

Computational Counting

Edited by

Ivona Bezáková¹, Leslie Ann Goldberg², and Mark R. Jerrum³

¹ Rochester Institute of Technology, US, ib@cs.rit.edu

² University of Oxford, GB, leslie.goldberg@cs.ox.ac.uk

³ Queen Mary University of London, GB, m.jerrum@qmul.ac.uk

Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 17341 “Computational Counting”. The seminar was held from 20th to 25th August 2017, at Schloss Dagstuhl – Leibniz Center for Informatics. A total of 43 researchers from all over the world, with interests and expertise in different aspects of computational counting, actively participated in the meeting.

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1 Executive Summary

Ivona Bezáková

Leslie Ann Goldberg

Mark Jerrum

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Computational counting problems arise in practical applications in many fields such as statistical physics, information theory and machine learning. In such a problem, the goal is to compute or to estimate a weighted sum. Some typical computational counting problems include evaluating a probability, the expectation of a random variable, a partition function, or an integral.

The study of the computational complexity of counting problems requires a coherent set of techniques which are different in flavour from those employed in other algorithmic branches of computer science. Relevant techniques include the analysis of Markov chains, the analysis of correlation decay, parameterised algorithms and complexity, and dichotomy techniques for constructing detailed classifications.

Most computational problems are intractable when considered from the perspective of classical complexity, so it is important to find ways to cope with intractability. These include approximation, randomisation, as well as viewing computational counting through the lens of parameterised complexity, where the goal is to find algorithms that are efficient when some key parameter is “small”. Intractability thresholds often relate to “phase transitions” as these key parameters vary. Great progress has been made in recent years towards understanding



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the complexity of approximate counting, based largely on a connection with these phase transitions.

Specific themes identified for consideration at the meeting included:

- *Exact counting*, including classifications, quasi-polynomial and/or moderately exponential algorithms for intractable problems, and parameterised algorithms; also complexity-theoretic limitations to obtaining exact solutions.
- *Approximate counting* including Markov Chain Monte Carlo (*MCMC*) algorithms, and algorithms based on decay of correlations; also complexity-theoretic limitations to obtaining approximate solutions.
- The interplay between *phase transitions* and computational tractability.
- *Constraint satisfaction* problems and the more general *Holant* framework. The partition functions of many models in statistical physics are included within this setting.

In the event, the talks ranged more widely than this list suggests.

Although the topic of Computational Counting has been explored at various meetings, including at Dagstuhl, for a number of years, it continues to retain its freshness. New approaches are found, new insights are gained, and new connections drawn with other areas both inside and outside computer science. Among the new directions that have emerged since the previous Dagstuhl Seminar in this series are the following.

- Results from quantum information theory applied to the apparently unrelated task of classifying the complexity of Holant problems. (Refer to the talk by Miriam Backens.)
- A new paradigm for designing polynomial-time algorithms for approximating partition functions with complex parameters. This is based on Taylor expansion in a zero-free region of the parameter space combined with an ingenious approach to enumerating small substructures. (Refer to talks by Alexander Barvinok, Jingcheng Liu, Viresh Patel and Guus Regts.)
- Emerging connections between the Lovász Local Lemma – specifically the Shearer condition and the Moser-Tardos algorithmic version – and sampling and approximate counting. (Refer to talks by Andreas Galanis and Heng Guo.)

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3 Overview of Talks

3.1 Holant problems and quantum information theory

Miriam Backens (University of Bristol, GB)

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Holant problems are a family of counting problems on graphs, parameterised by sets of complex-valued functions of Boolean inputs. While the Holant framework was originally inspired by ideas from quantum computation, this connection has not been exploited before to analyse their complexity. We show how to apply ideas from quantum information theory to the analysis of Holant problems, using the bijection that exists between n -ary complex-valued functions of Boolean inputs and states of qubits (i.e. quantum bits), which are described by vectors in \mathbb{C}^{2^n} . Many interesting families of functions in the Holant framework correspond to families of quantum states that are of independent interest in quantum information theory. We sketch the role these ideas played in the derivation of two new Holant dichotomies, which make use of methods and knowledge from the theory of quantum entanglement. One of the new results, the full dichotomy for Holant^c, solves a long-standing open problem.

3.2 TotP-completeness and a connection to the approximability of #SAT

Eleni Bakali (National Technical University of Athens, GR)

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Joint work of Eleni Bakali, Aggeliki Chalki, Aris Pagourtzis, Petros Pantavos and Stathis Zachos

TotP is a counting complexity class, containing all self-reducible problems in #P, with decision version in P. We study completeness under parsimonious reductions. One such TotP-complete problem is the #Clustered-Monotone-SAT, which is a version of #SAT restricted to formulas with the property that their set of satisfying assignments is connected via a tree structure. Finally we show that the general #SAT problem is reduced to #Clustered-Monotone-SAT under approximation preserving reductions.

3.3 Weighted counting of 0-1 vectors in a subspace

Alexander Barvinok (University of Michigan - Ann Arbor, US)

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Joint work of Alexander Barvinok, Guus Regts

Given n complex numbers w_1, \dots, w_n , we define the weight of a set $X \subset \{0, 1\}^n$ by

$$w(X) = \sum_{\substack{x \in X \\ x = (x_1, \dots, x_n)}} w_1^{x_1} \cdots w_n^{x_n} = \sum_{\substack{x \in X \\ x = (x_1, \dots, x_n)}} \prod_{j: x_j \neq 0} w_j.$$

