

Functoriality in Geometric Data

Edited by

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Abstract

This report provides an overview of the talks at the Dagstuhl Seminar 17021 “Functoriality in Geometric Data”. The seminar brought together researchers interested in the fundamental questions of *similarity* and *correspondence* across geometric data sets, which include collections of GPS traces, images, 3D shapes and other types of geometric data. A recent trend, emerging independently in multiple theoretical and applied communities, is to understand *networks* of geometric data sets through their relations and interconnections, a point of view that can be broadly described as exploiting the *functoriality* of data, which has a long tradition associated with it in mathematics. Functoriality, in its broadest form, is the notion that in dealing with any kind of mathematical object, it is at least as important to understand the transformations or symmetries possessed by the object or the family of objects to which it belongs, as it is to study the object itself. This general idea has led to deep insights into the structure of various geometric spaces as well as to state-of-the-art methods in various application domains. The talks spanned a wide array of subjects under the common theme of functoriality, including: the analysis of geometric collections, optimal transport for geometric datasets, deep learning applications and many more.

Seminar January 8–13, 2017 – <http://www.dagstuhl.de/17021>

1998 ACM Subject Classification I.3.5 Computational Geometry and Object Modeling, I.4 Image Processing and Computer Vision

Keywords and phrases computational geometry, data analysis, geometry processing

Digital Object Identifier 10.4230/DagRep.7.1.1

1 Summary

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Across science, engineering, medicine and business we face a deluge of data coming from sensors, from simulations, or from the activities of myriads of individuals on the Internet. The data often has a geometric character, as is the case with 1D GPS traces, 2D images, 3D scans, and so on. Furthermore, the data sets we collect are frequently highly correlated, reflecting information about the same or similar entities in the world, or echoing semantically



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Functoriality in Geometric Data, *Dagstuhl Reports*, Vol. 7, Issue 01, pp. 1–18

Editors: Mirela Ben-Chen, Frédéric Chazal, Leonidas J. Guibas, and Maks Ovsjanikov



Dagstuhl Reports

Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

important repetitions/symmetries or hierarchical structures common to both man-made and natural objects.

A recent trend, emerging independently in multiple theoretical and applied communities is to understand geometric data sets through their relations and interconnections, a point of view that can be broadly described as exploiting the *functoriality* of data, which has a long tradition associated with it in mathematics. Functoriality, in its broadest form, is the notion that in dealing with any kind of mathematical object, it is at least as important to understand the transformations or symmetries possessed by the object or the family of objects to which it belongs, as it is to study the object itself. This general idea been successfully applied in a large variety of fields, both theoretical and practical, often leading to deep insights into the structure of various objects as well as to elegant and efficient methods in various application domains, including computational geometry, computer vision and computer graphics.

This seminar brought together researchers and practitioners interested in notions of *similarity*, *correspondence* and, more generally, *relations* across geometric data sets. Mathematical and computational tools for the construction, analysis, and exploitation of such relational networks were the central focus of this seminar.

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3 Overview of Talks

3.1 Output sensitive algorithms for approximate incidences and their applications

Dror Aiger (Google Israel – Tel Aviv, IL), Haim Kaplan (Tel Aviv University, IL), and Micha Sharir (Tel Aviv University, IL)

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© Dror Aiger, Haim Kaplan, and Micha Sharir

