

# Parameterized Approximation: Algorithms and Hardness

Karthik C. S.<sup>\*1</sup>, Parinya Chalermsook<sup>\*2</sup>, Joachim Spoerhase<sup>\*3</sup>,  
Meirav Zehavi<sup>\*4</sup>, and Martin Herold<sup>†5</sup>

1 Rutgers University – New Brunswick, US. [karthik.0112358@gmail.com](mailto:karthik.0112358@gmail.com)

2 Aalto University, FI. [parinya.chalermsook@aalto.fi](mailto:parinya.chalermsook@aalto.fi)

3 University of Sheffield, GB. [j.spoerhase@sheffield.ac.uk](mailto:j.spoerhase@sheffield.ac.uk)

4 Ben Gurion University – Beer Sheva, IL. [zehavimeirav@gmail.com](mailto:zehavimeirav@gmail.com)

5 MPI für Informatik – Saarbrücken, DE. [mherold@mpi-inf.mpg.de](mailto:mherold@mpi-inf.mpg.de)

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## Abstract

Parameterization and approximation are two established approaches of coping with intractability in combinatorial optimization. In this Dagstuhl Seminar, we studied parameterized approximation as a relatively new algorithmic paradigm that combines these two popular research areas. In particular, we analyzed the solution quality (approximation ratio) as well as the running time of an algorithm in terms of a parameter that captures the “complexity” of a problem instance.

While the field has grown and yielded some promising results, our understanding of the area is rather ad-hoc compared to our knowledge in approximation or parameterized algorithms alone. In this seminar, we brought together researchers from both communities in order to bridge this gap by accommodating the exchange and unification of scientific knowledge.

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## 1 Executive Summary

*Joachim Spoerhase (University of Sheffield, GB)*

*Karthik C. S. (Rutgers University – New Brunswick, US)*

*Parinya Chalermsook (Aalto University, FI)*

*Meirav Zehavi (Ben Gurion University – Beer Sheva, IL)*

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Parameterization and approximation are two popular approaches of coping with intractability in combinatorial optimization. They have gained substantial maturity over the last decades, leading to tight bounds for many fundamental computational problems and beautiful algorithmic techniques that show surprising interplay between algorithms and various areas of mathematics. In this Dagstuhl Seminar, we studied parameterized approximation as a relatively new algorithmic paradigm that combines these two popular research areas. In particular, we analyzed the solution quality (approximation ratio) as well as the running

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\* Editor / Organizer

† Editorial Assistant / Collector



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time of an algorithm in terms of a parameter that captures the “complexity” of a problem instance.

While the field has grown and yielded some promising results, our understanding of the area is rather ad-hoc compared to our knowledge in approximation or parameterized algorithms alone. In this seminar, we brought together researchers from both communities in order to bridge this gap by accommodating the exchange and unification of scientific knowledge.

Our first goal was to foster a transfer of techniques between the classic fields of approximation and parameterization. We discussed how recent developments in one research area can be transferred to another. Towards this, we organized five invited one-hour tutorials (one on each morning) that were delivered by leading experts from the fields and which we believe helped exchange of state-of-the-art techniques. The tutorial topics covered parameterized or polynomial time approximation algorithms for graph edit and network design problems, for clustering problems, matroid constrained maximization problems, as well as the hardness of parameterized approximation of problems arising in error correcting codes.

Our second goal was to systematically identify important research directions and concrete open problems in research areas that are relevant to parameterized approximation. Towards this, we organized a panel discussion on the first seminar day led by experts from parameterized algorithms, approximation algorithms, hardness of approximation, but also from neighboring areas such as fine-grained complexity and coding theory. A vibrant discussion ensued between the moderator, panelists, but also the other participants. Moreover, we organized two open discussion sessions, which were open to any participant to bring up a topic they would like to discuss with the other participants: This could be, for example, concrete open problems, more general research directions, or highlights. Besides several concrete open problems suggested by participants, there were two contributions to the sessions that gave overviews over a coherent collection of open questions on hardness of parameterized approximation under Gap-ETH and beyond, as well as on the parameterized (in-)approximability of clustering problems.