We prove that if $X \subset \{0,1\}^n$ is the set of solutions to a system of homogeneous linear equations with integer coefficients such that each equation contains at most $r \geq 2$ variables and each variable is contained in at most $c \geq 1$ equations then $w(X) \neq 0$ whenever

$$|w_j| \leq \frac{0.43}{r\sqrt{c}} \quad \text{for } j = 1, \dots, n.$$

Consequently, $w(X)$ can be approximated within a relative error $0 < \epsilon < 1$ in $(rc)^{O(\ln n - \ln \epsilon)}$ time provided

$$|w_j| \leq \frac{0.42}{r\sqrt{c}} \quad \text{for } j = 1, \dots, n.$$

We apply the result to approximately count perfect matchings that are close in the Hamming distance to a given perfect matching in a given hypergraph. Namely, let $H = (V, E)$ be a hypergraph with set V of vertices, set $E \subset 2^V$ of edges, with degrees of the vertices not exceeding $r \geq 2$ and the cardinality of every edge not exceeding $c \geq 1$. Given a perfect matching M_0 in H , we consider the sum

$$\sum_M w^{\text{dist}(M, M_0)},$$

where the sum is taken over all perfect matchings M in H and $\text{dist}(M, M_0)$ is the number of edges of H where the matchings differ. We prove that if

$$|w| \leq \frac{0.42}{r\sqrt{c}}$$

then the sum can be approximated within a relative error $0 < \epsilon < 1$ in $(rc)^{O(\ln |E| - \ln \epsilon)}$ time.

3.4 Spatial Mixing and non-local Markov chains

Antonio Blanca (Georgia Institute of Technology - Atlanta, US)

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Joint work of Antonio Blanca, Pietro Caputo, Alistair Sinclair, Eric Vigoda

In this talk, we consider the effects that the strong spatial mixing condition (SSM) has on the rate of convergence to equilibrium distribution of *non-local* Markov chains for spin systems on the integer lattice graph \mathbb{Z}^d . We show that SSM implies that the relaxation time (i.e., the inverse spectral gap) of general block dynamics is $O(r)$, where r is the number of blocks. We also proved that SSM implies an $O(1)$ bound for the relaxation time of the Swendsen-Wang dynamics in d -dimensional cubes of \mathbb{Z}^d . For *monotone* spin systems, we establish that the mixing time of systematic scan dynamics is $O(\log n (\log \log n)^2)$, provided that SSM holds. Our proofs use a variety of techniques for the analysis of Markov chains including coupling, functional analysis and linear algebra.

3.5 A fully polynomial randomized approximation scheme for Kostka numbers of two-row diagrams

Cornelius Brand (*Universität des Saarlandes, DE*)

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Joint work of Markus Bläser, Cornelius Brand, Sandip Sinha

Kostka numbers are a central quantity in the representation theory of the symmetric group, where they appear as multiplicities in the isotypic decomposition of permutation modules. They also arise as a special case of other representation-theoretic coefficients, such as Littlewood-Richardson and Kronecker coefficients. Computing these coefficients is of high practical relevance for the Geometric Complexity Theory program, and the scarcity of results in this area make the special case of Kostka numbers a very natural place to start. Young's rule gives them a combinatorial interpretation: Kostka numbers count certain arrangements of boxes, filled with a given contingent of numbers. Narayanan [1] proved them $\#P$ -complete, even if the Kostka numbers come from two-rowed box arrangements. Although their algorithmic aspects have been thoroughly studied (see Rassart's thesis [2] and the article by Barvinok and Fomin [3]), all known algorithmic results are exact and thus, by Narayanan's result, inherently inefficient. Narayanan posed the complexity of efficiently approximating these numbers as an open problem. We make a first step towards resolving the open problem: We exhibit an FPRAS for the aforementioned subclass of Kostka numbers arising from two-rowed arrangements.

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3.6 Finite duality, connectivity of homomorphisms, spatial mixing and Gibbs measures

Andrei A. Bulatov (*Simon Fraser University - Burnaby, CA*)

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Joint work of Andrei Bulatov, Victor Dalmau, Raimundo Briceño

We consider the properties and objects associated with a finite relational structure H and listed in the title:

- structure H has finite duality
- for any G the set of homomorphisms $\text{hom}(G, H)$ is connected, that is, any homomorphism can be transformed to any other homomorphism through a sequence of homomorphisms such that any two consequent ones only differ in bounded number of points
- for any G the set $\text{hom}(G, H)$ satisfies the strong spatial mixing property defined here in terms of extendibility of partial homomorphisms
- for any G there exist parameters (activities) such that $\text{hom}(G, H)$ has a unique Gibbs measure. We show that for any H these properties are closely related.

3.7 Dyadic Tilings and Phase Transitions

Sarah Cannon (Georgia Institute of Technology - Atlanta, US)

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Joint work of Sarah Cannon, Sarah Miracle, David Levin, Dana Randall, Alexandre Stauffer

We consider the weighted edge-flip Markov chain for dyadic tilings, show it exhibits a phase transition, and give rigorous results about its behavior at the phase transition. A dyadic tiling of size n is a tiling of the unit square by n non-overlapping dyadic rectangles, each of area $1/n$, where a dyadic rectangle is any rectangle that can be written in the form $[a2^{-s}, (a+1)2^{-s}] \times [b2^{-t}, (b+1)2^{-t}]$ for $a, b, s, t \in \mathbb{Z}_{\geq 0}$. The edge-flip Markov chain selects a random edge of the tiling and replaces it with its perpendicular bisector if doing so yields a valid dyadic tiling. We consider a weighted version of this Markov chain where, given a parameter $\lambda > 0$, we would like to generate each dyadic tiling σ with probability proportional to $\lambda^{|\sigma|}$, where $|\sigma|$ is the total edge length. We give results for both the mixing time (t_{mix}) and relaxation time (t_{rel}) of this chain. Specifically, we show there is a phase transition: when $\lambda < 1$, the edge-flipping chain has $t_{mix} = O(n^2 \log n)$ and $t_{rel} = O(n \log n)$, while when $\lambda > 1$, both t_{mix} and t_{rel} are at least $\exp(\Omega(n^2))$. At the critical point $\lambda = 1$, we show that $t_{rel} = O(n^{4.09})$, which implies that $t_{mix} = O(n^{5.09})$, the first such polynomial bounds. We complement this by showing that both t_{mix} and t_{rel} are at least $\Omega(n^{1.38})$, improving upon the previously best lower bound of $\Omega(n \log n)$ coming from the diameter of the chain. For relaxation time, this means the behavior at the critical point $\lambda = 1$ is provably different than on either side of the critical point, providing support for a well-known statistical physics conjecture.