An ϵ -approximate incidence between a point and some geometric object (line, circle, plane, sphere) occurs when the point and the object lie at distance at most ϵ from each other. Given a set of points and a set of objects, computing the approximate incidences between them is a major step in many database and web-based applications in computer vision and graphics. Two important such applications are robust model fitting, where we want to find models (lines, planes, etc.) that lie near many “interesting” points of an image (or a 3D point cloud), and point pattern matching, where we are given two sets of points A and B , and we want to find a large subset A' of A for which there exists a rigid transformation which places each point of A' ϵ -close to some point of B . Typically, approximate incidences are used to find candidate transformations (between model and data or between A and B), which are then tested against the whole input to filter those that have a large match. In a typical approximate incidence problem of this sort, we are given a set P of m points in two or three dimensions, a set S of n objects (lines, circles, planes, spheres), and an error parameter $\epsilon > 0$, and our goal is to report all pairs $(p, s) \in P \times S$ that lie at distance at most ϵ from one another. We present efficient output-sensitive approximation algorithms for several cases, including points and lines or circles in the plane, and points and planes, spheres, lines, or circles in three dimensions. Some of these cases arise in the applications mentioned above. Our algorithms report all pairs at distance $\leq \epsilon$, but may also report additional pairs, all of which are guaranteed to be at distance at most $\alpha\epsilon$, for some constant $\alpha > 1$. Our algorithms are based on simple primal and dual grid decompositions and are easy to implement. We analyze our algorithms and prove guaranteed upper bounds on their running time and on the “distortion” parameter α . Furthermore, we present experimental results on real and random data that demonstrate the advantage of our methods compared to other methods and to the naive implementation, commonly used in practice in these applications.

3.2 Joint denoising and distortion correction of atomic scale scanning transmission electron microscopy images

Benjamin Berkels (RWTH Aachen, DE) and Benedikt Wirth (Universität Münster, DE)

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© Benjamin Berkels and Benedikt Wirth

Main reference B. Berkels, B. Wirth, “Joint denoising and distortion correction of atomic scale scanning transmission electron microscopy images”, arXiv:1612.08170v1 [cs.CV], 2016.


URL <https://arxiv.org/abs/1612.08170v1>

Nowadays, modern electron microscopes deliver images at atomic scale. The precise atomic structure encodes information about material properties. Thus, an important ingredient in the image analysis is to locate the centers of the atoms shown in micrographs as precisely as possible. Here, we consider scanning transmission electron microscopy (STEM), which

acquires data in a rastering pattern, pixel by pixel. Due to this rastering combined with the magnification to atomic scale, movements of the specimen even at the nanometer scale lead to random image distortions that make precise atom localization difficult. Given a series of STEM images, we derive a Bayesian method that jointly estimates the distortion in each image and reconstructs the underlying atomic grid of the material by fitting the atom bumps with suitable bump functions. The resulting highly non-convex minimization problems are solved numerically with a trust region approach. Well-posedness of the reconstruction method and the model behavior for faster and faster rastering are investigated using variational techniques. The performance of the method is finally evaluated on both synthetic and real experimental data.

3.3 Consistent discretization and minimization of the L1 norm on manifolds

Alex M. Bronstein (Technion – Haifa, IL)

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The L1 norm has been tremendously popular in signal and image processing in the past two decades due to its sparsity-promoting properties. More recently, its generalization to non-Euclidean domains has been found useful in shape analysis applications. For example, in conjunction with the minimization of the Dirichlet energy, it was shown to produce a compactly supported quasi-harmonic orthonormal basis, dubbed as compressed manifold modes. The continuous L1 norm on the manifold is often replaced by the vector l1 norm applied to sampled functions. We show that such an approach is incorrect in the sense that it does not consistently discretize the continuous norm and warn against its sensitivity to the specific sampling. We propose two alternative discretizations resulting in an iteratively-reweighed l2 norm. We demonstrate the proposed strategy on the compressed modes problem, which reduces to a sequence of simple eigendecomposition problems not requiring non-convex optimization on Stiefel manifolds and producing more stable and accurate results.

3.4 Geometric deep learning

Michael M. Bronstein (University of Lugano, CH)


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Many scientific fields study data that have an underlying structure that is non-Euclidean space. Some examples include social networks in computational social sciences, sensor networks in communications, functional networks in brain imaging, regulatory networks in genetics, and meshed surfaces in computer graphics. In many applications, such geometric data are large and complex (in the case of social networks, on the scale of billions), and are natural targets for machine learning techniques. In particular, we would like to use deep neural networks, which have recently proven to be powerful tools for a broad range of problems from computer vision, natural language processing, and audio analysis. However, these tools have been most successful on data with an underlying Euclidean or grid-like structure, and in cases where the invariances of these structures are built into networks used to model them. I will

discuss “Geometric deep learning”, a class of emerging techniques attempting to generalize deep neural models to non-Euclidean domains such as graphs and manifolds. I will show applications from the domains of vision, graphics, and network analysis.

