Report from Dagstuhl Seminar 18231

The Constraint Satisfaction Problem: Complexity and Approximability

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Abstract

Constraint satisfaction has always played a central role in computational complexity theory; appropriate versions of CSPs are classical complete problems for most standard complexity classes. CSPs constitute a very rich and yet sufficiently manageable class of problems to give a good perspective on general computational phenomena. For instance, they help to understand which mathematical properties make a computational problem tractable (in a wide sense, e.g., polynomial-time solvable, non-trivially approximable, fixed-parameter tractable, or definable in a weak logic). In the last decade, research activity in this area has significantly intensified and hugely impressive progress was made. The Dagstuhl Seminar 18231 "The Constraint Satisfaction Problem: Complexity and Approximability" was aimed at bringing together researchers using all the different techniques in the study of the CSP so that they can share their insights obtained during the past three years. This report documents the material presented during the course of the seminar.

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1 Executive Summary

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The *constraint satisfaction problem*, or CSP in short, provides a unifying framework in which it is possible to express, in a natural way, a wide variety of computational problems dealing with mappings and assignments, including satisfiability, graph colourability, and systems of equations. The CSP framework originated 30–35 years ago independently in artificial intelligence, database theory, and graph theory under three different guises, and it was realised only in the late 1990s that these are in fact different faces of the same fundamental problem. Nowadays, the CSP is extensively used in theoretical computer science, being a mathematical object with very rich structure that provides an excellent laboratory both for classification methods and for algorithmic techniques; while in AI and more applied areas of computer science, this framework is widely regarded as a versatile and efficient way of modelling and solving a variety of real-world problems, such as planning and scheduling, software verification, and natural language comprehension, to name just a few. An instance of the CSP consists of a set of variables, a set of values for the variables, and a set of constraints that restrict the combinations of values that certain subsets of variables may take. Given such an instance, the possible questions include (a) deciding whether there is an assignment of values to the variables so that every constraint is satisfied, or optimising such assignments in various ways, or (b) finding an assignment satisfying as many constraints as possible. There are many important modifications and extensions of this basic framework, e.g., those that deal with counting assignments or involve soft or global constraints.

Constraint satisfaction has always played a central role in computational complexity theory; appropriate versions of CSPs are classical complete problems for most standard complexity classes. CSPs constitute a very rich and yet sufficiently manageable class of problems to give a good perspective on general computational phenomena. For instance, they help to understand which mathematical properties make a computational problem tractable (in a wide sense, e.g., polynomial-time solvable, non-trivially approximable, fixed-parameter tractable, or definable in a weak logic). One of the most striking features of this research direction is the variety of different branches of mathematics (including universal algebra and logic, combinatorics and graph theory, probability theory and mathematical programming) that are used to achieve deep insights in the study of the CSP. In the last decade, research activity in this area has significantly intensified and hugely impressive progress was made.

The recent flurry of activity on the topic of the seminar is witnessed by four previous Dagstuhl seminars, titled "Complexity of constraints" (06401) and "The CSP: complexity and approximability" (09441, 12541, 15301), that were held in 2006, 2009, 2012, and 2015 respectively. This seminar was a follow-up to the 2009, 2012, and 2015 seminars. Indeed, the exchange of ideas at the 2009, 2012, and 2015 seminars has led to ambitious new research projects and to establishing regular communication channels. There is clearly the potential for further systematic interaction that will keep on cross-fertilising the areas and opening new research directions. The 2018 seminar brought together 47 researchers from different highly advanced areas of constraint satisfaction and involved many specialists who use universal-algebraic, combinatorial, geometric, and probabilistic techniques to study CSP-related algorithmic problems. The participants presented, in 24 talks, their recent results on a number of important questions concerning the topic of the seminar. One particular feature of this seminar is a significant increase in the number of talks involving multiple subareas and approaches within its research direction – a definite sign of the growing synergy, which is one of the main goals of this series of seminars.

Concluding remarks and future plans: The seminar was well received as witnessed by the high rate of accepted invitations and the great degree of involvement by the participants. Because of a multitude of impressive results reported during the seminar and active discussions between researchers with different expertise areas, the organisers regard this seminar as a great success. With steadily increasing interactions between such researchers, we foresee another seminar focusing on the interplay between different approaches to studying the complexity and approximability of the CSP. Finally, the organisers wish to express their gratitude to the Scientific Directors of the Dagstuhl Centre for their support of the seminar.