3.8 Information-theoretic thresholds

Amin Coja-Oghlan (Goethe-Universität - Frankfurt am Main, DE)

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Joint work of Charilaos Efthymiou, Nor Jaafari, Tobias Kapetanopoulos, Mihyun Kang, Florent Krzakala, Amin Coja-Oghlan, Will Perkins, Lenka Zdeborová

This talk deals with the problem of inferring a given signal from a noisy observation. The question is: for what signal-to-noisy ratio is a non-trivial recovery of the signal information-theoretically possible? The first result concerns low-density generator matrix codes, for which we obtain a formula both for the information-theoretic threshold and for the mutual information. The second result is about the stochastic block model, where we obtain the information-theoretic threshold in the disassortative case with a general number of colors. Up to the information-theoretic threshold the stochastic block model and the Erdos-Renyi model are mutually contiguous. The proofs are based on some general results on discrete probability distributions, which may be of independent interest.

3.9 The complexity of graph motif parameters

Radu Curticapean (Hungarian Academy of Sciences - Budapest, HU)

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Joint work of Radu Curticapean, Holger Dell, Dániel Marx

We consider the problem of counting small patterns in large graphs. Many such problems can be expressed as linear combinations of induced subgraph numbers: For fixed graphs H , let Ind_H be the function that maps input graphs G to the number of induced subgraphs of G that are isomorphic to H . Now consider the function space obtained by taking finite linear combinations of these Ind_H functions. We call this the space of “graph motif parameters”.

The set $\{Ind_H|H\}$ forms a basis for this space. But it is known that so does the set $\{Sub_H|H\}$ of functions that count (not necessarily induced) subgraphs, as well as the set $\{Hom_H|H\}$ of functions that count homomorphisms from fixed H to input graphs G . In particular, the numbers of induced subgraphs, subgraphs, or homomorphisms from fixed H can be expressed as graph motif parameters.

We are interested in the computational complexity of evaluating fixed graph motif parameters f on input graphs. To this end, we define the complexity $C(f)$ as the infimum over all c such that the evaluation of f has an $O(n^c)$ time algorithm. This number is finite for every f , as we can always evaluate f by brute-force.

Our main result is that for any graph motif parameter f , the complexity $C(f)$ is precisely the maximum $C(Hom_F)$ over all F that have a non-zero coefficient when expressing f in the Hom -basis. From previous results, we have a relatively good grip on the complexity of homomorphisms from small patterns. Using this, we gained a more comprehensive understanding of the complexity of other classes of graph motif parameters, with a focus on counting subgraphs, where we obtain much simpler proofs of existing dichotomies, together with tight lower bounds under the Exponential Time Hypothesis that previously seemed out of reach.

3.10 A Fixed-Parameter Perspective on #BIS

Holger Dell (Universität des Saarlandes, DE)

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Joint work of Radu Curticapean, Holger Dell, Fedor Fomin, Leslie Ann Goldberg, John Lapinskas

The problem of (approximately) counting the independent sets of a bipartite graph (#BIS) is the canonical approximate counting problem that is complete in the intermediate complexity class #RHI₁. It is believed that #BIS does not have an efficient approximation algorithm but also that it is not NP-hard. We study the robustness of the intermediate complexity of #BIS by considering variants of the problem parameterised by the size of the independent set. We exhaustively map the complexity landscape for three problems, with respect to exact computation and approximation and with respect to conventional and parameterised complexity. The three problems are counting independent sets of a given size, counting independent sets with a given number of vertices in one vertex class and counting maximum independent sets amongst those with a given number of vertices in one vertex class. Among other things, we show that all of these problems are NP-hard to approximate within any polynomial ratio. (This is surprising because the corresponding problems without the size

parameter are complete in $\#RHI_1$, and hence are not believed to be NP-hard.) We also show that the first problem is $\#W[1]$ -hard to solve exactly but admits an FPTRAS, whereas the other two are $W[1]$ -hard to approximate even within any polynomial ratio. Finally, we show that, when restricted to graphs of bounded degree, all three problems have efficient exact fixed-parameter algorithms.

3.11 Matchings and the switch chain

Martin Dyer (University of Leeds, GB)

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Joint work of Martin Dyer, Heiko Müller, Mark Jerrum

The switch chain is a simple Markov chain for generating a random perfect matching in a graph. This chain was studied in a 2016 paper for certain classes of bipartite graphs. We will describe extensions of that work to classes of nonbipartite graphs.

3.12 Improved bounds for sampling colorings of sparse random graphs

Charis Efthymiou (Goethe-Universität - Frankfurt a. M., DE)

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Joint work of Charilaos Efthymiou, Tom Hayes, Daniel Štefankovič, Eric Vigoda

We study the mixing properties of the single-site Markov chain known as the Glauber dynamics for sampling k -colorings of a sparse random graph $G(n, d/n)$ for constant d . The best known rapid mixing results for general graphs are in terms of the maximum degree Δ of the input graph G and hold when $k > 11\Delta/6$ for all G . Improved results hold when $k > \alpha\Delta$ for graphs with girth ≥ 5 and Δ sufficiently large where $\alpha \approx 1.7632\dots$ is the root of $\alpha = \exp(1/\alpha)$; further improvements on the constant α hold with stronger girth and maximum degree assumptions.