3.5 A convex representation for the Elastica functional.

Antonin Chambolle (*Ecole Polytechnique – Palaiseau, FR*)

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Joint work of Antonin Chambolle, Thomas Pock

In this talk, we have described a joint (ongoing) work with Thomas Pock (TU Graz, Austria) where we show how to design convex relaxation to curvature-dependent functionals such as $\Gamma \mapsto \int_{\Gamma} f(\kappa) ds$ where κ is the curvature and f is a convex function, with $f(t) \geq \sqrt{1+t^2}$. Our relaxation involves a lifting of the curve into the roto-translation group, and is easily extended as an energy of scalar functions in the plane. We can show that it is tight when Γ is the boundary of a C^2 set, and that it is a variant of a recent relaxation proposed by Bredies, Pock, Wirth [1]. It is however strictly below the standard lower semi-continuous relaxations of the Elastica which have been studied for many years (for boundaries, in the L^1 topology, see for instance [2] and the references therein).

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3.6 Regularized Optimal Transport


Marco Cuturi (*CREST – Malakoff, FR*)

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The optimal transport problem, first studied by Monge and later by Kantorovich, has drawn the attention of both pure and applied mathematicians for several years now. An important tool that arises from optimal transport theory is the definition of a versatile geometry that can be used to compare probability measures, the Wasserstein geometry. Because probability measures are widely used to model social and natural phenomena, the toolbox of optimal transport has been increasingly adopted in a wide array of applied fields, such as economy, fluid dynamics, quantum chemistry, computer vision or graphics. The main motivation behind my work was to design new machine learning methodologies built upon the optimal transport geometry. The main obstacle to this goal was computational, since optimal transport is notoriously costly to compute. To avoid that issue, I have proposed 3 years ago a very efficient numerical scheme to solve optimal transport problems that can scale up to large scales and use recent progresses in hardware, namely GPGPUs. This breakthrough has inspired several works in the span of 3 years, in machine learning and beyond, which I tried to survey in this Dagstuhl seminar talk.

3.7 Ollivier Ricci curvature on network data and applications

Jie Gao (Stony Brook University, US)

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In the talk I will introduce our recent work on Ollivier Ricci curvature for graphs and its applications in network analysis and graph mining. Ollivier Ricci curvature extends the notion of Ricci curvature from continuous setting to the case of graphs. For each edge xy , the curvature is defined by comparing the edge length and the optimal transport distance from x 's neighbors to y 's neighbors. We show that the Ollivier Ricci curvature exhibits interesting properties in real world graphs as those found for the Internet. Backbone edges, with negative curvature, connect communities of nodes which are connected by edges of positive curvature.

3.8 Model reduction for shape processing

Klaus Hildebrandt (TU Delft, NL)

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Joint work of Klaus Hildebrandt, Christopher Brandt, Christoph von Tycowicz, Christian Schulz
Main reference C. von Tycowicz, C. Schulz, H.-P. Seidel, K. Hildebrandt, “Real-Time Nonlinear Shape Interpolation”, *ACM Transactions on Graphics*, 34(3):34:1–34:10, ACM, 2015.
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URL <http://dx.doi.org/10.1111/cgf.12832>

The optimization of deformable, flexible or non-rigid shapes is essential for many tasks in geometric modeling and processing. In this talk, I will introduce model reduction techniques that can be used to construct fast approximation algorithms for shape optimization problems. The goal is to obtain run times that are independent of the resolution of the discrete shapes to be optimized. As examples, we will discuss methods for real-time elasticity-based shape interpolation and the efficient integration of a geometric flow of curves in shape space.