Description of the Topics of the Seminar

Classical computational complexity of CSPs. Despite the provable existence of intermediate problems (say, between P and NP-complete, assuming P $\neq$ NP), research in computational complexity has produced a widely known informal thesis that "natural problems are almost always complete for standard complexity classes". CSPs have been actively used to support and refine this thesis. More precisely, several restricted forms of the CSP have been investigated in depth. One of the main types of restrictions is the *constraint language* restriction, i.e. a restriction on the available types of constraints. By choosing an appropriate constraint language, one can obtain many well-known computational problems from graph theory, logic, and algebra. The study of the constraint language restriction was driven by the CSP *Dichotomy Conjecture* of Feder and Vardi which states that, for each fixed constraint language, the corresponding CSP is either in P or NP-complete. There are similar dichotomy conjectures concerning other complexity classes (e.g., L and NL). Recent breakthroughs in the complexity of the CSP have been made possible by the introduction of the universal-algebraic approach, which extracts algebraic structure from the constraint language and uses it to analyse problem instances. The above conjectures have algebraic versions which also predict in algebraic terms where the boundary between harder problems and easier problems lies. The algebraic approach has been applied to prove the Dichotomy Conjecture in many important special cases (e.g., Bulatov's dichotomy theorems for 3-valued and conservative CSPs), culminating in two independent proofs of the general conjecture announced in 2017 by Bulatov and Zhuk.

- Bulatov and Zhuk gave detailed talks on the main insights into their proofs.
- Kolmogorov described an algorithm for Boolean CSPs under the restriction that every variable appears in exactly two constraints and all constraints are even Δ-matroids.

The valued CSP (VCSP) is a significant generalisation of the CSP that involves both feasibility and optimisation aspects. While the computational complexity of finite-domain VCSPs is by now well understood, the infinite-domain VCSPs are fairly unexplored.

- Viola gave a talk on submodular VCSPs on infinite domains.
- Kazda presented his results on the structure of weighted clones, which are intimately related to the computational complexity of VCSPs.

Approximability of CSPs. The use of approximation algorithms is one of the most fruitful approaches to coping with NP-hardness. Hard optimisation problems, however, exhibit diverse behavior with respect to approximability, making it an exciting research area that is by now well-developed but far from fully understood.

An emerging topic bridging the complexity of the CSP with approximation aspects is *promise constraint satisfaction* (PCSP). The PCSP is a generalization of the CSP in which the constraints come in pairs of "stricter" and "weaker" versions. In a PCSP instance, the task is to find an assignment satisfying the weaker constraints under the promise that there is an assignment satisfying the strict constraints.

- Brakensiek gave an introductory talk to this exciting research direction and also presented a dichotomy classification for symmetric Boolean PCSPs.
- Opršal explained the very recently introduced algebraic approach to the computational complexity of PCSPs.
- Barto presented his results on PCSPs and cyclic operations.

Many approximation algorithms for CSPs are based on convex relaxations.

- Berkholz gave an overview on relaxations for Boolean CSPs based on algebraic methods.
- Schramm explained the power of semidefinite programming relaxations for random CSPs.
- Tulsiani presented results on the limits of linear programming relaxations for CSPs.
- Makarychev showed how to obtain an integrality gap for the Călinescu-Karloff-Rabani linear programming relaxation of the Multiway-Cut problem.
- Austrin established the currently best known inapproximability result for Min UnCut, which is a special Boolean CSP.

Some of the most exciting developments in approximability in the last decade revolve around the *unique games conjecture*, or UGC, of Khot (2002). This bold conjecture asserts that, for CSPs with a certain constraint language over a large enough domain, it is NP-hard to distinguish almost satisfiable instances from those where only a small fraction of constraints can be satisfied. This conjecture is known to imply tight inapproximability results for many classical optimisation problems. Moreover, if the UGC is true, then, as shown by Raghavendra in 2008, a simple algorithm based on semidefinite programming provides the best possible approximation for *all* CSPs (though the exact quality of this approximation is unknown).

- Moshkovitz presented recent developments on the so-called 2-to-2 PCP theorem, which covers important special cases of the UGC.

Logic and the complexity of CSPs. Logic has been used in two distinct ways in the study of the CSP. One of them, starting from earlier work of Kolaitis and Vardi, is *descriptive complexity*, where one tries to classify CSPs as classes of instances with respect to definability in a given logic. The other way is to use logic to specify CSP instances, which can be done very naturally. The latter direction leads to generalisations such as the quantified CSP (QCSP), as well as to the study of CSPs over infinite domains, where important links with the algebraic approach were found.

- Roy presented a dichotomy theorem for the inverse satisfiability problem.
- Bodirsky gave a talk on two methods of reducing infinite-domain CSPs to finite-domain CSPs.
- Pinsker explained recent results on the algebraic approach to infinite-domain CSPs. These results are related to the so-called loop conditions, which were in more detail discussed by Kozik.
- Kompatscher presented a proof of the equivalence of two dichotomy conjectures for infinite-domain CSPs.
- Mottet gave a new proof of the dichotomy for MMSNP and discussed consequences for infinite-domain CSPs.
- Martin described recent results for temporal and spatial problems, which are special cases of infinite-domain CSPs.

