Report from Dagstuhl Seminar 17081

Computability Theory

Edited by

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Abstract

Computability is one of the fundamental notions of mathematics and computer science, trying to capture the effective content of mathematics and its applications. Computability Theory explores the frontiers and limits of effectiveness and algorithmic methods. It has its origins in Gödel's Incompleteness Theorems and the formalization of computability by Turing and others, which later led to the emergence of computer science as we know it today. Computability Theory is strongly connected to other areas of mathematics and theoretical computer science. The core of this theory is the analysis of relative computability and the induced degrees of unsolvability; its applications are mainly to Kolmogorov complexity and randomness as well as mathematical logic, analysis and algebra. Current research in computability theory stresses these applications and focuses on algorithmic randomness, computable analysis, computable model theory, and reverse mathematics (proof theory). Recent advances in these research directions have revealed some deep interactions not only among these areas but also with the core parts of computability theory. The goal of this Dagstuhl Seminar is to bring together researchers from all parts of computability theory and related areas in order to discuss advances in the individual areas and the interactions among those.

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1 Executive Summary

Klaus Ambos-Spies
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Computability theory grew from work to understand effectiveness in mathematics. Sophisticated tools have been developed towards this task. For a while, the area tended to be concerned with internal considerations such as the structure of the various hierarchies developed for the tasks of calibrations. More recently, this machinery has seen modern applications into areas such as model theory, algorithmic randomness, analysis, ergodic theory, number theory and the like; and the tools have been used to answer several classical questions. Seminar 17081 was an opportunity for researchers in several areas of modern computability theory to get together and interact.

The format was for 2–3 lectures in the morning with at least one being an overview, and a similar number of 3–4 in the afternoon, with Wednesday afternoon and Friday afternoon free. The weather being miserable, participants opted to stay at the Schloss Wednesday afternoon, and quite a bit of work was done in pairs in the time left free, on the Wednesday afternoon in particular. At least one problem seen as significant in the area was solved (one concerning the strength of Ramsey's Theorem for Pairs in reverse mathematics), and the organizers know of several other papers in preparation resulting from the seminar.

The lectures were from various areas, but the greatest concentration was around

- classification tools in computable analysis (the Weihrauch Lattice) and Reverse Mathematics (on what proof-theoretic strength is needed to establish results in mathematics), and these areas' relationship with generating algorithms, such as in proof mining;
- computable model theory (looking at structures such as groups, rings, or abstract algebraic structures, endowing them with effectivity and seeing what else is algorithmic). Notable was the new work on effective uncountable structures such as uncountable free groups, and on topological groups;
- algorithmic randomness: Here one seeks to give meaning to randomness for individual strings and infinite sequences. Talks given explored the relationship of calibrations of randomness to computational power, and online notions of randomness.

Of course, these are not separate areas but are inter-related, and the talks reflected these inter-relationships.

Currently, computability theory is quite vibrant with many new applications being found, and a number of young and talented researchers entering the field. This was reflected in the age of the presenters of many of the lectures, as well as the significant number of people we could have invited in addition.

All in all, the meeting was a great success and should have significant impact on the development of the area.

2 Table of Contents

3 Overview of Talks

3.1 Machines running on random tapes and the probabilities of events

George Barmpalias (Victoria University – Wellington, NZ)

Joint work of Andrew Lewis-Pye, George Barmpalias, Douglas Cenzer, Christopher P. Porter
URL http://barmpalias.net

Probabilistic Turing machines have been studied since the 1940s, when it was shown that the probability of a machine with a random (as in probability theory) oracle computing any fixed non-computable real is 0. Chaitin's halting probability is the probability that a universal Turing machine halts on a random oracle (with empty numerical input) and was characterized in terms of algorithmic randomness and computable approximations. In general, one can ask the same question with respect to any property that a computation of a universal Turing machine may have when it reads a random oracle:

1. Will it compute a total function?
2. Will it enumerate a co-finite set (say, as the domain of a partial function that it computes)?
3. Will it enumerate a set which computes the halting problem?
4. Will it compute an incomputable function?
5. Will it halt with an output inside a certain set A?

Can we give characterizations of these probabilities in terms of algorithmic randomness and effectiveness properties? We show that this is possible, but we do not always get the expected answer.