Our third goal was to bolster the creation of new collaborations between researchers in the two communities by encouraging the participants to actively discuss the suggested directions for open problems. It was therefore a particular priority for us to provide sufficient time for collaboration. Towards this, we reserved time slots for six collaboration sessions and one session for progress reports.

We thank Martin Herold for collecting the abstracts from the participants and for his assistance with creating and editing this report.

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### 3 Overview of Talks

#### 3.1 Hardness of Approximation in P via Short Cycle Removal: Cycle Detection, Distance Oracles, and Beyond

*Amir Aboud (Weizmann Institute – Rehovot, IL)*

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**Joint work of** Amir Aboud, Karl Bringmann, Seri Khoury, Or Zamir

**Main reference** Amir Aboud, Karl Bringmann, Seri Khoury and Or Zamir: Hardness of Approximation in P via Short Cycle Removal: Cycle Detection, Distance Oracles, and Beyond. STOC 2022.

**URL** <https://doi.org/10.1145/3519935.3520066>

This talk will overview a new technique for gap amplification called “short cycle removal” and its applications for hardness of approximation for polynomial time problems. In particular, we will present lower bounds on the approximation factor of distance oracles that preprocess a graph in almost-linear time and answer distance queries in almost-constant time. Based on joint works with Karl Bringmann, Nick Fischer, Seri Khoury, and Or Zamir.

#### 3.2 Baby PIH: Parameterized Inapproximability of Min CSP

*Venkatesan Guruswami (University of California – Berkeley, US)*

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The Parameterized Inapproximability Hypothesis (PIH) is the analog of the PCP theorem in the world of parameterized complexity. It asserts that no FPT algorithm can distinguish a satisfiable 2CSP instance from one which is only  $(1 - \varepsilon)$ -satisfiable (where the parameter is the number of variables) for some constant  $0 < \varepsilon < 1$ .

We consider a minimization version of CSPs (Min-CSP), where one may assign  $r$  values to each variable, and the goal is to ensure that every constraint is satisfied by some choice among the  $r \times r$  pairs of values assigned to its variables (call such a CSP instance  $r$ -list-satisfiable). We prove the following strong parameterized inapproximability for Min CSP: For every  $r \geq 1$ , it is W[1]-hard to tell if a 2CSP instance is satisfiable or is not even  $r$ -list-satisfiable. We refer to this statement as “Baby PIH”, following the recently proved Baby PCP Theorem. Our proof adapts the combinatorial arguments underlying the Baby PCP theorem, overcoming some significant obstacles that arise in the parameterized setting.

An extension of our result to an average-version of Baby PIH would prove the inapproximability of parameterized  $k$ -ExactCover, a notorious open problem.

### 3.3 A $(3/2 + \epsilon)$ -Approximation for Multiple TSP with a Variable Number of Depots

Matthias Kaul (TU Hamburg, DE)

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**Joint work of** Max Deppert, Matthias Kaul, Matthias Mnich

**Main reference** Max Deppert, Matthias Kaul, Matthias Mnich: “A  $(3/2 + \epsilon)$ -Approximation for Multiple TSP with a Variable Number of Depots”, in Proc. of the 31st Annual European Symposium on Algorithms, ESA 2023, September 4-6, 2023, Amsterdam, The Netherlands, LIPIcs, Vol. 274, pp. 39:1–39:15, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2023.