Exact exponential complexity of CSPs. The area of parameterised complexity is closely related to the area of exact exponential complexity, in which the goal is to design the most efficient exponential-time algorithms. There has been significant progress on the exact exponential complexity of CSPs.

- Golovnev presented results that give optimal lower bounds on the running time of algorithms for deciding if there is a homomorphism from one graph to another.
- The complexity of counting solutions for CSPs and related problems from statistical physics were presented by Goldberg and Jerrum.

2 Table of Contents

3 Overview of Talks

3.1 Improved NP-hardness of approximating Max Cut

Per Austrin (KTH Royal Institute of Technology – Stockholm, SE)

Joint work of Per Austrin, Mårten Wiman

We show that the Min Uncut problem is NP-hard to approximate within 1.4568. The previous best bound was $11/8 - \epsilon$ [1].

References

1 J. Håstad, S. Huang, R. Manokaran, R. O'Donnell, and J. Wright. Improved NP-Inapproximability for 2-Variable Linear Equations. *Theory of Computing*, 13(1):1–51, 2017.

3.2 Cyclic operations in promise constraint satisfaction problems

Libor Barto (Charles University – Prague, CZ)

The promise constraint satisfaction problem (PCSP) is a generalization of the constraint satisfaction problem (CSP), where the aim is to distinguish between instances satisfiable over a fixed constraint language and instances that are not satisfiable over another fixed, weaker, constraint language. I will talk about two modest applications of cyclic operations (which proved useful in CSP) in PCSP. The first one is a negative result saying that the tractability of a PCSP cannot be always explained by a tractable finite-domain CSP, in a certain precise sense. The second one shows that monotone Boolean PCPs with enough cyclic polymorphisms are tractable.

3.3 On algebraic and semi-algebraic relaxations for Boolean CSPs

Christoph Berkholz (HU Berlin, DE)

Main reference Christoph Berkholz: "The Relation between Polynomial Calculus, Sherali-Adams, and Sum-of-Squares Proofs", in Proc. of the 35th Symposium on Theoretical Aspects of Computer Science, STACS 2018, February 28 to March 3, 2018, Caen, France, LIPIcs, Vol. 96, pp. 11:1–11:14, Schloss Dagstuhl – Leibniz-Zentrum fuer Informatik, 2018.
URL http://dx.doi.org/10.4230/LIPIcs.STACS.2018.11

In this talk I will give an overview on relaxations for Boolean CSPs based on algebraic methods (such as Hilbert's Nullstellensatz and the Gröbner basis Algorithm) as well as relaxations based on linear and semi-definite programming (such as the Sherali-Adams and the sum-of-squares hierarchy). A particular focus will be on comparing these methods with respect to their complexity, including recent findings from the main reference.

3.4 Reducing Infinite-Domain CSPs to Finite-Domain CSPs

Manuel Bodirsky (TU Dresden, DE)

In this talk I present two fundamentally different techniques to reduce infinite-domain CSPs to finite-domain CSPs, and give many examples of CSPs that can be solved in polynomial time using these methods.

3.5 Promise Constraint Satisfaction

Joshua Brakensiek (Carnegie Mellon University – Pittsburgh, US)

Joint work of Joshua Brakensiek, Venkatesan Guruswami
Main reference Joshua Brakensiek, Venkatesan Guruswami: "Promise Constraint Satisfaction: Algebraic Structure and a Symmetric Boolean Dichotomy", CoRR, Vol. abs/1704.01937, 2017.
URL http://arxiv.org/abs/1704.01937

With the recent resolution of the dichotomy conjecture, much focus is now on understanding generalizations of CSPs. We propose a generalization of CSPs called Promise CSPs. In a Promise CSP, constraints come in pairs: "stricter" and "weaker" versions (possibly over different domains). The main computational question is: given that there is an assignment satisfying the strict constraints, find an assignment to the weaker versions. Besides capturing ordinary CSPs, Promise CSPs also capture other problems in the literature, such as approximate graph coloring and $(2+\epsilon)$-SAT [1].

Promise CSPs can still be studied with polymorphic techniques, but they differ substantial from their CSP cousins. First, due to the gap between "strict" and "weak" versions, polymorphisms are no longer closed under composition (in particular they do not form a clone). Thus, tractability can no longer be explained by a single polymorphism but requires understanding an infinite sequence of polymorphisms simultaneously. In particular, it seems like topological methods rather than algebraic methods may be of greater use in understanding Promise CSPs.

