei and Vasilis Gkatzelis and Ruta Mehta, Nash Social Welfare Approximation for Strategic Agents. arxiv (<http://arxiv.org/abs/1607.01569>) 2016

3.9 Matroids and Allocation of Indivisible Goods

Laurent Gourves (University Paris-Dauphine, FR)

License © Creative Commons BY 3.0 Unported license
© Laurent Gourves

Joint work of Laurent Gourves, Carlos A. Martinhon, Jerome Monnot, Lydia Tlilane

We propose an extension of the allocation of indivisible goods to matroids in the sense that the agents get elements that form a base of a matroid. We present some exchange properties that can be used for a matroid extension to MMS and the Cut and Choose protocol, together with an expansion of a matroid that helps to maximize the utilitarian social welfare, with upper bounds on the number of elements that each agent receives.

3.10 The redesign of the Israeli medical internship lottery

Avinatan Hassidim (Bar-Ilan University – Ramat Gan, IL)

License © Creative Commons BY 3.0 Unported license
© Avinatan Hassidim

Joint work of Arnon Afek, Noga Alon, Slava Bronfmann, Avinatan Hassidim, Assaf Romm

Acquiring an Israeli m.d. requires performing an internship in one of the hospitals in Israel. In the past, interns were assigned using a variant of Random Serial Dictatorship. We redesigned the market to use a proprietary algorithm achievement a benefit in satisfaction.

3.11 Procedural Justice in Simple Bargaining Games

Dorothea Herreiner (Loyola Marymount University, US)

License © Creative Commons BY 3.0 Unported license
© Dorothea Herreiner

Giving an affected person some control in a decision-making process generally increases the satisfaction with the outcome because participation contributes to procedural justice. Empowering a receiver in a simple bargaining game by providing the option to reject a proposal (ultimatum game) instead of imposing a proposal (dictator game) leads to more equitable outcomes as Shor (2007) shows. Whether empowerment itself matters, i.e. the fact that the receiver can influence outcomes, or the implicit recognition by the proposer that the receiver is disadvantaged, i.e. the intention behind the empowerment, remains an open questions addressed in this experimental study. Several variants of Shor’s empowerment game (choice between ultimatum and dictator game) are considered where the choice to empower the receiver is made by the proposer, randomly, or a third party. Significant differences emerge between proposals depending on the empowerment of the receiver and in the frequency with which the receiver is empowered; the intentionality behind the empowerment decisions, however, does not seem to make a significant difference.

3.12 Inheritance Game

Mehmet Ismail (Maastricht University, NL)

License  Creative Commons BY 3.0 Unported license
© Mehmet Ismail

A couple delegates a person, D, to divide a cake (their inheritance) of unit length among their children, players A and B. Players are in separate rooms and each have half of the cake on the table in front of them. Each chooses (e.g., by cutting) a piece from the cake, $[0, 1/2]$. The rules are as follows:

1. If some of the players disagree with the rules, nobody will receive anything. The choice 0 expresses disagreement.
2. Otherwise, players receive their own piece. And, if there is some piece left from either player, D will pay each an extra 1 unit of money (as he'd like to taste the cake and convince them to agree with this rule).


The unique Nash equilibrium is $(0, 0)$, which resembles a Bertrand duopoly outcome since 0 is weakly dominated by any strategy. Unlike in the duopoly game, however, all strategies but $1/2$ are dominated. Thus, the only undominated strategy profile is $(1/2, 1/2)$, which is also the unique maximin equilibrium [1]. Each has a profitable deviation from this profile, but the deviator would receive a smaller piece than the non-deviator, which gives incentives to free ride on the deviators: a social dilemma situation.

References

- 1 Mehmet Ismail. *Maximin Equilibrium: A Minimal Extension of Maximin Strategies*. 2014.

3.13 Making the Rules of Sports Fairer

Mehmet Ismail (Maastricht University, NL) and Steven J. Brams

License  Creative Commons BY 3.0 Unported license
© Mehmet Ismail and Steven J. Brams

Main reference S. J. Brams, M. S. Ismail, "Making the Rules of Sports Fairer," SSRN, 2016.