3.9 Similarity and correspondence problems for 3D shapes in motion

Franck Hétroy-Wheeler (INRIA – Grenoble, FR)

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Joint work of Jinlong Yang, Aurela Shehu, Jean-Sébastien Franco, Franck Hétroy-Wheeler, Stefanie Wuhrer
Main reference J. Yang, J.-S. Franco, F. Hétroy-Wheeler, S. Wuhrer, “Estimation of Human Body Shape in Motion with Wide Clothing”, in *Computer Vision – ECCV 2016, LNCS, Vol. 9908*, pp. 439–454, Springer, 2016.
URL http://dx.doi.org/10.1007/978-3-319-46493-0_27

In this talk I focus on moving 3D shapes, represented as sequences of meshes or point clouds without explicit temporal coherence. I review two recent works involving human shapes in wide clothing. The first work [2] estimates the body shape under the clothing with the help of a shape space. The second one [1] tracks the surface of the cloth over time, using the assumption that the deformation is locally near-isometric. Both methods involve different functoriality tools, namely statistical shape spaces and deformation models.

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3.10 Part Structures in Large Collections of 3D Models

Vladimir G. Kim (Adobe Systems Inc. – Seattle, US)

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Joint work of Vladimir G. Kim, Minhyuk Sung, Li Yi, Leonidas Guibas, Duygu Ceylan, I-Chao Shen, Mengyan Yan, Hao Su, Cewu Lu, Qixing Huang, Alla Sheffer, Roland Angst

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URL <http://dx.doi.org/10.1145/2816795.2818094>

As large repositories of 3D shape collections grow, understanding the geometric data, especially encoding the inter-model similarity, their variations, semantics and functionality, is of central importance. My research addresses the challenge of deriving probabilistic models that capture common structure in large, unorganized, and diverse collections of 3D polygonal shapes. We present a part-based model for for encoding structural variations in collections of man-made shapes, and demonstrate its applications to shape completion, exploration, organization, analysis, and synthesis of geometric data. Our part based model is trained on collections of segmented 3D models with labeled parts. To facilitate creation of these data with rich annotations, we also propose an active tool for acquiring detailed region labels from crowd workers. Our tool models human behavior and explicitly minimizes human effort.

3.11 Invariants and learning for geometry analysis

Ron Kimmel (Technion – Haifa, IL)

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Joint work of Ron Kimmel, Gautam Pai

In my presentation I discussed the relation between axiomatic understanding of geometric forms and structures and modern learning by convolutional networks. We focused on planar curves for which we demonstrated how Cartan’s theorem indicates the sufficient invariants that could serve as a signature indicating the number of features required for matching curves. We then suggested a Siamese architecture for training such a network and demonstrated feature extortion with that network. Relation to curvature were demonstrated. Dealing with the Euclidean, affine, and similarity groups was presented. As for surfaces, we reviewed recent

geometries related to equi-affine, similarity, and full affine groups of transformations with some recent matching results known for these transformations, for which Gromov distances, treating surfaces as metric spaces, were approximated.

3.12 Fully Spectral Partial Shape Matching

Or Litany (Tel Aviv University, IL), Alex M. Bronstein (Technion – Haifa, IL), Michael M. Bronstein (University of Lugano, CH), and Emanuele Rodolà (University of Lugano, CH)

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Joint work of Or Litany, Emanuele Rodolà, Alex M. Bronstein, and Michael M. Bronstein

Main reference O. Litany, E. Rodolà, A. M. Bronstein, M. M. Bronstein, “Fully Spectral Partial Shape Matching”, Computer Graphics Forum (CGF) – Proc. EUROGRAPHICS 2017, Vol. 36(2), 2017; preprint available from author’s webpage.