Besides this general overview of Promise CSPs, some new results are discussed. On the algorithmic side, families of polymorphisms known as "threshold-periodic polymorphisms" are discussed whose corresponding co-clones seem to necessarily require both bounded-width and linear equation methods in order to solve. On the hardness side, the standard paradigm in hardness of approximation (long code testing + a suitable label cover variant) is sufficient to give a complete classification of Boolean Promise CSPs with symmetric, folded predicates.

References

1 P. Austrin, V. Guruswami, and J. Håstad. $(2+\epsilon)$-Sat Is NP-hard. *SIAM Journal on Computing*, 46(5):1554–1573, 2017.

3.6 CSP Dichotomy

Andrei A. Bulatov (Simon Fraser University – Burnaby, CA)

Main reference Andrei A. Bulatov: "A Dichotomy Theorem for Nonuniform CSPs", in Proc. of the 58th IEEE Annual Symposium on Foundations of Computer Science, FOCS 2017, Berkeley, CA, USA, October 15-17, 2017, pp. 319–330, IEEE Computer Society, 2017.
URL http://dx.doi.org/10.1109/FOCS.2017.37
Main reference Andrei A. Bulatov: "A dichotomy theorem for nonuniform CSPs", CoRR, Vol. abs/1703.03021, 2017.
URL http://arxiv.org/abs/1703.03021

In 1993 Feder and Vardi posed a conjecture that suggests that every Constraint Satisfaction problem (CSP) parametrized by its target structure is either solvable in polynomial time or is NP-complete [2]. Later this conjecture was made more precise by delineating the exact condition that separates the polynomial case from the NP-complete one [1]. The hardness part of the conjecture has been known for long time. In this work we present a polynomial time algorithm that solves all CSPs satisfying the condition, thus, confirming Feder-Vardi conjecture.

References

1 A. A. Bulatov, P. Jeavons, and A. A. Krokhin. Classifying the complexity of constraints using finite algebras. *SIAM Journal on Computing*, 34(3):720–742, 2005.
2 T. Feder and M. Y. Vardi. Monotone monadic SNP and constraint satisfaction. *STOC*, pages 612–622, 1993.

3.7 Boolean approximate counting CSPs with weak conservativity

Leslie Ann Goldberg (University of Oxford, GB)

Joint work of Miriam Backens, Andrei A. Bulatov, Leslie Ann Goldberg, Colin McQuillan, Stanislav Živný
Main reference Miriam Backens, Andrei Bulatov, Leslie Ann Goldberg, Colin McQuillan, Stanislav Živný: "Boolean approximate counting CSPs with weak conservativity, and implications for ferromagnetic two-spin", CoRR, Vol. abs/1804.04993, 2018.
URL http://arxiv.org/abs/1804.04993

We analyse the complexity of approximate counting constraint satisfactions problems $\#\mathrm{CSP}(F)$, where F is a set of nonnegative rational-valued functions of Boolean variables. A complete classification is known in the conservative case, where F is assumed to contain arbitrary unary functions. We strengthen this result by fixing any permissive strictly increasing unary function and any permissive strictly decreasing unary function, and adding only those to F: this is weak conservativity. The resulting classification is employed to characterise the complexity of a wide range of two-spin problems, fully classifying the ferromagnetic case.

3.8 Tight Bounds for the Graph Homomorphism Problem

Alexander Golovnev (Yahoo! Research – New York, US)

Joint work of Marek Cygan, Fedor V. Fomin, Alexander Golovnev, Alexander S. Kulikov, Ivan Mihajlin, Jakub Pachocki, Arkadiusz Socala
Main reference Marek Cygan, Fedor V. Fomin, Alexander Golovnev, Alexander S. Kulikov, Ivan Mihajlin, Jakub Pachocki, Arkadiusz Socala: "Tight Lower Bounds on Graph Embedding Problems", J. ACM, Vol. 64(3), pp. 18:1–18:22, 2017.
URL http://dx.doi.org/10.1145/3051094

We prove that unless the Exponential Time Hypothesis (ETH) fails, deciding if there is a homomorphism from graph G to graph H cannot be done in time $|V(H)|^{o(|V(G)|)}$. We also show an exponential-time reduction from Graph Homomorphism to Subgraph Isomorphism. This rules out (subject to ETH) a possibility of $|V(H)|^{o(|V(H)|)}$-time algorithm deciding if graph G is a subgraph of H. For both problems our lower bounds asymptotically match the running time of brute-force algorithms trying all possible mappings of one graph into another. Thus, our work closes the gap in the known complexity of these problems.

Moreover, as a consequence of our reductions, conditional lower bounds follow for other related problems such as Locally Injective Homomorphism, Graph Minors, Topological Graph Minors, Minimum Distortion Embedding and Quadratic Assignment Problem.