Moreover we answer one of the last remaining questions from the BSL 2006 list of open questions in randomness (by Miller and Nies), by showing that the probability that the universal machine halts and outputs a number in a non-empty Π^0_1 set is always left-c.e. and ML-random. Intuitively, this says that if we code arithmetical sentences into numbers, the probability that the universal machine outputs an undecidable sentence (in PA) can be effectively approximated from below!

My talk is mainly based on the following recent work:

- The probability of a computable output from a random oracle
  George Barmpalias, Douglas Cenzer and Christopher P. Porter
  https://arxiv.org/abs/1612.08537
- Differences of halting probabilities
  George Barmpalias and Andy Lewis-Pye
  https://arxiv.org/abs/1604.00216
- Random numbers as probabilities of machine behaviour
  George Barmpalias, Douglas Cenzer and Christopher P. Porter
  https://arxiv.org/abs/1605.05838

3.2 Deep Π_1^0 classes

Laurent Bienvenu (University of Montpellier & CNRS, FR)

Joint work of Laurent Bienvenu, Christopher P. Porter
Main reference L. Bienvenu, C. P. Porter, "Deep Π_1^0 classes", Bulletin of Symbolic Logic, 22(2):249–286, 2016.
URL https://arxiv.org/abs/1403.0450v3

We will present the concept of deep Π_1^0 classes, which can be thought of as those classes whose paths are uniformly 'hard to generate probabilistically' and discuss the many interesting properties those classes enjoy. In particular we will see that they behave quite similarly to the class of PA degrees in their interactions with algorithmic randomness.

3.3 Finding bases of uncountable free abelian groups is hard

Noam Greenberg (Victoria University – Wellington, NZ)

Joint work of Noam Greenberg, Dan Turetsky, Linda Brown Westrick

We use admissible computability to discuss effective properties of uncountable free abelian groups. Assuming $V = L$, for all regular uncountable κ there is a κ-computable free abelian group with no κ-computable basis, indeed no κ-arithmetical basis, and usually one can avoid any lower cone below a $\Delta_1^1(L_\kappa)$ degree. On the other hand, not much can be coded into bases of groups: a forcing construction shows that the most that can be coded is $\emptyset'$ or $\emptyset''$, depending on κ (for example, if it is a successor of a singular cardinal, or inaccessible). The index-set of κ-computable free abelian groups is $\Sigma_1^1(L_\kappa)$-complete, unless κ is weakly compact.

3.4 Monte Carlo Computability

Rupert Hölzl (Universität der Bundeswehr – München, DE)

Joint work of Vasco Brattka, Rupert Hölzl, Rutger Kuyper
Main reference V. Brattka, R. Hölzl, R. Kuyper, "Monte Carlo Computability", Proc. of the 34th Symp. on Theoretical Aspects of Computer Science (STACS 2017), LIPIcs, Vol. 66, pp. 17:1–17:14, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2017.
URL http://dx.doi.org/10.4230/LIPIcs.STACS.2017.17

We introduce Monte Carlo computability as a probabilistic concept of computability on infinite objects and prove that Monte Carlo computable functions are closed under composition. We then mutually separate the following classes of functions from each other: the class of multi-valued functions that are non-deterministically computable, that of Las Vegas computable functions, and that of Monte Carlo computable functions. We give natural examples of computational problems witnessing these separations. As a specific problem which is Monte Carlo computable but neither Las Vegas computable nor non-deterministically computable, we study the problem of sorting infinite sequences.

3.5 Strong and non-strong degrees of categoricity

Iskander Kalimullin (Kazan Federal University – Kazan, RU)