URL <http://dx.doi.org/10.2139/ssrn.2737672>

In the beginning of my presentation, I ran a mini tournament on Catch-Up [1], which is a two-person game in which players alternate removing numbers from an initial set $\{1, 2, \dots, n\}$. Players begin with scores of 0, and the acting player removes numbers (which are added to his score), one by one, until his score *equals* or *exceeds* the opponent's score. If the scores are tied, the game is drawn; otherwise, the player with the higher score wins.

I then presented "Making the Rules of Sports Fairer," which is a joint work with Steven J. Brams. In this paper, we argue that the rules of many sports are not fair – they do not ensure that equally skilled competitors have the same probability of winning. As an example, the penalty shootout in soccer, wherein a coin toss determines which team kicks first on all five penalty kicks, gives a substantial advantage to the first-kicking team, both in theory and practice. We show that a so-called Catch-Up Rule for determining the order of kicking would not only make the shootout fairer but also is essentially strategy proof. By contrast, the so-called Standard Rule now used for the tiebreaker in tennis is fair. We briefly consider several other sports, all of which involve scoring a sufficient number of points to win, and show how they could benefit from certain rule changes, which would be straightforward to implement.

References

- 1 Aaron Isaksen, Mehmet Ismail, Steven J. Brams, Andy Nealen. *Catch-Up: A Game in Which the Lead Alternates*. *Game & Puzzle Design* 1, no. 2:38–49. 2015.

3.14 Open Problems: Rules That Make Service Sports More Competitive

Mehmet Ismail (Maastricht University, NL), Steven J. Brams, D. Marc Kilgour, and Walter Stromquist (Swarthmore College, US)

License © Creative Commons BY 3.0 Unported license
© Mehmet Ismail, Steven J. Brams, D. Marc Kilgour, and Walter Stromquist

The Standard Rule – presently used in badminton, racquetball, squash, and volleyball – says that the player who *won* the last point serves for the next point, whereas the so-called Catch-Up Rule says that the player who *lost* the last point serves for the next.

The open problem was that the probability of the first-serving player winning is the same under both Standard Rule and Catch-Up Rule, which was solved by Walter Stromquist, one of the participants at Dagstuhl Seminar.

3.15 Direct algorithms for balanced two-person fair division of indivisible items: A computational study

Marc Kilgour (Wilfrid Laurier University, CA) and Rudolf Vetschera

License © Creative Commons BY 3.0 Unported license
© Marc Kilgour and Rudolf Vetschera

Direct algorithms for the balanced fair division of indivisible items between two persons are assessed computationally. Several algorithms are applied to all possible fair-division problems with 4, 6, 8, and 10 items to determine how well the algorithms do at achieving various fairness properties such as envy-freeness, Pareto-optimality, and maximality.

3.16 Maximin Envy-Free Division of Indivisible Items


Christian Klamler (Universität Graz, AT), Steven J. Brams, and Marc Kilgour (Wilfrid Laurier University, CA)

License © Creative Commons BY 3.0 Unported license
© Christian Klamler, Steven J. Brams, and Marc Kilgour

Assume that two players have strict rankings over an even number of indivisible items. We propose two algorithms to find balanced allocations of these items that are maximin – maximize the minimum rank of the items that the players receive – and are envy-free and Pareto-optimal if such allocations exist. To determine whether an envy-free allocation exists, we introduce a simple condition on preference profiles; in fact, our condition guarantees the existence of a maximin, envy-free, and Pareto-optimal allocation. Although not strategy-proof, our algorithms would be difficult to manipulate unless a player has complete information about its opponent’s ranking. We assess the applicability of the algorithms to real-world problems, such as allocating marital property in a divorce or assigning people to committees or projects.