3.9 Approximately counting list H-colourings: a complexity classification

Mark R. Jerrum (Queen Mary University of London, GB)

Joint work of Andreas Galanis, Leslie Ann Goldberg, Mark R. Jerrum
Main reference Andreas Galanis, Leslie Ann Goldberg, Mark Jerrum: "A Complexity Trichotomy for Approximately Counting List H-Colorings", TOCT, Vol. 9(2), pp. 9:1–9:22, 2017.
URL http://dx.doi.org/10.1145/3037381

Suppose H is a fixed graph. H-colourings of a graph G (a.k.a. homomorphisms from G to H) generalise familiar proper vertex colourings of G. More than 15 years ago, Dyer and Greenhill considered the computational complexity of counting H-colourings, and demonstrated a dichotomy, in terms of the graph H, between polynomial time and #P-complete.

That result was for exact counting, and, even now, there is only a partial complexity classification for approximate counting. However, the classification problem becomes tractable if we look instead at *list* H-colourings. In this talk, I'll present a classification (in fact a trichotomy) for the problem of approximately counting list H-colourings of a graph. It turns out that some interesting hereditary graph classes come into play in describing and proving the trichotomy result.

3.10 Reflections for VCSP

Alexandr Kazda (Charles University – Prague, CZ)

Take a weighted constraint language Gamma on a finite domain. The complexity of VCSP(Γ) depends only on the clone of weighted polymorphisms of Γ. Kolmogorov, Rolínek and Krokhin gave a complete classification of this complexity in 2015 (for the journal version, see [1]); however there is still room for improvement of our understanding of weighted clones.

In our talk, we gave an overview of VCSP and conjectured that the theory of weighted clones could be made more elegant if one could make the idea of a reflection from [2] work for weighted clones and VCSPs.

References

1 V. Kolmogorov, A. Krokhin, and M. Rolínek. The Complexity of General-Valued CSPs. *SIAM Journal on Computing*, 46(3):1087–1110, 2017.
2 L. Barto, J. Opršal, and M. Pinsker. The wonderland of reflections. *Israel Journal of Mathematics*, 223(1):363–398, 2018.

3.11 Even Delta-Matroids and the Complexity of Planar Boolean CSPs

Vladimir Kolmogorov (IST Austria – Klosterneuburg, AT)

Joint work of Alexandr Kazda, Vladimir Kolmogorov, Michal Rolínek
Main reference Alexandr Kazda, Vladimir Kolmogorov, Michal Rolínek: "Even Delta-Matroids and the Complexity of Planar Boolean CSPs", CoRR, Vol. abs/1602.03124, 2016.
URL http://arxiv.org/abs/1602.03124

We study Boolean CSPs where each variable appears in exactly two constraints and all constraints come from some language Γ (we call it "EdgeCSP(Γ)"). If Γ contains the two constant unary relations then the only interesting case is when all constraints in Γ are "Δ-matroids" (otherwise EdgeCSP(Γ) and CSP(Γ) have the same complexities, as shown by Feder). I will present a polynomial-time algorithm for "Even Δ-matroids", as well as an extension to some non-even Δ-matroids. One consequence of this result is settling the complexity classification of planar Boolean CSPs started by Dvořák and Kupec.

3.12 The Equivalence of Two Dichotomy Conjectures for Infinite Domain CSPs

Michael Kompatscher (Charles University – Prague, CZ)

Joint work of Libor Barto, Michael Kompatscher, Miroslav Olsák, Trung Van Pham, Michael Pinsker
Main reference Libor Barto, Michael Kompatscher, Miroslav Olsák, Michael Pinsker, Van Trung Pham: "Equations in oligomorphic clones and the Constraint Satisfaction Problem for ω-categorical structures", CoRR, Vol. abs/1612.07551, 2016.
URL http://arxiv.org/abs/1612.07551

CSPs of reducts of finitely bounded homogeneous structures form a natural generalization of finite CSPs and also allow us to also apply the universal algebraic approach. As for finite structures, non-triviality of the equational structure of the corresponding polymorphism clone is conjectured to be a/the source of tractability. In my talk I would like to discuss to which extend it is enough to consider only non-trivial equations of height 1.

3.13 Finite pseudoloops

Marcin Kozik (Jagiellonian University – Kraków, PL)

The loop lemma states: if a finite digraph has no sources and no sinks, contains a closed walk with one more forward than backward edge and has a Taylor polymorphism then it contains a loop. This result is closely connected to algebraic investigations of CSPs with finite templates.

When investigating a structure of CSPs with omega-categorical templates the question becomes: take a graph with no sources and no sinks, and an oligomorphic subgroup of its automorphism group: if the quotient of the graph modulo the orbit equivalence contains a walk as in the loop lemma and the graph does not pp-interpret 3-clique with parameters does the quotient necessarily contain a loop? We prove this statement true in the finite case.