Joint work of Nikolay Bazhenov, Iskander Kalimullin, Mars Yamaleev

A computable structure $\mathcal{A}$ has a *degree of categoricity* $\mathbf{x}$ if $\mathbf{x}$ is the least Turing degree such that $\mathcal{A}$ is $\mathbf{x}$-computably categorical. A degree of categoricity $\mathbf{x}$ is *strong* if there are two computable copies $\mathcal{B} \cong \mathcal{C} \cong \mathcal{A}$ such that $\mathbf{x} \leq_{\mathrm{T}} f$ for every isomorphism $f\colon \mathcal{B} \to \mathcal{C}$. Answering a question from [1] on the existence of non-strong degrees of categoricity we introduce the notion of spectral dimension of a computable structure: the *spectral dimension* of a computable structure $\mathcal{A}$ with a degree of categoricity $\mathbf{x}$ is equal to an ordinal $n \leq \omega$ if n is the least ordinal such that there are computable copies $\mathcal{B}_i \cong \mathcal{C}_i \cong \mathcal{A}$, $i < n$, such that $\mathbf{x} \leq_{\mathrm{T}} \bigoplus_{i<n} f_i$ for every choice of isomorphisms $f_i\colon \mathcal{B}_i \to \mathcal{C}_i, i < n$ (considering categoricity spectra the notion of spectral dimension can be easily adapted to the case when a structure has no degree of categoricity). We show that for every $n < \omega$ there is a rigid computable structure of the degree of categoricity $\mathbf{0}'$ having spectral dimension n. The original question from [1] now can be updated to the form: is there a computable structure with a degree of categoricity having spectral dimension ω? Such a structure, if it exists, can not be rigid.

References

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3.6 Topological aspects of enumeration degrees

Takayuki Kihara (University of California – Berkeley, US)

Joint work of Takayuki Kihara, Steffen Lempp, Keng Meng Ng, Arno Pauly

Pauly and the speaker introduced a general way of assigning a degree structure to each admissibly represented space. From this perspective, the enumeration degrees can be thought of as the degree structure of a universal second-countable T_0 space. This idea enable us to classify enumeration degrees in terms of general topology. For instance, the Turing degrees (total e-degrees) are the "finite dimensional metrizable e-degrees", and the continuous degrees are the "metrizable e-degrees". We can then talk about the existences of a Hausdorff (T_2) e-degree which is not an Urysohn ($T_{2.5}$) e-degree, of a Frechet (T_1) e-degree which is Hausdorff-quasiminimal, etc.

Note that the admissibly represented spaces form a cartesian closed category, which is far larger than the category of second-countable T_0 spaces. In general, the degree structure of a non-second-countable space is not a substructure of the enumeration degrees. For instance, one can show that the de Groot dual of the Baire space (which is not second-countable) has a point having non-enumeration degree.

3.7 The Scott Isomorphism Theorem

Julia Knight (University of Notre Dame, US)

Joint work of Rachael Alvir, Julia Knight, Charles McCoy

Scott [5] proved that for a countable structure $\mathcal{A}$ for a countable language L, there is a sentence of $L_{\omega_1\omega}$ (a *Scott sentence*) whose countable models are just the isomorphic copies of $\mathcal{A}$. The complexity of an optimal *Scott sentence* measures the internal complexity of the structure. I will describe some recent results on the complexity of Scott sentences. I had conjectured that every finitely generated group has a d-Σ_2 Scott sentence, and every computable finitely generated group has a computable d-Σ_2 Scott sentence. Recently, Harrison-Trainor and Ho [2] showed that both conjectures are false. Alvir, McCoy and I [1] applied a result of Montalbán [4] and one of A. Miller [3] to show that a finitely generated group has a d-Σ_2 Scott sentence iff there is a generating tuple whose orbit is defined by a Π_1 formula. Using effective versions of the results of Montalbán and A. Miller, we get the fact that a computable finitely generated group has a computable d-Σ_2 Scott sentence iff there is a generating tuple whose orbit is defined by a computable Π_1 formula.

References

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4 A. Montalbán, "A robuster Scott rank", Proceedings of the AMS, Vol. 143, pp. 5427-5436, 2015.
5 D. Scott, "Logic with denumerably long formulas and finite strings of quantifiers", In The Theory of Models, ed. by J. Addison, L. Henkin, and A. Tarski, pp. 329–341, North-Holland, 1965.

3.8 Computability, Proof Mining and Metric Regularity

Ulrich Kohlenbach (TU Darmstadt, DE)

Concepts of metric regularity and weak sharp minima which are generalizations of quantitative notions of strong uniqueness to problems with non-unique solutions play an important role in convex optimization. We will discuss computability and proof theoretic aspects of this as well as applications to minimization problems, fixed points of resolvents and zeros of subdifferentials (partly joint work with Genaro Lopez-Acedo). We also present recent applications of 'proof mining' to convex feasibility problems [2, 3]. In particular, we give a polynomial rate of asymptotic regularity [4] for Bauschke's solution of the minimal displacement conjecture [1], that is, for Picard iterates of compositions of metric projections in Hilbert space (without the assumption of the existence of a fixed point).