3.17 What is the highest guaranteed maximin approximation?

David Kurokawa (Carnegie Mellon University – Pittsburgh, US)

License  Creative Commons BY 3.0 Unported license
© David Kurokawa

Joint work of David Kurokawa, Ariel D. Procaccia, Junxing Wang

The maximin share guarantee is one of the few well-established notions of fairness in the setting of fairly dividing indivisible goods. Although believed to always exist, [Procaccia and Wang, Fair Enough: Guaranteeing Approximate Maximin Shares, EC 2014] showed that in very intricately constructed examples, the property is not guaranteeable – but were only able to demonstrate the absence in examples with high approximations to the maximin share guarantee. In the same work, they showed that a $2/3$ approximation does always exist. This leads to a natural question of what is the highest guaranteed maximin approximation? We explore previous techniques of approximation and examine where they break down to improve the bound and also touch upon finding examples with worse guarantees.

3.18 Fair Division under Additive Utilities: good and bad news

Hervé J. Moulin (University of Glasgow, GB)

License  Creative Commons BY 3.0 Unported license
© Hervé J. Moulin

Joint work of Anna Bogomolnaia, Herve Moulin

Modern economic analysis mostly dismisses additive utilities that ignore complementarities between commodities. But recent work on the practical implementation of fair division rules in user-friendly websites (Spliddit, Adjusted Winner) gives a central role to this simple preference domain for compelling practical reasons, and brings back into sharp focus the 1959 results of Eisenberg and Gale on linear economies. Think of distributing the family heirlooms between siblings, splitting the assets of a divorcing couple, or allocating job shifts between substitutable workers: most people cannot form sophisticated preferences described by general utility functions, just like participants in a combinatorial auction do not form a complete ranking of all subsets of objects. Thus individual preferences are elicited by a simple bidding system: you distribute 100 points over the different goods, and these weights define your additive utility. The proof of the pudding is in the eating: thousands of visitors use these sites every month, fully aware that their bid is interpreted as their additive utility. Fairness as equal opportunities is achieved by the familiar Competitive Equilibrium with Equal Incomes. When dividing goods this rule is normatively compelling. Because it also maximizes the Nash product of utilities, it is unique utility-wise, continuous in the utility matrix, and easy to compute. It also guarantees that more manna to divide is never bad news for any participant (Resource Monotonicity), that by raising my bid on a certain good I cannot end up with a smaller share of that good (Responsive Shares), and that the size of my bids for the goods I do not eat is irrelevant (Independence of Lost Bids). The latter property is characteristic. When dividing bads, the Competitive Equilibria with Equal Incomes captures all the critical points of the Nash product of utilities, and is still characterized by Invariance of Lost Bids. It can be severely multi-valued: up to $2^{\min\{n,p\}} - 1$ distinct utility profiles with n agents and p goods. Moreover any single-valued efficient division rule attempting to implement equal opportunities faces two severe impossibility results: no such rule can be resource monotonic and guarantee the fair share; no such rule can be Envy-Free and continuous in the utility parameters. The fair division of bads is not a piece of cake.

3.19 Strategy-Proofness of Scoring Allocation Correspondences for Indivisible Goods

Nhan-Tam Nguyen (Heinrich-Heine-Universität Düsseldorf, DE)

License © Creative Commons BY 3.0 Unported license
© Nhan-Tam Nguyen

Joint work of Nhan-Tam Nguyen, Dorothea Baumeister, Joerg Rothe
Main reference N. Nguyen, D. Baumeister, J. Rothe, “Strategy-Proofness of Scoring Allocation Correspondences for Indivisible Goods”, in Proc. of the 24th Int’l Joint Conf. on Artificial Intelligence (IJCAI’15), pp. 1127–1133, AAAI Press, 2015.
URL <http://ijcai.org/Abstract/15/163>

We study resource allocation in a model due to Brams and King [1] and further developed by Baumeister et al. [2]. Resource allocation deals with the distribution of resources to agents. We assume resources to be indivisible, nonshareable, and of single-unit type. Agents have ordinal preferences over single resources. Using scoring vectors, every ordinal preference induces a utility function. These utility functions are used in conjunction with utilitarian social welfare to assess the quality of allocations of resources to agents. Then allocation correspondences determine the optimal allocations that maximize utilitarian social welfare.