**URL** <https://doi.org/10.4230/LIPICS.ESA.2023.39>

One of the most studied extensions of the famous Traveling Salesperson Problem (TSP) is the Multiple TSP: a set of  $m \geq 1$  salespersons collectively traverses a set of  $n$  cities by  $m$  non-trivial tours, to minimize the total length of their tours. This problem can also be considered to be a variant of Uncapacitated Vehicle Routing, where the objective is to minimize the sum of all tour lengths. When all  $m$  tours start from and end at a single common depot  $v_0$ , then the metric Multiple TSP can be approximated equally well as the standard metric TSP, as shown by Frieze (1983)[1]. The metric Multiple TSP becomes significantly harder to approximate when there is a set  $D$  of  $d \geq 1$  depots that form the starting and end points of the  $m$  tours. For this case, only a  $(2 - 1/d)$ -approximation in polynomial time is known, as well as a  $3/2$ -approximation for constant  $d$  which requires a prohibitive run time of  $n^{\Theta(d)}$  (Xu and Rodrigues, INFORMS J. Comput., 2015)[3]. A recent work of Traub, Vygen and Zenklusen (STOC 2020)[2] gives another approximation algorithm for metric Multiple TSP with run time  $n^{\Theta(d)}$ , which reduces the problem to approximating metric TSP. In this paper we overcome the  $n^{\Theta(d)}$  time barrier: we give the first efficient approximation algorithm for Multiple TSP with a variable number  $d$  of depots that yields a better-than-2 approximation. Our algorithm runs in time  $(1/\epsilon)^{O(d \log d)} \cdot n^{O(1)}$ , and produces a  $(3/2 + \epsilon)$ -approximation with constant probability. For the graphic case, we obtain a deterministic  $3/2$ -approximation in time  $2^d \cdot n^{O(1)}$ .

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- 3 Zhou Xu and Brian Rodrigues. A  $3/2$ -approximation algorithm for the multiple TSP with a fixed number of depots. *INFORMS J. Comput.*, 27(4):636–645, 2015.

### 3.4 Parameterized Approximability of $F$ -Deletion Problems

Euiwoong Lee (University of Michigan – Ann Arbor, US)

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For a fixed class  $F$  of graphs, the  $F$ -Deletion problem, given a graph  $G$ , asks to remove the minimum number of vertices so that the resulting graph belongs to the class  $F$ . The study of various  $F$ -Deletion problems has led to interesting connections between approximation

algorithms and parameterized algorithms, ultimately leading to parameterized approximation algorithms. In this talk, we survey known results for subgraph, induced subgraph, and minor deletion problems.

### 3.5 The Cut Covering Lemma

*Jason Li (University of California – Berkeley, US)*

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**Main reference** Stefan Kratsch, Magnus Wahlström: “Compression via Matroids: A Randomized Polynomial Kernel for Odd Cycle Transversal”, *ACM Trans. Algorithms*, Vol. 10(4), pp. 20:1–20:15, 2014.

**URL** <https://doi.org/10.1145/2635810>

We present the cut covering lemma, a classic result by Kratsch and Wahlström (2012)[1] in FPT literature with many applications to kernelization. Loosely speaking, the lemma states that on any graph with  $k$  terminal vertices, there is a smaller graph on  $O(k^3)$  vertices that preserves the complete cut structure of the graph. We discuss connections to the field of fast graph algorithms, in particular the implications of an approximate version of the cut covering lemma with  $O(k)$  vertices.

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### 3.6 Parameterized Approximation Schemes for Clustering with General Norm Objectives

*Dániel Marx (CISPA – Saarbrücken, DE)*

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**Joint work of** Fateme Abbasi, Sandip Banerjee, Jaroslav Byrka, Parinya Chalermsook, Ameet Gadekar, Kamyar Khodamoradi, Roohani Sharma, Joachim Spoerhase

**Main reference** Fateme Abbasi, Sandip Banerjee, Jaroslav Byrka, Parinya Chalermsook, Ameet Gadekar, Kamyar Khodamoradi, Dániel Marx, Roohani Sharma, Joachim Spoerhase: “Parameterized Approximation Schemes for Clustering with General Norm Objectives”, *CoRR*, Vol. abs/2304.03146, 2023.