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2 M. A. A. Khan, U. Kohlenbach, "Quantitative image recovery theorems", Nonlinear Analysis, Vol. 106, pp. 138–150, 2014.
3 U. Kohlenbach, "On the quantitative asymptotic behavior of strongly nonexpansive mappings in Banach and geodesic spaces", Israel Journal of Mathematics, Vol. 216, pp. 215–246, 2016.
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3.9 A peek at the higher levels of the Weihrauch hierarchy

Alberto Marcone (University of Udine, IT)

Joint work of Andrea Cettolo, Alberto Marcone

Weihrauch reducibility and the ensuing Weihrauch hierarchy have been successfully used to refine reverse mathematics results for statements which are provable in ACA_0 and below. The study the Weihrauch hierarchy for functions arising from statements lying at higher levels (such as ATR_0) of the reverse mathematics spectrum was suggested by the author in the open problem session of Dagstuhl Seminar 15392 in September 2015.

We start this study, obtaining in some cases the expected finer classification, but in others observing a collapse of statements that are not equivalent with respect to provability in subsystems of second order arithmetic. This is in part due to the increased syntactic complexity of the statements.

Our preliminary results deal with comparability of well-orderings, $\boldsymbol{\Sigma}^1_1$-separation, and $\boldsymbol{\Delta}^1_1$-comprehension.

3.10 Randomness notions in Muchnik and Medvedev degrees

Kenshi Miyabe (Meiji University – Kawasaki, JP)

The main question of this talk is whether one can construct a more random set from a random set. This question can be formalized by mass problems, that is, Muchnik and Medvedev degrees. As an example, computable randomness is strictly below ML-randomness in Muchnik degrees because there exists a high minimal Turing degree, which contains a computably random set but no ML-random set is Turing below it. Similar arguments can be applied for other pairs. There are two interesting pairs of randomness notions that have the same Muchnik degree. One pair is the one of ML-randomness and difference random. This is because, for ML-random set $X + Y$, at least one of X or Y should be difference random. In contrast, ML-randomness and difference random have different Medvedev degrees. In other words, one can not compute a difference random from a ML-random uniformly. The proof uses the Levin-Kautz theorem and no-randomness-from-nothing for ML-randomness. The other pair is the one of Schnorr randomness and computable randomness. They have the same Muchnik degree but different Medvedev degrees. The proof extends the method separating Schnorr randomness and computable randomness using Levy's zero-one law from probability theory.

3.11 Stopping time complexity

Alexander Shen (University of Montpellier & CNRS, FR)

Joint work of Mikhail Andreev, Gleb Posobin, Alexander Shen

Consider a bit string x written on the input one-directional tape of some Turing machine. We want the machine to stop reading the tape exactly when x is read. How much information should be communicated to this machine? We may call this amount "stopping time complexity" of x.

This quantity (in the context of prediction theory) was considered by Vovk and Pavlovic (see https://arxiv.org/abs/1603.04283), and we try to perform a more systematic analysis of it in the language of Kolmogorov complexity.

One can consider the plain version of stopping time complexity (minimal plain complexity of a Turing machine that stops at x). It turns out to be equivalent to monotone-conditional complexity $C(x|x*)$ where the condition x is considered as a prefix of the string. There is also a quantitative characterization as a minimal upper semicomputable function such that on every path there is at most 2^n points where the function drops below n.

We show also that one should be careful: for the general case of $C(x|y*)$ we should consider monotone (prefix-stable), not prefix-free functions of y.

A similar theory can be constructed for prefix versions of stopping time complexity. We answer the question asked by Vovk–Pavlovic and show that the minimal prefix complexity of a program stopping at x, the quantity $\mathrm{K}(x|x*)$ and the logarithm of stopping time semimeasure, introduced by Vovk and Pavlovic, are all different. Also we show that the stopping time semimeasure has a natural probabilistic interpretation while for the general case $m(x|y*)$ the natural interpretation is no longer valid.