URL <http://dx.doi.org/10.1111/cgf.13123>

URL http://vision.in.tum.de/_media/spezial/bib/litany-eg17.pdf

We propose an efficient procedure for calculating partial dense intrinsic correspondence between deformable shapes performed entirely in the spectral domain. Our technique relies on the recently introduced partial functional maps formalism and on the joint approximate diagonalization (JAD) of the Laplace-Beltrami operators previously introduced for matching non-isometric shapes. We show that a variant of the JAD problem with an appropriately modified coupling term (surprisingly) allows to construct quasi-harmonic bases localized on the latent corresponding parts. This circumvents the need to explicitly compute the unknown parts by means of the cumbersome alternating minimization used in the previous approaches, and allows performing all the calculations in the spectral domain with constant complexity independent of the number of shape vertices. We provide an extensive evaluation of the proposed technique on standard non-rigid correspondence benchmarks and show state-of-the-art performance in various settings, including partiality and the presence of topological noise.

3.13 Efficient Deformable 2D-to-3D Shape Matching

Zorah Löhner (TU München, DE), Michael M. Bronstein (University of Lugano, CH), Daniel Cremers (TU München, DE), Emanuele Rodolà (University of Lugano, CH), and Frank R. Schmidt (TU München, DE)

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© Zorah Löhner, Michael M. Bronstein, Daniel Cremers, Emanuele Rodolà, and Frank R. Schmidt

Main reference Z. Löhner, E. Rodolà, F. R. Schmidt, M. M. Bronstein, D. Cremers, “Efficient Globally Optimal 2D-to-3D Deformable Shape Matching”, in Proc. of IEEE Conference on Computer Vision and Pattern Recognition (CVPR 2016), pp. 2185–2193, IEEE, 2016; preprint available from author’s webpage.

URL <http://dx.doi.org/10.1109/CVPR.2016.240>

URL <http://vision.in.tum.de/~laehner/Elastic2D3D/>

In this talk we present an algorithm for non-rigid 2D-to-3D shape matching, where the input is a 2D query shape as well as a 3D target shape and the output is a continuous matching curve represented as a closed contour on the 3D shape. We cast the problem as finding the shortest circular path on the product 3-manifold of the two shapes. Quantitative and qualitative evaluation confirms that the method provides excellent results for sketch-based deformable 3D shape retrieval.

3.14 Multiscale Methods for Dictionary Learning, Regression and Optimal Transport for data near low-dimensional sets

Mauro Maggioni (Johns Hopkins University – Baltimore, US)

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Joint work of Mauro Maggioni, Wenjing Liao, Stefano Vigogna, Sam Gerber

Main reference W. Liao, M. Maggioni, S. Vigogna, “Learning adaptive multiscale approximations to data and functions near low-dimensional sets”, in Proc. of the 2016 IEEE Information Theory Workshop (ITW 2016), pp. 226–230, IEEE, 2016.

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We discuss a family of ideas, algorithms, and results for analyzing various new and classical problems in the analysis of high-dimensional data sets. These methods we discuss perform well when data is (nearly) intrinsically low-dimensional. They rely on the idea of performing suitable multiscale geometric decompositions of the data, and exploiting such decompositions to perform a variety of tasks in signal processing and statistical learning. In particular, we discuss the problem of dictionary learning, where one is interested in constructing, given a training set of signals, a set of vectors (dictionary) such that the signals admit a sparse representation in terms of the dictionary vectors. We then discuss the problem of regressing a function on a low-dimensional unknown manifold. For both problems we introduce a multiscale estimator, fast algorithms for constructing it, and give finite sample guarantees for its performance, and discuss its optimality. Finally, we discuss an application of these multiscale decompositions to the fast calculation of optimal transportation plans, introduce a multiscale version of optimal transportation distances, and discuss preliminary applications.