**URL** <https://doi.org/10.48550/ARXIV.2304.03146>

We consider the well-studied algorithmic regime of designing a  $(1+\epsilon)$ -approximation algorithm for a  $k$ -clustering problem that runs in time  $f(k, \epsilon)\text{poly}(n)$ . Our main contribution is a clean and simple EPAS that settles more than ten clustering problems (across multiple well-studied objectives as well as metric spaces) and unifies well-known EPASes. Our algorithm gives EPASes for a large variety of clustering objectives (for example,  $k$ -means,  $k$ -center,  $k$ -median, priority  $k$ -center,  $\ell$ -centrum, ordered  $k$ -median, socially fair  $k$ -median aka robust  $k$ -median, or more generally monotone norm  $k$ -clustering) and metric spaces (for example, continuous high-dimensional Euclidean spaces, metrics of bounded doubling dimension, bounded treewidth metrics, and planar metrics). Key to our approach is a new concept that we call bounded  $\epsilon$ -scatter dimension – an intrinsic complexity measure of a metric space that is a relaxation of the standard notion of bounded doubling dimension.

### 3.7 FPT Approximation Schemes for Matroid-constrained Problems

Hadas Shachnai (Technion – Haifa, IL)

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**Joint work of** Ilan Doron Arad, Ariel Kulik, Hadas Shachnai

**Main reference** Ilan Doron Arad, Ariel Kulik, Hadas Shachnai: “Budgeted Matroid Maximization: a Parameterized Viewpoint”, CoRR, Vol. abs/2307.04173, 2023.


**URL** <https://doi.org/10.48550/ARXIV.2307.04173>

We study budgeted variants of well known maximization problems with multiple matroid constraints. Given an  $\ell$ -matchoid  $\mathcal{M}$  on a ground set  $E$ , a profit function  $p$  and a cost function  $c$  on  $E$ , and a budget  $B$ , the goal is to find in the  $\ell$ -matchoid a feasible set  $S$  of maximum profit  $p(S)$  subject to the budget constraint, i.e.,  $c(S) \leq B$ . The *budgeted  $\ell$ -matchoid* (BM) problem includes as special cases budgeted  $\ell$ -dimensional matching and budgeted  $\ell$ -matroid intersection. A strong motivation for studying BM from parameterized viewpoint comes from the APX-hardness of unbudgeted  $\ell$ -dimensional matching (i.e.,  $B = \infty$ ) already for  $\ell = 3$ . Nevertheless, while there are known FPT algorithms for the unbudgeted variants of the above problems, the *budgeted* variants are studied here for the first time through the lens of parameterized complexity.

We show that BM parametrized by solution size is  $W[1]$ -hard, already with a degenerate single matroid constraint. Thus, an exact parameterized algorithm is unlikely to exist, motivating the study of *FPT-approximation schemes* (FPAS). Our main result is an FPAS for BM (implying an FPAS for  $\ell$ -dimensional matching and budgeted  $\ell$ -matroid intersection), relying on the notion of representative set – a small cardinality subset of elements which preserves the optimum up to a small factor. We also give a lower bound on the minimum possible size of a representative set which can be computed in polynomial time.

### 3.8 Approximating Weighted Connectivity Augmentation below Factor 2

Vera Traub (Universität Bonn, DE)

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**Joint work of** Vera Traub, Rico Zenklusen

**Main reference** Vera Traub, Rico Zenklusen: “A  $(1.5 + \epsilon)$ -Approximation Algorithm for Weighted Connectivity Augmentation”, in Proc. of the 55th Annual ACM Symposium on Theory of Computing, STOC 2023, Orlando, FL, USA, June 20-23, 2023, pp. 1820–1833, ACM, 2023.