For sparse random graphs the maximum degree is a function of n and the goal is to obtain results in terms of the expected degree d . The following rapid mixing results for $G(n, d/n)$ hold with high probability over the choice of the random graph for sufficiently large constant d . Mossel and Sly [1] proved rapid mixing for constant k , and Efthymiou [2] improved this to k linear in d . The condition was improved to $k > 3d$ by Yin and Zhang [3] using non-MCMC methods.

Here we prove rapid mixing when $k > \alpha d$ where $\alpha \approx 1.7632\dots$ is the same constant as above. Moreover we obtain $O(n^3)$ mixing time of the Glauber dynamics, while in previous rapid mixing results the exponent was an increasing function in d . As in previous results for random graphs our proof analyzes an appropriately defined block dynamics to “hide” high-degree vertices. One new aspect in our improved approach is utilizing so-called local uniformity properties for the analysis of block dynamics. To analyze the “burn-in” phase we prove a concentration inequality for the number of disagreements propagating in large blocks.

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3.13 The Complexity of Counting Surjective Homomorphisms and Compactions

Jacob Focke (University of Oxford, GB)

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Joint work of Jacob Focke, Leslie Ann Goldberg, Stanislav Živný

A homomorphism from a graph G to a graph H is a function from the vertices of G to the vertices of H that preserves edges. A homomorphism is *surjective* if it uses all of the vertices of H and it is a *compaction* if it uses all of the vertices of H and all of the non-loop edges of H . Hell and Nešetřil gave a complete characterisation of the complexity of deciding whether there is a homomorphism from an input graph G to a fixed graph H . A complete characterisation is not known for surjective homomorphisms or for compactions, though there are many interesting results. Dyer and Greenhill gave a complete characterisation of the complexity of counting homomorphisms from an input graph G to a fixed graph H . In this paper, we give a complete characterisation of the complexity of counting surjective homomorphisms from an input graph G to a fixed graph H and we also give a complete characterisation of the complexity of counting compactions from an input graph G to a fixed graph H .

3.14 Inapproximability of the independent set polynomial below the Shearer threshold

Andreas Galanis (University of Oxford, GB)

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Joint work of Andreas Galanis, Leslie Ann Goldberg, Daniel Štefankovič

For a graph G , let $p_G(\lambda) = \sum \lambda^{|I|}$ where the sum ranges over all the independent sets I of G . For which values of λ can we approximate $p_G(\lambda)$ on graphs G of max degree D ?

For $\lambda > 0$, breakthrough results of Weitz and Sly established a computational transition from easy to hard at the tree uniqueness threshold from statistical physics, given by $\lambda_c(D) = (D-1)^{(D-1)}/(D-2)^D$.

For $\lambda < 0$, the evaluation of the independent set polynomial is connected to the conditions of the Lovasz Local Lemma. Shearer identified the threshold $\lambda^*(D) = (D-1)^{(D-1)}/D^D$ as the maximum value q such that every family of events with failure probability at most q and whose dependency graph has max degree D has nonempty intersection. Very recently, Patel and Regts, and Harvey et al. have independently designed FPTASes for approximating the value of the independent set polynomial whenever $0 > \lambda > -\lambda^*(D)$.

Our main result establishes for the first time a computational transition at the Shearer threshold. We show that for all $D \geq 3$, for all $\lambda < -\lambda^*(D)$, it is NP-hard to approximate the

independent set polynomial on graphs of maximum degree D , even within an exponential factor. In fact, we now have the following picture for evaluating the independent set polynomial on graphs G of max degree D .

- For $-\lambda^*(D) < \lambda < \lambda_c(D)$, the problem admits an FPTAS.
- For $\lambda < -\lambda^*(D)$ or $\lambda > \lambda_c(D)$, the problem is NP-hard to approximate.

3.15 Uniqueness of Gibbs Measures for Continuous Hardcore Models

David Gamarnik (MIT - Cambridge, US)

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Joint work of David Gamarnik, Kavita Ramanan

We formulate a continuous version of the well known discrete hardcore (or independent set) model on a locally finite graph, parameterized by the so-called activity parameter $\lambda > 0$. In this version, the state or “spin value” x_u of any node u of the graph lies in the interval $[0, 1]$, the hardcore constraint $x_u + x_v \leq 1$ is satisfied for every edge (u, v) of the graph, and the space of feasible configurations is given by a convex polytope. When the graph is a regular tree, we show that there is a unique Gibbs measure associated to each activity parameter $\lambda > 0$. Our result shows that, in contrast to the standard discrete hardcore model, the continuous hardcore model does not exhibit a phase transition on the infinite regular tree. We also consider a family of continuous models that interpolate between the discrete and continuous hardcore models on a regular tree when $\lambda = 1$ and show that each member of the family has a unique Gibbs measure, even when the discrete model does not. In each case, the proof entails the analysis of an associated Hamiltonian dynamical system that describes a certain limit of the marginal distribution at a node. Furthermore, given any sequence of regular graphs with fixed degree and girth diverging to infinity, we apply our results to compute the asymptotic limit of suitably normalized volumes of the corresponding sequence of convex polytopes of feasible configurations. In particular, this yields an approximation for the partition function of the continuous hard core model on a regular graph with large girth in the case $\lambda = 1$.