3.15 On equilibrium shapes, Michell structures and “smoothness” of polyhedral surfaces

Martin Kilian (TU Wien, AT), Davide Pellis (TU Wien, AT), Johannes Wallner (TU Graz, AT), and Helmut Pottmann (TU Wien, AT)

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Pure geometric shape modeling is not sufficient to achieve an efficient digital workflow from design to production, since it tends to result in costly feedback loops between design, engineering and fabrication. To overcome this problem, recent research incorporates key aspects of function and fabrication into an intelligent shape modeling process. In the present talk, we will illustrate this trend at hand of our ongoing research on shape modeling in the presence of structural and fabrication constraints. We discuss (i) 2D trusses with minimum weight (Michell structures), (ii) form-finding for architectural freeform shells and (iii) kink-minimizing polyhedral surfaces: All these themes are related to the minimization of total absolute curvature (integral of sum of absolute values of principal curvatures) of surfaces in various geometries and thus can be computationally treated with the same methodology.

3.16 Smooth Interpolation of Key Frames in a Riemannian Shell Space

Martin Rumpf (Universität Bonn, DE)

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Joint work of Pascal Huber, Ricardo Perl, Martin Rumpf

Splines and subdivision curves are flexible tools in the design and manipulation of curves in Euclidean space. In this paper we study generalizations of interpolating splines and subdivision schemes to the Riemannian manifold of shell surfaces in which the associated metric measures both bending and membrane distortion. The shells under consideration are assumed to be represented by Loop subdivision surfaces. This enables the animation of shells via the smooth interpolation of a given set of key frame control meshes. Using a variational time discretization of geodesics efficient numerical implementations can be derived. These are based on a discrete geodesic interpolation, discrete geometric logarithm, discrete exponential map, and discrete parallel transport. With these building blocks at hand discrete Riemannian cardinal splines and three different types of discrete, interpolatory subdivision schemes are defined. Numerical results for two different subdivision shell models underline the potential of this approach in key frame animation.

3.17 Reduction and reconstruction of complex spatio-temporal data

Konstantin Mischaikow (Rutgers University – Piscataway, US)

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It is almost cliché at this point to note that high dimensional data is being collected from experiments or generated through numerical simulation at an unprecedented rate and that this rate will continue rising extremely rapidly for the foreseeable future. Our interest is in data associated with high dimensional nonlinear complex spatiotemporal dynamics. The focus of this talk is on our efforts to use persistent homology both as a dimension reduction technique and a technique for reconstructing structures of the underlying dynamical system. I will present some results associated with dynamics of fluid convection and dense granular media and will try to highlight open questions.

3.18 Optimal transport between a simplex soup and a point cloud

Quentin Mérigot, Jocelyn Meyron, and Boris Thibert

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In this talk, we are interested in solving an optimal transport problem between a measure supported on a simplex soup and a measure supported on a finite point set for the quadratic cost in \mathbb{R}^d . Similarly as in (Aurenhammer, Hoffman, Aronov, Algorithmica, 1998), this optimal transport problem can be recast as finding a Power diagram supported on the finite point set such that the Power cells intersected with the simplex soup have a prescribed measure.

We show the convergence with linear speed of a damped Newton algorithm to solve this non-linear problem. The convergence relies on a genericity condition on the point cloud with respect to the simplex soup and on a connectedness condition for the support of the measure

defined on the simplex soup. Finally, we apply our algorithm in \mathbb{R}^3 to compute optimal transport plans between a measure supported on a triangulation and a discrete measure.

3.19 Partial Functional Correspondence

Emanuele Rodolà (University of Lugano, CH), Alex M. Bronstein (Technion – Haifa, IL), Michael M. Bronstein (University of Lugano, CH), Luca Cosmo (University of Venice, IT), Daniel Cremers (TU München, DE), Or Litany (Tel Aviv University, IL), Jonathan Masci (IDSIA, CH), and Andrea Torsello (University of Venice, IT)

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In this talk we present our recent line of work on (deformable) partial shape correspondence in the spectral domain. We first introduce Partial Functional Maps (PFM), showing how to robustly formulate the shape correspondence problem under missing geometry with the language of functional maps. We use perturbation analysis to show how removal of shape parts changes the Laplace-Beltrami eigenfunctions, and exploit it as a prior on the spectral representation of the correspondence. We show further extensions to deal with the presence of clutter (deformable object-in-clutter) and multiple pieces (non-rigid puzzles). The resulting algorithms yield state-of-the-art results on challenging correspondence benchmarks in the presence of partiality and topological noise.