3.14 Integrality Gap For Minimum Multiway Cut

Yury Makarychev (TTIC – Chicago, US)

Joint work of Haris Angelidakis, Yury Makarychev, Pasin Manurangsi
Main reference Haris Angelidakis, Yury Makarychev, Pasin Manurangsi: "An Improved Integrality Gap for the Călinescu-Karloff-Rabani Relaxation for Multiway Cut", in Proc. of the Integer Programming and Combinatorial Optimization - 19th International Conference, IPCO 2017, Waterloo, ON, Canada, June 26-28, 2017, Proceedings, Lecture Notes in Computer Science, Vol. 10328, pp. 39–50, Springer, 2017.
URL http://dx.doi.org/10.1007/978-3-319-59250-3_4

We consider the Călinescu–Karloff–Rabani linear programming relaxation for Minimum Multiway Cut. We prove that its integrality gap is at least $\alpha_k = 6/(5 + 1/(k-1))$, where k is the number of terminals. Our result improves the previous best known gap lower bound of $8/(7 + 1/(k-1))$, which was established by Freund and Karloff in 2000. Our result implies that it is NP-hard to get an $\alpha_k - \epsilon$ approximation for the problem (for every $\epsilon > 0$), if the Unique Games Conjecture is true.

3.15 Classification transfer for qualitative reasoning problems

Barnaby Martin (Durham University, GB)

Joint work of Manuel Bodirsky, Peter Jonsson, Barnaby Martin, Antoine Mottet
Main reference Barnaby Martin, Peter Jonsson, Manuel Bodirsky, Antoine Mottet: "Classification transfer for qualitative reasoning problems", CoRR, Vol. abs/1805.02038, 2018.
URL http://arxiv.org/abs/1805.02038

We study formalisms for temporal and spatial reasoning in the modern, algebraic and model-theoretic, context of infinite-domain Constraint Satisfaction Problems (CSPs). We show how questions on the complexity of their subclasses can be solved using existing results via the powerful use of primitive positive (pp) interpretations and pp-homotopy.

We demonstrate the methodology by giving a full complexity classification of all constraint languages that are first-order definable in Allen's Interval Algebra and contain the basic relations (s) and (f). In the case of the Rectangle Algebra we answer in the affirmative the old open question as to whether ORD-Horn is a maximally tractable subset among the (disjunctive, binary) relations. We then generalise our results for the Rectangle Algebra to the r-dimensional Block Algebra.

3.16 Small Set Expansion in The Johnson Graph

Dana Moshkovitz (University of Texas – Austin, US)

Joint work of Subhash Khot, Dor Minzer, Dana Moshkovitz, Shmuel Safra
Main reference S. Khot, D. Minzer, D. Moshkovitz, and M. Safra: "Small Set Expansion in The Johnson Graph". *Electronic Colloquium on Computational Complexity (ECCC)*, 25:78, 2018.
URL https://eccc.weizmann.ac.il/report/2018/078

We study expansion properties of the (generalized) Johnson Graph. For natural numbers $t < l < k$, the nodes of the graph are sets of size l in a universe of size k. Two sets are connected if their intersection is of size t. The Johnson graph arises often in combinatorics and theoretical computer science: it represents a "slice" of the noisy hypercube, and it is the graph that underlies direct product tests as well as a candidate hard unique game.

We prove that any small set of vertices in the graph either has near perfect edge expansion or is not pseudorandom. Here "not pseudorandom" means that the set becomes denser when conditioning on containing a small set of elements. In other words, we show that slices of the noisy hypercube – while not small set expanders like the noisy hypercube – only have non-expanding small sets of a certain simple structure.

This paper is related to a recent line of work establishing the 2-to-2 Theorem in PCP. The result was motivated, in part, by [1] which hypothesized and made partial progress on similar result for the Grassmann graph. In turn, our result for the Johnson graphs served as a crucial step towards the full result for the Grassmann graphs completed subsequently in [2].

References

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2 S. Khot, D. Minzer, and M. Safra. On Independent Sets, 2-to-2 Games and Grassmann Graphs. *Electronic Colloquium on Computational Complexity (ECCC)*, 23:124, 2016.