3.16 A threshold result for loose Hamiltonicity in random regular uniform hypergraphs

Catherine Greenhill (UNSW Sydney, AU)

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Joint work of Catherine Greenhill, Daniel Altman, Mikhail Isaev, Reshma Ramadurai


A hypergraph is s -uniform if every edge contains s vertices. Robinson and Wormald [1, 2] proved that for any constant $r \geq 3$, a uniformly random r -regular graph on n vertices is Hamiltonian with probability which tends to 1 as $n \rightarrow \infty$. We extend this result to loose Hamilton cycles in uniformly random s -uniform r -regular hypergraphs, finding the degree threshold (as a function of s) which guarantees existence.

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3.17 Uniform Sampling through the Lovász Local Lemma

Heng Guo (*Queen Mary University of London, GB*)

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Joint work of Heng Guo, Mark Jerrum, Jingcheng Liu

We propose a new algorithmic framework, called “partial rejection sampling”, to draw samples exactly from a product distribution, conditioned on none of a number of bad events occurring. Our framework builds new connections between the variable framework of the Lovász Local Lemma and some classical sampling algorithms such as the “cycle-popping” algorithm for rooted spanning trees by Wilson. Among other applications, we discover new algorithms to sample satisfying assignments of k -CNF formulas with bounded variable occurrences.

3.18 Counting graph homomorphisms modulo 2

Andreas Göbel (*Hasso-Plattner-Institut - Potsdam, DE*)

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The complexity of modular counting was introduced by Papadimitriou and Zachos and it has been pioneered by Valiant who famously introduced a problem for which counting modulo 7 is easy where counting modulo 2, is intractable. A characteristic feature of modular counting is that cancellations make wider classes of instances tractable than is the case for exact (non-modular) counting.

A homomorphism from a graph G to a graph H is a function from $V(G)$ to $V(H)$ that preserves edges. Many combinatorial structures that arise in mathematics and in computer science can be represented naturally as graph homomorphisms and as weighted sums of graph homomorphisms. Modular counting provides a rich setting in which to study the structure of homomorphism problems.

In this talk we will discuss the complexity of counting graph homomorphisms modulo 2. Faben and Jerrum are the first to study the complexity of this problem. They show that for a specific family of target graphs the problem of counting homomorphisms to a graph modulo 2 is computationally easy and conjecture that for every other target graph the problem is computationally hard. They also prove their conjecture when H is a tree. Our results build upon the work of Faben and Jerrum. We first prove their conjecture for the class of cactus graphs, which are connected graphs in which every edge belongs to at most one cycle. We then show that for all graphs that contain no 4-cycles the problem of counting graph homomorphisms modulo 2 is either easy to compute or hard to compute, and that there are no target graphs H for which the problem has intermediate complexity.

3.19 Fine-grained reductions from approximate counting to decision

John Lapinskas (University of Oxford, GB)

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Joint work of Holger Dell, John Lapinskas

Main reference Holger Dell and John Lapinskas. Fine-grained reductions from approximate counting to decision. Preprint.

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The main problems in fine-grained complexity are CNF-SAT, the Orthogonal Vectors problem, 3SUM, and the Negative-Weight Triangle problem (which is closely related to All-Pairs Shortest Path). In this paper, we consider the approximate counting version of each problem; thus instead of simply deciding whether a witness exists, we attempt to (multiplicatively) approximate the number of witnesses. In each case, we provide a fine-grained reduction from the approximate counting form to the usual decision form. For example, if there is an $O(c^n)$ -time algorithm that solves k -SAT for all k , then we prove there is an $O((c + o(1))^n)$ -time algorithm to approximately count the satisfying assignments. Similarly, we get that the exponential time hypothesis (ETH) is equivalent to an approximate counting version. This mirrors a result of Sipser [2] and Stockmeyer [3], who proved such a result in the classical polynomial-time setting, and a similar result due to Müller [1] in the FPT setting.


Our algorithm for polynomial-time problems applies in a general setting in which we approximately count edges of a bipartite graph to which we have limited access. In particular, this means it can be applied to problem variants in which significant improvements over the conjectured running time bounds are already known. For example, the Orthogonal Vectors problem over $\text{GF}(m)^d$ for constant m can be solved in time $n \cdot \text{poly}(d)$ by a result of Williams and Yu [5]; our result implies that we can approximately count the number of orthogonal pairs with essentially the same running time. Moreover, our overhead is only polylogarithmic, so it can be applied to subpolynomial improvements such as the $n^3 / \exp(\Theta(\sqrt{\log n}))$ time algorithm for the Negative-Weight Triangle problem due to Williams [4].

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3.20 The Ising Partition Function: Zeros and Deterministic Approximation

Jingcheng Liu (University of California - Berkeley, US)

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Joint work of Jingcheng Liu, Alistair Sinclair, Piyush Srivastava

We study the problem of approximating the partition function of the ferromagnetic Ising model in graphs and hypergraphs. Our first result is a *deterministic* approximation scheme (an FPTAS) for the partition function in bounded degree graphs that is valid over the entire range of parameters β (the interaction) and λ (the external field), except for the case $|\lambda| = 1$ (the “zero-field” case). A *randomized* algorithm (FPRAS) for all graphs, and all β, λ , has long been known. Unlike most other deterministic approximation algorithms for problems in statistical physics and counting, our algorithm does not rely on the “decay of correlations” property. Rather, we exploit and extend machinery developed recently by Barvinok, and Patel and Regts, based on the location of the complex zeros of the partition function, which can be seen as an algorithmic realization of the classical Lee-Yang approach to phase transitions. Our approach extends to the more general setting of the Ising model on hypergraphs of bounded degree and edge size, where no previous algorithms (even randomized) were known for a wide range of parameters. In order to achieve this extension, we establish a tight version of the Lee-Yang theorem for the Ising model on hypergraphs, improving a classical result of Suzuki and Fisher.