3.12 Genericity, randomness, and differentiable functions

Sebastiaan A. Terwijn (Radboud University Nijmegen, NL)

Joint work of Rutger Kuyper, Sebastiaan A. Terwijn
Main reference R. Kuyper, S. A. Terwijn, "Effective genericity and differentiability", Journal of Logic and Analysis, 6(4):1–14, 2014.
URL http://logicandanalysis.org/index.php/jla/article/view/215/94

We present a theorem characterizing the notion of 1-genericity in terms of differentiable functions. We compare this to a recent characterization of the notion of 1-randomness, also in terms of differentiability. We also discuss variations about n-genericity, multiple differentiability, and polynomial time computability.

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2 R. Kuyper, S. A. Terwijn, "Effective genericity and differentiability", Journal of Logic and Analysis, Vol. 6(4), pp. 1–14, 2014.

3.13 Stochasticity in Algorithmic Statistics for Polynomial Time

Nikolay K. Vereshchagin (Moscow State University, RU & National Research University Higher School of Economics – Moscow, RU) and Alexey Milovanov

Main reference A. Milovanov, N. Vereshchagin, "Stochasticity in Algorithmic Statistics for Polynomial Time", Report TR17-043, ECCC, 2017.
URL https://eccc.weizmann.ac.il/report/2017/043/download

A fundamental notion in Algorithmic Statistics is that of a stochastic object, that is, an object having a simple plausible explanation. Informally, a probability distribution is a plausible explanation for x if it looks likely that x was drawn at random with respect to that distribution. In this paper, we suggest three definitions of a plausible statistical hypothesis for Algorithmic Statistics with polynomial time bounds, which are called *acceptability, plausibility and optimality*. Roughly speaking, a probability distribution μ is called an acceptable explanation for x, if x possesses all properties decidable by short programs in a short time and shared by almost all objects (with respect to μ). Plausibility is a similar notion, however this time we require x to possess all properties T decidable even by long programs in a short time and shared by almost all objects. To compensate the increase in program length, we strengthen the notion of 'almost all' – the longer the program recognizing the property is, the more objects must share the property. Finally, a probability distribution μ is called an optimal explanation for x if $\mu(x)$ is large (close to $2^{-C^{\mathrm{poly}}(x)}$).

Almost all our results hold under some plausible complexity theoretic assumptions. Our main result states that for acceptability and plausibility there are infinitely many non-stochastic objects, that is, objects that do not have simple plausible (acceptable) explanations. We explain why we need assumptions – our main result implies that P $\neq$ PSPACE. In the proof of that result, we use the notion of an *elusive set*, which is interesting in its own right. Using elusive sets, we show that the distinguishing complexity of a string x can be super-logarithmically less than the conditional complexity of x with condition r for almost all r (for polynomial time bounded programs). Such a gap was known before, however only in the case when both complexities are conditional, or both complexities are unconditional.

It follows from the definition that plausibility implies acceptability and optimality. We show that there are objects that have simple acceptable but implausible and non-optimal explanations. We prove that for strings whose distinguishing complexity is close to Kolmogorov complexity (with polynomial time bounds) plausibility is equivalent to optimality for all simple distributions, a fact that can be considered a justification of the Maximal Likelihood Estimator.

3.14 Turing-, tt-, and m-reductions for functions in the Baire hierarchy

Linda Brown Westrick (University of Connecticut – Storrs, US)

Joint work of Adam Day, Rod Downey, Linda Brown Westrick

For arbitrary functions $f\colon [0,1] \to \mathbb{R}$, (including in particular highly non-continuous functions), what would be the right notion of Turing reducibility and its variants? We present a computationally motivated definition of $\leq_{\mathrm{T}}$, $\leq_{\mathrm{tt}}$, and $\leq_{\mathrm{m}}$ for such functions, and show that within the Baire hierarchy, the linearly ordered $\leq_{\mathrm{T}}$-equivalence classes correspond precisely to the proper Baire classes. Further, within the Baire 1 functions, the $\leq_{\mathrm{tt}}$ and $\leq_{\mathrm{m}}$ equivalence classes enjoy a natural correspondence with levels of the α rank on Baire 1 functions considered in Kechris and Louveau (1990).

4 Working groups

4.1 Summary of the open problems session

- **Question (Barmpalias)** What is the measure of minimal covers in the Turing degrees?
- **Conjecture (Barmpalias)** A K-degree has uncountably many predecessors if and only if it is not infinitely often K-trivial.