3.17 A universal-algebraic proof of the dichotomy for MMSNP

Antoine Mottet (TU Dresden, DE)

Joint work of Manuel Bodirsky, Florent Madelaine, Antoine Mottet
Main reference Manuel Bodirsky, Florent R. Madelaine, Antoine Mottet: "A universal-algebraic proof of the complexity dichotomy for Monotone Monadic SNP", in Proc. of the 33rd Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2018, Oxford, UK, July 09-12, 2018, pp. 105–114, ACM, 2018.
URL http://dx.doi.org/10.1145/3209108.3209156

The logic MMSNP is a restricted fragment of existential second-order logic that was discovered by Feder and Vardi, who showed that every MMSNP sentence is computationally equivalent to a finite-domain CSP; the involved probabilistic reductions were derandomized by Kun using explicit constructions of expander structures. I will present a new proof based on the universal-algebraic method and recent Ramsey-theoretic results by Hubička and Nešetřil. This new proof allows us to verify the infinite-domain dichotomy conjecture by Bodirsky and Pinsker for infinite-domain CSPs in MMSNP.

3.18 Algebraic view on PCSP and hardness of coloring a d-colorable graph with $2d - 1$ colors

Jakub Opršal (TU Dresden, DE)

Joint work of Jakub Bulín, Andrei Krokhin, Jakub Opršal

The talk focuses on fixed template promise constraint satisfaction problem (PCSP). A template of PCSP is a pair of finite relational structures **A** and **B** with a homomorphism between them. The goal is to decide, given third structure **C**, whether **C** maps homomorphically to **A**, or it does not even map to **B**. Recently, polymorphisms have been successfully used by Austrin, Håstad, Guruswami and Brakiensiek to provide several results on the complexity of PCSPs.

We show that the complexity depends only on minor (height 1) identities satisfied by the polymorphisms. Further, we use this result to give a reduction from promise 3-uniform hypergraph coloring to promise graph coloring providing that deciding whether a graph is d-colorable or not even $(2d - 1)$-colorable is NP-hard for any $d > 2$.

3.19 Algebraic structure of polymorphism clones of infinite CSP templates

Michael Pinsker (TU Wien, AT)

Joint work of Pierre Gillibert, Julius Jonušas, Michael Pinsker

We survey recent results on the algebraic structure of polymorphism clones of infinite CSP templates. Satisfaction of identities in such clones is reflected by the satisfaction of so-called loop conditions, which are in some sense fixed point properties of actions of the clone on graphs. We compare the relative strength of such conditions.

3.20 A Dichotomy Theorem for the Inverse Satisfiability Problem

Biman Roy (Linköping University, SE)

Joint work of Victor Lagerqvist, Biman Roy
Main reference Victor Lagerkvist, Biman Roy: "A Dichotomy Theorem for the Inverse Satisfiability Problem", in Proc. of the 37th IARCS Annual Conference on Foundations of Software Technology and Theoretical Computer Science, FSTTCS 2017, December 11-15, 2017, Kanpur, India, LIPIcs, Vol. 93, pp. 39:39–39:14, Schloss Dagstuhl – Leibniz-Zentrum fuer Informatik, 2017.
URL http://dx.doi.org/10.4230/LIPIcs.FSTTCS.2017.39

The inverse satisfiability problem over a set of Boolean relations Γ (Inv-SAT(Γ)) is the computational decision problem of, given a relation R, deciding whether there exists a SAT(Γ) instance with R as its set of models. This problem is co-NP-complete in general and a dichotomy theorem for finite Γ containing the constant Boolean relations was obtained by Kavvadias and Sideri. In this paper we remove the latter condition and prove that Inv-SAT(Γ) is always either tractable or co-NP-complete for all finite sets of relations Γ, thus solving a problem open since 1998. Very few of the techniques used by Kavvadias and Sideri are applicable and we have to turn to more recently developed algebraic approaches based on partial polymorphisms. We also consider the case when Γ is infinite, where the situation differs markedly from the case of SAT. More precisely, we show that there exists infinite Γ such that Inv-SAT(Γ) is tractable even though there exists finite $\Delta \subset \Gamma$ such that Inv-SAT(Δ) is co-NP-complete.

References

1 P. Jonsson, V. Lagerkvist, and G. Nordh. Constructing NP-intermediate problems by blowing holes with parameters of various properties. *Theoretical Computer Science*, 581:67–82, 2015.
2 P. Jonsson, V. Lagerkvist, G. Nordh, and B. Zanuttini. Strong partial clones and the time complexity of SAT problems. *Journal of Computer and System Sciences*, 84:52–78, 2017.
3 D. Kavvadias and M. Sideri. The inverse satisfiability problem. *SIAM Journal on Computing*, 28:152–163, 1998.

3.21 Refuting Random CSPs: Survey + NAE-3SAT

Tselil Schramm (University of California – Berkeley, US)

Joint work of Yash Deshpande, Andrea Montanari, Ryan O'Donnell, Tselil Schramm, Subhabrata Sen
Main reference Yash Deshpande, Andrea Montanari, Ryan O'Donnell, Tselil Schramm, Subhabrata Sen: "The threshold for SDP-refutation of random regular NAE-3SAT", CoRR, Vol. abs/1804.05230, 2018.
URL http://arxiv.org/abs/1804.05230

In this talk I'll give a survey of recent advances in our understanding of the power of Semidefinite Programs for refuting random CSPs, then describe a recent result in which we establish the exact basic SDP threshold for refuting random regular NAE-3SAT.