3.21 Approximately counting small witnesses when there aren’t too many

Kitty Meeks (University of Glasgow, GB)

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Suppose we want to know how many witnesses of size k exist in a universe of size n . If we can guarantee that the total number of witnesses is sufficiently large, we can apply well-known random sampling techniques in an obvious way to obtain a good estimate of the number of witnesses. Here, we consider the other extreme: if we know that the number of witnesses is very small (so in particular an efficient counting algorithm could in fact examine all witnesses), can we obtain an efficient algorithm to (approximately) count the number of witnesses? We answer this question in the affirmative for a large family of problems, building on previous work on enumerating small witnesses [1] to show that, so long as there is an fpt-algorithm which solves corresponding decision problem correctly with probability greater than $1/2$, we can obtain an FPTRAS for the counting problem. We also consider a more challenging version of the problem in which we wish to count all minimal witnesses having size at most k , and discuss how to solve this in one specific example.

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3.22 Mixing Times of Markov Chains for Self-Organizing Lists and Biased Permutations

Sarah Miracle (University of St. Thomas - St. Paul, US)

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Joint work of Prateek Bhakta, Sarah Miracle, Dana Randall, Amanda Streib

In this talk I discuss a biased version of the nearest-neighbor transposition Markov chain on the set of permutations where neighboring elements i and j are placed in order (i, j) with probability $p_{i,j}$. The goal is to identify the class of parameter sets $\mathbf{P} = \{p_{i,j}\}$ for which this Markov chain is rapidly mixing.

In joint work with Prateek Bhakta, Dana Randall and Amanda Streib, we use a reduction from biased permutations to Asymmetric Simple Exclusion Processes (ASEPs) to show that the chain may be slow in the most general setting, even if \mathbf{P} is positively biased (i.e., $1 \geq p_{i,j} \geq 1/2$ for all $i < j$). We then prove the chain is rapidly mixing for two new classes. The first is “Choose Your Weapon,” where we are given r_1, \dots, r_{n-1} with $1 \geq r_i \geq 1/2$ and $p_{i,j} = r_i$ for all $i < j$. In the second class “League Hierarchies,” the probabilities are based on binary trees.

Finally, in joint work with Amanda Streib, we consider distributions arising from k -class particle processes, where the elements are divided into k classes and the probability of exchanging neighboring elements depends on the particular classes the elements are in. We further require that k is a constant, and all probabilities between elements in different classes are bounded away from $1/2$. These particle processes arise in the context of self-organizing lists and our result also applies beyond permutations to the setting where all particles in a class are indistinguishable. Additionally we show that a broader class of distributions based on trees is also rapidly mixing.

3.23 Zero-free regions and approximation algorithms for graph polynomials

Viresh Patel (University of Amsterdam, NL)

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Joint work of Viresh Patel, Guus Regts

In this talk, I discuss a new way of constructing deterministic polynomial-time approximation algorithms for computing complex-valued evaluations of a large class of graph polynomials on bounded degree graphs. In particular, our approach works for the Tutte polynomial and independence polynomial, as well as partition functions of complex-valued spin and edge-coloring models. Our work builds on a recent line of work initiated by Barvinok, which provides a new algorithmic approach besides the existing Markov chain Monte Carlo method and the correlation decay method for these types of problems.

3.24 Extremal bounds on partition functions via observables

Will Perkins (University of Birmingham, GB)

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Joint work of Ewan Davies, Matthew Jenssen, Barnaby Roberts, Guillem Perarnau, Will Perkins

We present a new method for proving extremal bounds on the partition function of statistical physics models on various classes of bounded degree graphs. As an example, consider the hard core model of a random independent set from a graph G , with $\Pr(I) = \lambda^{|I|}/Z_G(\lambda)$, where $Z_G(\lambda)$ is the partition function of the model. Let $\bar{\alpha}_G(\lambda) = \lambda \cdot (\log Z_G(\lambda))' / |V(G)|$ be the occupancy fraction, or the expected fraction of vertices appearing in the random independent set. Using a linear programming relaxation, we show that for any d -regular graph G and any $\lambda > 0$, $\bar{\alpha}_G(\lambda) \leq \bar{\alpha}_{K_{d,d}}(\lambda)$. This strengthens results of Kahn, Zhao, Galvin, and Tetali on the partition function of the hard core model on regular graphs, as the bound on the occupancy fraction can be integrated to obtain the corresponding bound on the log partition function. We describe a general framework for this method in terms of local weak convergence of Gibbs measures on sparse graphs and describe further applications to matchings and to independent sets in regular graphs with girth constraints.

3.25 Slow Convergence via Free Energy for the Ising Model

Dana Randall (Georgia Institute of Technology - Atlanta, US)


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Joint work of David Gillman, Dana Randall

We consider mixing times of Glauber dynamics for the ferromagnetic Ising model on an $n \times n$ square lattice region. If we fix the spins on the top and bottom sides of the square to be $+$ and the left and right sides to be $-$, a standard Peierls argument shows that below some critical temperature t_c , any local Markov chain M requires time exponential in n to mix. The argument works by showing that each state in the cut has small energy (and therefore probability). We consider a family of “balanced mixed boundary conditions” formed by rotating the $+$ sides and $-$ sides pn lattice steps clockwise around the square, $0 \leq p < 1$. The Peierls argument can be extended to this family; however, one finds that the critical temperature $t_c(p)$ approaches 0 as p approaches $1/2$, and the argument breaks down when $p = 1/2$. Here we show that there is a universal temperature T below which \mathcal{M} will be slow for any balanced mixed boundary (defined as above). The novelty of our argument is showing that there is an exponentially small cut indicated by the *free energy*; we show that the free energy is smaller for a certain cut set due to the energy term when p is close to 0 or 1 and due to the entropy term when p is close to $1/2$, with all other cases interpolating between these two extremes.