3.20 A Selection of Categorical Viewpoints on Shape Matching

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While “shape” is an important concept in order to compare, detect and classify geometric data, the definition of a shape depends very much on the specific application.

Whether we understand a shape as a mere set, a topological or a metric space drives the development of different shape matching approaches.

In this presentation I talk about a selection of different categorical viewpoints on shape and shape matching in order to explore the strengths and weaknesses of these approaches from a theoretical and a practical point of view.

3.21 Towards a Geometric Functionality Descriptor


Ariel Shamir (The Interdisciplinary Center – Herzliya, IL)

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The American architect Louis Sullivan coined the phrase “form follows function” in architecture. However, almost in all areas of design as well as in nature, one of the key aspects that define shapes is their functionality. In this talk I will describe several efforts to study the functionality of 3D objects by examining their shape and the connection between them. First, by trying to learn about functionality from interactions with other objects in scenes, then by trying to co-analyze similar shapes and their interactions and lastly, by studying movements and motion of shape parts.

3.22 Regularized Optimal Transport on Graphs: Rank-1 Hessian Updates for Quadratic Regularization

Justin Solomon (MIT – Cambridge, US) and Montacer Essid (New York University, US)

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Optimal transportation provides a means of lifting distances between points on a geometric domain to distances between signals over the domain, expressed as probability distributions. On a graph, transportation problems such as computation of the Wasserstein distance between distributions can be used to express challenging tasks involving matching supply to demand with minimal shipment expense; in discrete language, these problems become “multi-commodity network flow” problems. Regularization typically is needed to ensure uniqueness for the linear ground distance case and to improve optimization convergence; state-of-the-art techniques employ entropic regularization on the transportation matrix.

In this work, we explore a quadratic alternative to entropic regularization for transport with linear ground distance over a graph. The dual of this problem exhibits elegant second-order structure that we leverage to derive an easily-implemented Newton-type algorithm with fast convergence. The end result is state-of-the-art performance compared with much more involved large-scale convex optimization machinery.

3.23 Common Manifold Learning with Alternating Diffusion

Ronen Talmon (*Technion – Haifa, IL*)

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Joint work of Roy Lederman, Ronen Talmon, Hau-tieng Wu

Main reference R. R. Lederman, R. Talmon, “Learning the geometry of common latent variables using alternating-diffusion”, in *Applied and Computational Harmonic Analysis*, Elsevier, 2015.

URL <http://dx.doi.org/10.1016/j.acha.2015.09.002>

We consider the problem of hidden common manifold extraction from multiple data sets, which have observation-specific distortions and artifacts. A new manifold learning method is presented based on alternating products of diffusion operators and local kernels. We provide theoretical analysis showing that our method is able to build a variant of the Laplacian of the hidden common manifold, while suppressing the observation-specific artifacts. The generality of this method is demonstrated in data analysis applications, where different types of devices are used to measure the same activity. In particular, we present applications to problems in biomedicine, neuroscience, and audio analysis.

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3.24 Optimal transport between a point cloud and a simplex soup

Boris Thibert (*Laboratoire Jean Kuntzmann – Grenoble, FR*)

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I will consider in this talk the computation of an optimal transport map between a measure supported on a simplex soup and a measure supported on a finite point set for the quadratic cost in \mathbb{R}^d . Similarly as in work of Aurenhammer, Hoffman and Aronov, this optimal transport problem can be recast as finding a Power diagram supported on the finite point set such that the Power cells intersected with the simplex soup have a prescribed measure. I will show the convergence with linear speed of a damped Newton algorithm to solve this nonlinear problem and also apply our algorithm in 3D to compute optimal transport plans between a measure supported on a triangulation and a discrete measure. This work is in collaboration with Jocelyn Meyron and Quentin Mérigot.