Since agents may have an incentive to misreport their true preferences, the question of strategy-proofness is important to resource allocation. We assume that a manipulator has responsive preferences over the power set of the resources. We use extension principles (from social choice theory, such as the Kelly and the Gardenfors extension) for preferences to study manipulation of allocation correspondences. We characterize strategy-proofness of the utilitarian allocation correspondence: It is Gardenfors/Kelly-strategy-proof if and only if the number of different values in the scoring vector is at most two or the number of occurrences of the greatest value in the scoring vector is larger than half the number of goods.

References

- 1 Brams S, King D (2005) Efficient fair division: Help the worst off or avoid envy? *Rationality and Society* 17(4):387–421
- 2 Baumeister D, Bouveret S, Lang J, Nguyen N, Nguyen T, Rothe J (2014) Scoring rules for the allocation of indivisible goods. In: *Proceedings of the 21st European Conference on Artificial Intelligence*, IOS Press, pp. 75–80

3.20 The Single-Peaked Domain Revisited: A Simple Global Characterization

Clemens Puppe (KIT – Karlsruher Institut für Technologie, DE)

License © Creative Commons BY 3.0 Unported license
© Clemens Puppe

Main reference C. Puppe, “The Single-Peaked Domain Revisited: A Simple Global Characterization,” Manuscript, March 2016.
URL <http://micro.econ.kit.edu/downloads/Charact-SP.pdf>

It is proved that, among all restricted preference domains that guarantee consistency (i.e. transitivity) of pairwise majority voting, the single-peaked domain is the only minimally rich and connected domain that contains two completely reversed strict preference orders. It is argued that this result explains the predominant role of single-peakedness as a domain restriction in models of political economy and elsewhere. The main result has a number of corollaries, among them a dual characterization of the single- dipped domain; it also

implies that a single-crossing (‘order-restricted’) domain can be minimally rich only if it is a subdomain of a single-peaked domain. The conclusions are robust as the results apply both to domains of strict and of weak preference orders, respectively.

3.21 Preferences over Allocation Mechanisms and Recursive Utility

Uzi Segal (Boston College, US) and David Dillenberger

License © Creative Commons BY 3.0 Unported license
© Uzi Segal and David Dillenberger

We deal with a simple problem: There are n units of two types that need to be allocated among n people, one per person. Preferences are stochastic. Each person prefers good 1 with probability q and good 2 with probability $1 - q$. These probabilities are independent across individuals. We analyze several allocation mechanisms with different levels of knowledge and show that:

1. Mechanisms may be identical from an ex-post point of view, but not ex-ante, as individuals are not indifferent between them.
2. Preferences over some well known mechanisms are linked to different forms of rejection and acceptance of ambiguity.
3. Both the well known top-cycle and serial-dictatorship mechanisms are inefficient.

3.22 Fair Division of Land

Erel Segal-Halevi (Bar-Ilan University – Ramat Gan, IL)

License © Creative Commons BY 3.0 Unported license
© Erel Segal-Halevi
Joint work of Yonatan Aumann, Avinatan Hassidim, Shmuel Nitzan, Erel Segal-Halevi, Balazs Sziklai
URL <http://erelsgl.github.io/topics/en/fairness/>

The talk is a short summary of my Ph.D. research (2013–2016). The goal of this research is to apply cake-cutting algorithms for dividing land. I present several issues that have to be addressed.

1. Geometry: When dividing land, in contrast to cake, the two-dimensional geometric shape of the pieces is important.
2. Redivision: Dividing land, in contrast to cake, is an on-going process. Land often has to be re-divided. The re-division process should be fair both for the old and for the new agents.
3. Group ownership: The ownership of land, in contrast to cake, is often shared among several individuals, such as family members. Each of these members may have different preferences.
4. Land-value data: For land, in contrast to cake, there exist detailed value maps, which can be used to test the performance of cake-cutting algorithms.