3.22 From Weak to Strong LP Gaps for all CSPs

Madhur Tulsiani (TTIC – Chicago, US)

Joint work of Mrinalkanti Ghosh, Madhur Tulsiani
Main reference M. Ghosh and M. Tulsiani: "From Weak to Strong Linear Programming Gaps for All Constraint Satisfaction Problems" *Computational Complexity Conference 2017*, Vol. 14, pages 1–33, Theory of Computing.
URL http://dx.doi.org/10.4086/toc.2018.v014a010

We study the approximability of constraint satisfaction problems (CSPs) by linear programming (LP) relaxations. We show that for every CSP, the approximation obtained by a basic LP relaxation, is no weaker than the approximation obtained using relaxations given by $\Omega(\log n/(\log\log n))$ levels of the of the Sherali-Adams hierarchy on instances of size n.

It was proved by Chan et al. [1] that any polynomial size LP extended formulation is no stronger than relaxations obtained by a super-constant levels of the Sherali-Adams hierarchy. Combining this with our result also implies that any polynomial size LP extended formulation is no stronger than the basic LP, which can be thought of as the base level of the Sherali-Adams hierarchy. This essentially gives a dichotomy result for approximation of CSPs by polynomial size LP extended formulations.

Using our techniques, we also simplify and strengthen the result by Khot et al. [2] on (strong) approximation resistance for LPs. They provided a necessary and sufficient condition under which $\Omega(\log\log n)$ levels of the Sherali-Adams hierarchy cannot achieve an approximation better than a random assignment. We simplify their proof and strengthen the bound to $\Omega(\log n/(\log\log n))$ levels.

References

1 S. O. Chan, J. R. Lee, P. Raghavendra, and D. Steurer. Approximate Constraint Satisfaction Requires Large LP Relaxations. *FOCS 2013*, pages 350–359.
2 S. Khot, M. Tulsiani, and P. Worah. A characterization of strong approximation resistance. *STOC 2014*, pages 634–643.

3.23 Submodular cost functions and valued constraint satisfaction problems over infinite domains

Caterina Viola (TU Dresden, DE)

Joint work of Manuel Bodirsky, Marcello Mamino, Caterina Viola
Main reference Manuel Bodirsky, Marcello Mamino, Caterina Viola: "Submodular Functions and Valued Constraint Satisfaction Problems over Infinite Domains", CoRR, Vol. abs/1804.01710, 2018.
URL http://arxiv.org/abs/1804.01710

Valued constraint satisfaction problems (VCSPs) are a generalisation of constraint satisfaction problems (CSPs) that capture combinatorial optimisation problems. Recently, the computational complexity of all VCSPs for finite sets of cost functions over finite domains has been classified completely. Many natural optimisation problems, however, cannot be formulated as VCSPs over a finite domain (e.g., the linear programming problem), but can be modelled as a VCSP over the domain of rational numbers, $\mathbb{Q}$.

I will focus a special class of VCSPs over $\mathbb{Q}$, namely the class of VCSPs for finite piecewise linear homogenerous (PLH) valued languages. A PLH valued language is a set of cost functions that admit a first-order definition over $\mathbb{Q}$, using the order relation, the element 1, and the scalar multiplication by rationals. For these languages the VCSP is solvable in polynomial time when the cost functions are additionally submodular. Moreover, the submodularity is a condition of maximal tractability for PLH valued languages.

3.24 An algorithm for Constraint Satisfaction Problem on finite set

Dmitriy Zhuk (Moscow State University, RU)

Main reference Dmitriy Zhuk: "A Proof of CSP Dichotomy Conjecture", in Proc. of the 58th IEEE Annual Symposium on Foundations of Computer Science, FOCS 2017, Berkeley, CA, USA, October 15-17, 2017, pp. 331–342, IEEE Computer Society, 2017.
URL http://dx.doi.org/10.1109/FOCS.2017.38
Main reference Dmitriy Zhuk: "The Proof of CSP Dichotomy Conjecture", CoRR, Vol. abs/1704.01914, 2017.
URL http://arxiv.org/abs/1704.01914

In 2017 two proofs of CSP Dichotomy Conjecture and correspondingly two polynomial algorithms for constraint languages admitting a weak near-unanimity polymorphism were developed. In the talk one of the algorithms will be discussed in detail. I will try to minimize the amount of algebraic notions and focus on the new methods and ideas.

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