**URL** <https://doi.org/10.1145/3564246.3585122>

The Weighted Connectivity Augmentation Problem (WCAP) asks to increase the edge-connectivity of a graph in the cheapest possible way by adding edges from a given set. It is one of the most elementary network design problems for which no better-than-2 approximation algorithm has been known, whereas 2-approximations can be easily obtained through a variety of well-known techniques.

In this talk, I will discuss an approach showing that approximation factors below 2 are achievable for WCAP, ultimately leading to a  $(1.5 + \epsilon)$ -approximation algorithm. Our approach is based on a highly structured directed simplification of WCAP with planar optimal solutions. We show how one can successively improve solutions of this directed simplification by moving to mixed-solutions, consisting of both directed and undirected edges. These insights can be leveraged in local search and relative greedy strategies, inspired by recent advances on the Weighted Tree Augmentation Problem, to obtain a  $(1.5 + \epsilon)$ -approximation algorithm for WCAP.

## 4 Panel discussions

### 4.1 Panel Discussion

*Frances A. Rosamond (University of Bergen, NO), Amir Abboud (Weizmann Institute – Rehovot, IL), Michael R. Fellows (University of Bergen, NO), Venkatesan Guruswami (University of California – Berkeley, US), Bingkai Lin (Nanjing University, CN), and Saket Saurabh (The Institute of Mathematical Sciences – Chennai, IN)*

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© Frances A. Rosamond, Amir Abboud, Michael R. Fellows, Venkatesan Guruswami, Bingkai Lin, and Saket Saurabh

The aim of the panel was fostering exchange between the various research communities (eg, parameterized and approximation algorithms as well as hardness of approximation) by identifying (i) general key challenges (meta research questions in a conceptual or technical sense) that are important to advance and promote the field, (ii) suitable taxonomy that allows to classify the possible algorithmic results, (iii) advance systematic understanding (in contrast to ad-hoc results), (iv) and concrete open research questions.

In particular the panelists discussed (under the moderation of Frances Rosamond) the following questions.

- In which direction would you like to see the field grow?
- What is a distinctive technical challenge in parameterized approximation? By “distinctive challenge” we mean an (exciting) technical challenge that (may) require an approach that goes beyond combining techniques that were previously already used in parameterization or approximation separately. We believe that it is crucial for the field to gain momentum and attract researchers that there are such unique challenges.
- What is your favorite result in the field?

After the panelists discussed the three questions, the floor was opened to all the participants of the Dagstuhl Seminar to share their ideas.

## 5 Open problems

### 5.1 TSP with line-neighborhoods in $\mathbb{R}^3$

*Antonios Antoniadis (University of Twente, NL)*

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The traveling salesperson problem (TSP) with line neighborhoods given a set of  $n$  lines in  $\mathbb{R}^3$  one seeks a shortest tour (closed curve)  $\mathcal{C}$  that visits each line. A line  $L$  is visited by  $\mathcal{C}$  if and only if  $\mathcal{C} \cap L$  is non-empty. In [1] an  $O(\log^3 n)$ -approximation algorithm was presented that is based on a reduction from TSP with line neighborhoods to Group Steiner Tree (at the loss of a constant factor in the approximation ratio). The setting where the lines are parallel is equivalent to solving a classical Euclidean instance in  $\mathbb{R}^2$  and thus the problem is NP-hard. It was also shown among other results in [2], that the problem is actually APX-hard and admits an  $O(\log^2 n)$ -approximation algorithm, albeit with a running time of  $n^{O(\log \log n)}$ . Given the large gap between the respective upper and lower bounds, the most important open question with respect to the problem is whether or not it admits a constant-approximation algorithm.



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## 5.2 Parameterized approximability of clustering problems

Vincent Cohen-Addad (Google Paris, FR)


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Given a set of points (clients)  $C$  and a set of facilities  $F$  in a metric space  $(C \cup F, \text{dist})$ , the classic  $k$ -Median problem asks to find a subset  $S \subseteq F$  of size  $k$  (the centers), such that the total distance  $d(S) := \sum_{p \in C} \min_{c \in S} \text{dist}(p, c)$  of points in  $C$  to the closest facility is minimized.