3.25 Product Manifold Filter – Towards a Continuity Prior for 3D shape correspondence

Matthias Vestner (*TU München, DE*)

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Many algorithms for the computation of correspondences between deformable shapes rely on some variant of nearest neighbor matching in a descriptor space. Such are, for example, various point-wise correspondence recovery algorithms used as a post-processing stage in

the functional correspondence framework. Such frequently used techniques implicitly make restrictive assumptions (e.g., near-isometry) on the considered shapes and in practice suffer from lack of accuracy and result in poor surjectivity. We propose an alternative recovery technique capable of guaranteeing a bijective correspondence and producing significantly higher accuracy and smoothness. Unlike other methods our approach does not depend on the assumption that the analyzed shapes are isometric. We derive the proposed method from the statistical framework of kernel density estimation and demonstrate its performance on several challenging deformable 3D shape matching datasets.

3.26 Physical Graphic Design

Wilmot Li (Adobe Systems Inc. – Seattle, US)

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Joint work of Wilmot Li, Danny Kaufman

Advances in digital fabrication technology enable an increasingly diverse spectrum of users to create physical artifacts. One emerging fabrication workflow involves the use of traditional graphic design (i.e., the design of layered 2.5D vector graphics) as a front end to produce physical objects. We refer to this process as physical graphic design. While creating physical artifacts via 2D design may seem like a solved problem (after all, there has been decades of research and commercial development on graphic design tools), the problem is more involved than it initially appears. In this talk, we discuss some of the fundamental challenges of physical graphic design, which stem from the complex interplay between design requirements and physical/fabrication constraints.

3.27 Data fitting tools in Riemannian spaces

Benedikt Wirth (Universität Münster, DE)

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Joint work of Pierre-Antoine Absil, Pierre-Yves Gouenbourger, Behrend Heeren, Martin Rumpf, Peter Schröder, Paul Striowski, Max Wardetzky


Main reference P.-A. Absil, P.-Y. Gouenbourger, P. Striowski, B. Wirth, “Differentiable piecewise-Bezier surfaces on Riemannian manifolds”, *SIAM Journal on Imaging Sciences*, 9(4):1788–1828, SIAM, 2016.

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Several computer vision, life science or imaging applications deal with data that can be viewed as points in a (possibly high-dimensional) Riemannian manifold. Many of these applications require data fitting of different types, for instance to perform statistical regression, interpolation, or data compression. Classical approaches to data fitting include computation of averages, of inter- and extrapolating curves, or fitting of low-dimensional manifolds. The talk presents some numerical techniques that can be employed to this end, in particular a nonlinear variational discretisation and different ways to define and compute curves or submanifolds in Riemannian spaces. The Riemannian space of (discrete) viscous shells, which can be used to model 3D geometries, serves as an example setting. Here, each point represents a (triangulated) surface embedded in 3D space. The Riemannian metric on this space takes in-plane stretching and bending of surfaces into account, which ultimately leads to elliptic PDEs in many of the data fitting problems.

3.28 Qualitative and Multi-Attribute Learning from Diverse Data Collections

Hao Zhang (Simon Fraser University – Burnaby, CA)

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When dealing with a large collection of data possessing much diversity, it can be difficult, if not impossible, to properly quantify all pairwise similarities between the data entities using a single numerical distance to allow a meaningful global analysis. In this talk, I will first go over a qualitative analysis method we developed for organizing a heterogeneous collection of shapes. The method turns a set of *quartet* query results into a single categorization tree, where each quartet query gathers a similarity ranking between data pairs from a data quadruplet and these rankings are best reflected by the number of “edge hops” in the categorization tree. Next, I will introduce a new problem which tries to account for the multiple perspectives people may draw upon when performing similarity rankings. We are interested in how much one can learn of the different perspectives the human subjects may have used to provide the ranking results, where the only available information to us is the ranking results. Our first attempt is to classify the similarity queries, with reasonable confidence, based on latent perspectives. That is, we never explicitly identify the perspectives or data attributes in the output (they remain latent) nor do we pre-select a superset of candidate attributes to start the analysis.

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