3.23 Fairness and False-Name-Proofness in Randomized Allocation of a Divisible Good


Taiki Todo (Kyushu University – Fukuoka, JP), Yuko Sakurai, and Makoto Yokoo

License  Creative Commons BY 3.0 Unported license
© Taiki Todo, Yuko Sakurai, and Makoto Yokoo

Cake cutting has been recognized as a fundamental model for allocating a divisible good in a fair manner, and several envy-free cake cutting algorithms have been proposed. Recent works reconsidered cake cutting from the perspective of mechanism design and developed strategy-proof cake cutting mechanisms; no agent has any incentive to cheat them by misrepresenting her utility function. In this talk I consider a different type of manipulations; each agent might create fake identities to cheat the mechanism. Such manipulations have been called Sybils or false-name manipulations, and designing robust mechanisms against them, i.e., false-name-proof, is a challenging problem. I first present an impossibility result, which states that no false-name-proof mechanism simultaneously satisfies envy-freeness and Pareto efficiency. I then present a new mechanism that is optimal in terms of worst-case loss among those that satisfy false-name-proofness, strong envy-freeness, and a weaker efficiency property. To improve the efficiency, I also provide another mechanism that satisfies a weaker notion of false-name-proofness, as well as strong envy-freeness and Pareto efficiency. Furthermore, I give a short discussion on the effect of introducing agents' costs for managing fake accounts.

3.24 Deceased Organ Matching in Australia and New Zealand.

Toby Walsh (UNSW – Sydney, AU)

License  Creative Commons BY 3.0 Unported license
© Toby Walsh

I discuss how we might match deceased organs to patients more effectively. One of the primary goals is to match the quality of the deceased organ and the patient due to the increasing age of donated kidneys. I formulate this as an online problem, discuss axiomatic properties and propose some novel mechanisms.

Participants

- Yonatan Aumann
Bar-Ilan University –
Ramat Gan, IL
- Haris Aziz
Data61 / NICTA – Sydney, AU
- Sylvain Bouveret
LIG - Grenoble, FR & Université
Grenoble-Alpes, FR
- Simina Brânzei
The Hebrew University of
Jerusalem, IL
- Katarina Cechlarova
Pavol Jozef Safarik University –
Kosice, SK
- Yann Chevaleyre
University of Paris North, FR
- Marco Dall'Aglio
LUISS Guido Carli – Rome, IT
- Edith Elkind
University of Oxford, GB
- Ulle Endriss
University of Amsterdam, NL
- Serge Gaspers
UNSW – Sydney, AU
- Vasilis Gkatzelis
Stanford University, US
- Laurent Gourves
University Paris-Dauphine, FR
- Avinatan Hassidim
Bar-Ilan University – Ramat
Gan, IL
- Dorothea Herreiner
Loyola Marymount Univ., US
- Mehmet Ismail
Maastricht University, NL
- Michael A. Jones
Mathematical Reviews – Ann
Arbor, US
- Marc Kilgour
Wilfrid Laurier University, CA
- Christian Klamler
Universität Graz, AT
- David Kurokawa
Carnegie Mellon University &
Pittsburgh, US
- Jérôme Lang
University Paris-Dauphine, FR
- Simon William Mackenzie
UNSW – Sydney, AU
- Hervé J. Moulin
University of Glasgow, GB
- Nhan-Tam Nguyen
Heinrich-Heine-Universität
Düsseldorf, DE
- Trung Thanh Nguyen
New York University –
Abu Dhabi, AE
- Ariel D. Procaccia
Carnegie Mellon University –
Pittsburgh, US
- Clemens Puppe
KIT – Karlsruher Institut für
Technologie, DE
- Jörg Rothe
Heinrich-Heine-Universität
Düsseldorf, DE
- Uzi Segal
Boston College, US
- Erel Segal-Halevi
Bar-Ilan University –
Ramat Gan, IL
- Walter Stromquist
Swarthmore College, US
- Taiki Todo
Kyushu Univ. – Fukuoka, JP
- Toby Walsh
UNSW – Sydney, AU