1. Find a  $(1 + 2/e)$ -approximation that runs in time  $2^{O(k)} \text{poly}(n)$  (where  $(C \cup F, \text{dist})$  is an arbitrary metric space).
2. Consider the continuous setting of  $k$ -median with  $L_\infty$  metrics (i.e.:  $C$  is a subset of  $\mathbb{R}^d$  and  $F$  is  $\mathbb{R}^d$ ). Find a 2-Approximation that runs in  $2^{O(k)} \text{poly}(nd)$  time.

## 5.3 Parameterized Approximation for the Santa Claus Problem

Fabrizio Grandoni (SUPSI – Lugano, CH)

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In the Santa Claus problem we are given a collection of presents and a collection of children. Each present  $i$  has a value  $v_{ij} \geq 0$  for child  $j$ . The happiness of a child  $j$  is the sum of the values of the presents that (s)he receives. Our goal is to assign the presents so as to maximize the minimum happiness of any child. Santa Claus turns out to be an extremely challenging problem in terms of approximation algorithms. The best known lower bound on the (polynomial-time) approximation ratio is 2, while the best known upper bound on the same ratio is polynomial in the number  $n$  of items.

Given the difficulty of this problem, it makes sense to consider FPT approximation algorithms. One natural parameter is the number  $k$  of children. I did ask if a constant approximation (or better) is possible in FPT time. Andreas Wiese made me notice that a parameterized approximation scheme (PAS) for this problem is implied by [1] (described in the form of an EPTAS for constant  $k$ )

An interesting open question that remains is to define alternative parameters that make sense in practice, and design FPT approximation algorithms with respect to them.

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## 5.4 FPT Inapproximability Results Beyond Gap-ETH

Pasin Manurangsi (Google Thailand – Bangkok, TH)

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While a number of parameterized problems are known to be hard to approximate under the Gap Exponential Time Hypothesis (Gap-ETH), certain results are still out of reach of the current techniques even under Gap-ETH. We discuss a few such questions, including:

1. Strong FPT Inapproximability of  $k$ -Set Cover. While it is known that approximating  $k$ -Set Cover to within  $g(k)$ -factor is W[1]-hard for any function  $g$ [6]. The situation is less clear when we allow dependency on  $n$  (the number of elements in the universe) in the approximation factor; the greedy algorithm yields  $O(\log n)$ -approximation while the best known ETH-hardness result is only  $\tilde{\Omega}(\log^{1/k} n)$ [4]. Open Question: Can we improve this hardness to, say,  $\Omega(\log^{0.99} n)$  under Gap-ETH?
2. Total FPT Inapproximability of **Exact**  $k$ -Set Cover. Exact  $k$ -Set Cover is a special case of the Set Cover problem where we are promised that there exists a set cover of size  $k$  such that the subsets in the solution are all disjoint. (Note here that the output solution only needs to cover the space but needs not be disjoint.) This version of Set Cover is often useful in subsequent reductions, e.g. to Coding-theoretic and Lattice problems. While the hardness of Exact Set Cover is achieved for free in the NP-hardness of approximation reduction for Set Cover of Feige[3], the parameterized hardness reductions do **not** give the hardness of such a version. This is due to the so-called “projection” property in Label Cover, which does **not** hold in the parameterized version of Label Cover used in the FPT hardness regime; in fact, it is not hard to see that requiring such projection properties will lead to at most  $2^k$  gap. To the best of our knowledge, the best FPT hardness of approximation of Exact Set Cover based on Gap-ETH is only  $k^{1/2-o(1)}$ , which follows from the result of Manurangsi and Dinur [2]. Open Question: Can we rule out all  $g(k)$ -approximation FPT algorithm for Exact  $k$ -Set Cover?
3. Total FPT Inapproximability of Densest  $k$ -Subgraph (with perfect completeness). The best known FPT hardness of approximation under Gap-ETH of Densest  $k$ -Subgraph has inapproximability factor of  $k^{o(1)}$  (where  $o(1)$  can be any function that converges to zero as  $k \rightarrow \infty$ )[1]. On the other hand, under a non-standard “Strongish Planted Clique Hypothesis”, this factor can be improved to  $o(k)$ [5]. Open Question: Can we prove the same hardness under Gap-ETH?