3.26 On a conjecture of Sokal concerning roots of the independence polynomial

Guus Regts (University of Amsterdam, NL)

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Joint work of Han Peters, Guus Regts

Sokal conjectured about 16 years ago that there exists a region D_Δ in the complex plane that contains the interval $[0, \lambda_c(\Delta))$, where $\lambda_c(\Delta) := \frac{(\Delta-1)^{\Delta-1}}{(\Delta-2)^\Delta}$ and $\Delta \geq 3$, such that for any graph G of maximum degree at most Δ the independence polynomial of G does not vanish on D_Δ . In joint work with Han Peters we have settled this conjecture using complex dynamics. In this talk I will explain the connection between the location of zeros of the independence polynomial and complex dynamical systems and give some ideas of our proof of the conjecture. I will also explain how, by work of Patel and myself, building on a line of work initiated by Barvinok, this gives a new FPTAS for approximating evaluations of the independence polynomial on D_Δ . I will end with stating some open problems that arise naturally from our work.

3.27 Functional Clones and Expressibility of Partition Functions

David Richerby (University of Oxford, GB)

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Joint work of Andrei Bulatov, Leslie Ann Goldberg, Mark Jerrum, David Richerby, Stanislav Živný

The complexity of counting CSPs depends very much on reductions between constraint languages, and whether functions in one constraint language can be expressed in terms of those in another. Functional clones are sets of functions closed under the appropriate notions of expressibility for CSPs. We give a tour of some relevant clones, including those related to the Ising model and matchgates, and the relationships between them.

3.28 Counting Restricted Homomorphisms via Möbius Inversion over Matroid Lattices

Marc Roth (Universität des Saarlandes, DE)

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We present a framework for the complexity classification of parameterized counting problems that can be formulated as the summation over the numbers of homomorphisms from small pattern graphs H_1, \dots, H_l to a big host graph G with the restriction that the coefficients correspond to evaluations of the Möbius function over the lattice of a graphic matroid. This generalizes the idea of Curticapean, Dell and Marx [1] who used a result of Lovász stating that the number of subgraph embeddings from a graph H to a graph G can be expressed as such a sum over the lattice of partitions of H . In the first step we introduce what we call graphically restricted homomorphisms that, inter alia, generalize subgraph embeddings as well as locally injective homomorphisms. We provide a complete parameterized complexity

dichotomy for counting such homomorphisms, that is, we identify classes of patterns for which the problem is fixed-parameter tractable (FPT), including an algorithm, and prove that all other pattern classes lead to $\#W[1]$ -hard problems. The main ingredients of the proof are the complexity classification of linear combinations of homomorphisms due to Curticapean, Dell and Marx [1] as well as a corollary of Rota's NBC Theorem which states that the sign of the Möbius function over a geometric lattice only depends on the rank of its arguments.

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3.29 Exact recovery in the Ising blockmodel

Piyush Srivastava (TIFR Mumbai, IN)

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Joint work of Quentin Berthet, Philippe Rigollet, Piyush Srivastava

We introduce the Ising blockmodel, a graphical model for community detection based on the classical Ising model from statistical mechanics. We establish tight bounds on the sample complexity for both information theoretic and algorithmically efficient recovery of the planted communities. We also show that the sample complexity exhibits a phase transition in the parameter space of the model.

3.30 Mixing time of random walks on dynamic graphs

Alexandre Stauffer (University of Bath, GB)

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Joint work of Yuval Peres, Alexandre Stauffer, Jeff Steif

In this talk I discuss the problem of random walks on dynamic graphs (that is, on graphs that changes at the same time that the walker moves). We initially focus on the model of random walk on dynamical percolation. In this model, the edges of a graph G are either open or closed, and refresh their status at rate μ . At the same time a random walker moves on G at rate 1 but only along edges which are open. The regime of interest here is when μ goes to zero as the number of vertices of G goes to infinity, since this creates long-range dependencies on the model. When G is the d -dimensional torus of side length n , we prove that in the subcritical regime, the mixing times is of order n^2/μ . We also obtain results concerning mean squared displacement and hitting times. We present new results in this model and discuss some more general questions regarding random walks on dynamic graphs.

3.31 Structure Learning of H -colorings

Daniel Štefankovič (University of Rochester, US)

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Joint work of Antonio Blanca, Zongchen Chen, Daniel Štefankovič, Eric Vigoda

We study the structure learning problem for graph homomorphisms, commonly referred to as H -colorings, including the weighted case which corresponds to spin systems with hard constraints. The learning problem is as follows: for a fixed (and known) constraint graph H with q colors and an unknown graph $G = (V, E)$ with n vertices, given uniformly random H -colorings of G , how many samples are required to learn the edges of the unknown graph G ? We give a characterization of H for which the problem is identifiable for every G , i.e., we can learn G with an infinite number of samples. We focus particular attention on the case of proper vertex q -colorings of graphs of maximum degree d where intriguing connections to statistical physics phase transitions appear. We prove that when $q > d$ the problem is identifiable and we can learn G in $\text{poly}(d, q) \times O(n^2 \log n)$ time. In contrast for soft-constraint systems, such as the Ising model, the best possible running time is exponential in d . When $q \leq d$ we prove that the problem is not identifiable, and we cannot hope to learn G . When $q < d - \sqrt{d} + \Theta(1)$ we prove that even learning an equivalent graph (any graph with the same set of H -colorings) is computationally hard – sample complexity is exponential in n in the worst-case. For the q -colorings problem, the threshold for efficient learning seems to be connected to the uniqueness/non-uniqueness phase transition at $q = d$. We explore this connection for general H -colorings and prove that under a well-known condition in statistical physics, known as Dobrushin uniqueness condition, we can learn G in $\text{poly}(d, q) \times O(n^2 \log n)$ time.