- **Question (Fouché)** The continuous action of a topological group G on a discrete set X is said to be a Ramsey action if for each finite subset F of X and each finite colouring of X, there is some $g \in G$ such that the colouring is monochromatic on gF. Such an action is necessarily transitive. A topological group is called Ramsey if all transitive actions on discrete sets are Ramsey. Let LO be the set of total orders on the natural numbers, viewed as a closed subspace of $\mathbb{N}^{\mathbb{N}\times\mathbb{N}}$. It is a deep fact that if G is a Ramsey group, then the logical action of G on LO has a fixed point. It is well-known that the space LO has a unique S_∞-invariant Radon measure μ. This is a computable measure. The problem proposed is to understand the fixed points of the action of a Ramsey group on LO from the viewpoint of algorithmic randomness relative to μ.

- **Question (Kalimullin; see Abstract 3.5)** Is there a computable structure with a degree of categoricity having the spectral dimension ω?
- **Question (Kalimullin)** Is there a computable structure $\mathcal{A}$ of computable dimension 2 with 2-element automorphism group such that two isomorphisms between its computable copies $\mathcal{A}_0$ and $\mathcal{A}_1$ have incomparable Turing degrees?

- **Question (Nies)** For K-trivial sets A and B we say that $A \leq_{\mathrm{ML}} B$ if every ML-random A computing B computes A. Is $\leq_{\mathrm{ML}}$ arithmetical? Note that by the Gandy basis theorem, if $A \not\leq_{\mathrm{ML}} B$ then there is a counterexample $Z \leq_{\mathrm{T}} \mathcal{O}$.
- **Question (Nies)** A K-trivial set A is called smart if every ML-random $Y \geq_{\mathrm{T}} A$ computes all the K-trivials. Is there a minimal pair of smart K-trivials?
- **Question (Nies)** Suppose A is K-trivial. Is A Turing below each LR-hard ML-random?
- **Question (Nies)** Is weak 2-randomness closed upward under $\leq_{\mathrm{K}}$?

- **Question (Yu)** For any real x and constant c, let $A_{x,c} = \{n \mid \mathrm{K}^x(n) \geq \mathrm{K}(n) - c\}$. Define $x \geq_{\mathrm{WLK}} y$ if for any c, if $A_{x,c}$ is infinite, then there is some d so that $A_{x,c} \subseteq A_{y,d}$. The question is whether for any weakly low for K real x, $x \geq_{\mathrm{WLK}} y$ implies $x \geq_{\mathrm{LK}} y$?
- **Question (Yu)** Is it true that under the assumption of PD, every uncountable Π^1_3 set A ranges over an upper cone of Q_3-degrees?

5 Impact

5.1 Preliminary results as reported by participants

During the seminar, numerous groups of researchers took advantage of the free time between talks to collaborate on their current research projects. While at the time of writing of this report it is too early to fully appreciate the impact of the seminar on the field of computability, the following very partial list describes concrete results already reported by participants.

- Willem Fouché reports that he and Arno Pauly finalised two papers during the seminar: "How constructive is constructing measures?" (Journal of Logic and Analysis, 9:c3, 1–30, 2017) and "Weihrauch-completeness for layerwise computability" (together with George Davie; submitted to Logical Methods in Computer Science). He furthermore reports that he and André Nies continued their work on the project "Computable profinite groups and randomness".
- Ulrich Kohlenbach reports that he finished his work on the article "A polynomial rate of asymptotic regularity for compositions of projections in Hilbert space" (submitted March 2017) during the seminar.
- Antonio Montalbán and Richard Shore report that they worked briefly on finishing their article "Conservativity and ultrafilters over subsystems of second order arithmetic" which is about to be submitted. They also worked extensively on a follow-up article tentatively titled "Iterated Hindman's Theorem, Gower's Fin_k Theorem, and the Infinite Hale-Jewett Theorem all peas in a pod".
- Linda Brown Westrick is reporting that after her presentation on the topic of a continuous reducibility for Borel functions she had fruitful discussions with Takayuki Kihara, Antonio Montalbán, and Arno Pauly, who shared with her some connections with their work. She also took advantage of the workshop to work on an ongoing project about Scott sets with Mariya Soskova, and on another ongoing project about a lightface version of reducibility for Borel functions with Rod Downey.

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