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## 5.5 $(2 - \epsilon)$ -approximation for the Capacitated Vehicle Routing Problem

Hang Zhou (*Ecole Polytechnique – Palaiseau, FR*)

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In the capacitated vehicle routing problem, we are given a metric space with a vertex called depot and a set of vertices called terminals. The goal is to find a minimum length collection of tours starting and ending at the depot such that each tour visits at most  $k$  terminals, and each terminal is visited by some tour. We consider this problem in the Euclidean plane. The best-to-date approximation ratio was  $2 + \epsilon$  using the iterated tour partitioning technique introduced by Haimovich and Rinnooy Kan. It is an open question whether there is a better-than-2 approximation.

## Participants

- Amir Abboud  
Weizmann Institute –  
Rehovot, IL
- Antonios Antoniadis  
University of Twente, NL
- Jarek Byrka  
University of Wroclaw, PL
- Karthik C. S.  
Rutgers University – New  
Brunswick, US
- Parinya Chalermsook  
Aalto University, FI
- Vincent Cohen-Addad  
Google – Paris, FR
- Klim Efremenko  
Ben Gurion University –  
Beer Sheva, IL
- Andreas Emil Feldmann  
University of Sheffield, GB
- Michael R. Fellows  
University of Bergen, NO
- Ameet Gadekar  
Aalto University, FI
- Fabrizio Grandoni  
SUPSI – Lugano, CH
- Carla Groenland  
Utrecht University, NL
- Venkatesan Guruswami  
University of California –  
Berkeley, US
- Martin Herold  
MPI für Informatik –  
Saarbrücken, DE
- Lawqueen Kanesh  
Indian Institute of Technology –  
Jodhpur, IN
- Matthias Kaul  
TU Hamburg, DE
- Madhumita Kundu  
University of Bergen, NO
- Euiwoong Lee  
University of Michigan –  
Ann Arbor, US
- Jason Li  
University of California –  
Berkeley, US
- Bingkai Lin  
Nanjing University, CN
- Pasin Manurangsi  
Google Thailand – Bangkok, TH
- Dániel Marx  
CISPA – Saarbrücken, DE
- Pranabendu Misra  
Chennai Mathematical  
Institute, IN
- Tobias Mömke  
Universität Augsburg, DE
- Danupon Nanongkai  
MPI für Informatik –  
Saarbrücken, DE
- Marcin Pilipczuk  
University of Warsaw, PL
- Nidhi Purohit  
University of Bergen, NO
- Rajiv Raman  
University Blaise Pascal –  
Aubiere, FR & IIIT Delhi –  
New Delhi, IN
- Frances A. Rosamond  
University of Bergen, NO
- Saket Saurabh  
The Institute of Mathematical  
Sciences – Chennai, IN
- Chris Schwiegelshohn  
Aarhus University, DK
- Hadas Shachnai  
Technion – Haifa, IL
- Roohani Sharma  
MPI für Informatik –  
Saarbrücken, DE
- Krzysztof Sornat  
SUPSI – Lugano, CH
- Joachim Spoerhase  
University of Sheffield, GB
- Vera Traub  
Universität Bonn, DE
- Daniel Vaz  
ENS – Paris, FR
- Andreas Wiese  
TU München, DE
- Meirav Zehavi  
Ben Gurion University –  
Beer Sheva, IL
- Hang Zhou  
Ecole Polytechnique –  
Palaiseau, FR