3.32 Approximating partition functions of bounded-degree Boolean counting Constraint Satisfaction Problems

Kuan Yang (University of Oxford, GB)

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Joint work of Andreas Galanis, Leslie Ann Goldberg, Kuan Yang

We study the complexity of approximate counting Constraint Satisfaction Problems ($\#CSPs$) in a bounded degree setting. Specifically, given a Boolean constraint language Γ and a degree bound Δ , we study the complexity of $\#CSP_{\Delta}(\Gamma)$, which is the problem of counting satisfying assignments to CSP instances with constraints from Γ and whose variables can appear at most Δ times. Our main result shows that: (i) if every function in Γ is affine, then $\#CSP_{\Delta}(\Gamma)$ is in FP for all Δ , (ii) otherwise, if every function in Γ is in a class called IM_2 , then for all sufficiently large Δ , $\#CSP_{\Delta}(\Gamma)$ is equivalent under approximation-preserving (AP) reductions to the counting problem $\#BIS$ (the problem of counting independent sets in bipartite graphs) (iii) otherwise, for all sufficiently large Δ , it is NP-hard to approximate the number of satisfying assignments of an instance of $\#CSP_{\Delta}(\Gamma)$, even within an exponential factor. Our result extends previous results, which apply only in the so-called “conservative” case.

3.33 What can be sampled locally?

Yitong Yin (Nanjing University, CN)

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Joint work of Weiming Feng, Yuxin Sun, Yitong Yin

The local computation of Linial [1] and Naor and Stockmeyer [2] asks whether a locally definable distributed computing problem can be solved locally: for a given local CSP whether a CSP solution can be constructed by a distributed algorithm using local information. In this paper, we consider the problem of sampling a uniform CSP solution by distributed algorithms, and ask whether a locally definable joint distribution can be sampled from locally. More broadly, we consider sampling from Gibbs distributions induced by weighted local CSPs in the LOCAL model. We give two Markov chain based distributed algorithms which we believe to represent two fundamental approaches for sampling from Gibbs distributions via distributed algorithms. The first algorithm generically parallelizes the single-site sequential Markov chain by iteratively updating a random independent set of variables in parallel, and achieves an $O(\Delta \log n)$ time upper bound in the LOCAL model, where Δ is the maximum degree, when the Dobrushin's condition for the Gibbs distribution is satisfied. The second algorithm is a novel parallel Markov chain which proposes to update all variables simultaneously yet still guarantees to converge correctly with no bias. It surprisingly parallelizes an intrinsically sequential process: stabilizing to a joint distribution with massive local dependencies, and may achieve an optimal $O(\log n)$ time upper bound independent of the maximum degree Δ under a stronger mixing condition. In addition, we show that almost all nontrivial Gibbs distributions require $\Omega(\log n)$ rounds to sample in the LOCAL model. We also show a strong $\Omega(\text{diam})$ lower bound for sampling independent set in graphs with maximum degree $\Delta \geq 6$. This lower bound holds even when every node is aware of the graph. This gives a strong separation between sampling and constructing locally checkable labelings.

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Participants

- Miriam Backens
University of Bristol, GB
- Eleni Bakali
National Technical University of Athens, GR
- Alexander Barvinok
University of Michigan – Ann Arbor, US
- Ivona Bezáková
Rochester Institute of Technology, US
- Markus Bläser
Universität des Saarlandes, DE
- Antonio Blanca
Georgia Institute of Technology – Atlanta, US
- Cornelius Brand
Universität des Saarlandes, DE
- Andrei A. Bulatov
Simon Fraser University – Burnaby, CA
- Sarah Cannon
Georgia Institute of Technology – Atlanta, US
- Amin Coja-Oghlan
Goethe-Universität – Frankfurt am Main, DE
- Radu Curticapean
Hungarian Academy of Sciences – Budapest, HU
- Holger Dell
Universität des Saarlandes, DE
- Martin Dyer
University of Leeds, GB
- Charis Efthymiou
Goethe-Universität – Frankfurt a. Main, DE
- Jacob Focke
University of Oxford, GB
- Andreas Galanis
University of Oxford, GB
- David Gamarnik
MIT – Cambridge, US
- Andreas Göbel
Hasso-Plattner-Institut – Potsdam, DE
- Leslie Ann Goldberg
University of Oxford, GB
- Catherine Greenhill
UNSW Sydney, AU
- Heng Guo
Queen Mary University of London, GB
- Thomas Hayes
University of New Mexico – Albuquerque, US
- Miki Hermann
Ecole Polytechnique – Palaiseau, FR
- Thore Husfeldt
IT University of Copenhagen, DK
- Mark R. Jerrum
Queen Mary University of London, GB
- John Lapinskas
University of Oxford, GB
- Jingcheng Liu
University of California – Berkeley, US
- Kitty Meeks
University of Glasgow, GB
- Sarah Miracle
University of St. Thomas – St. Paul, US
- Viresh Patel
University of Amsterdam, NL
- Will Perkins
University of Birmingham, GB
- Dana Randall
Georgia Institute of Technology – Atlanta, US
- Guus Regts
University of Amsterdam, NL
- David Richerby
University of Oxford, GB
- Marc Roth
Universität des Saarlandes, DE
- Piyush Srivastava
TIFR Mumbai, IN
- Alexandre Stauffer
University of Bath, GB
- Daniel Stefankovic
University of Rochester, US
- Mingji Xia
Chinese Academy of Sciences – Beijing, CN
- Kuan Yang
University of Oxford, GB
- Yitong Yin
Nanjing University, CN
- Chihao Zhang
The Chinese University of Hong Kong, HK
- Stanislav Zivny
University of Oxford, GB

