Report from Dagstuhl Seminar 24212

Classical-Quantum Synergies in the Theory and Practice of Quantum Error Correction

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Abstract

The Dagstuhl Seminar 24212 "Classical-Quantum Synergies in the Theory and Practice of Quantum Error Correction" was held on May 20–23, 2024, and brought together 30 participants from 13 countries. The seminar served as an interaction forum for senior and talented junior researchers, crossing boundaries between classical and quantum coding theory, and related areas of quantum technology and engineering problems. The topics covered by the seminar ranged from models of quantum noise to the theory and practice of quantum codes, including fault-tolerant error correction and fault-tolerant quantum computation, quantum error correction for specific technology constraints or noise models, decoding aspects of topological and quantum LDPC codes, and quantum error correction for scalable modular quantum computing architectures. The two and a half day program of the seminar consisted of 14 invited talks, and five breakout sessions, aimed at fostering an exchange of knowledge and viewpoints on challenges faced by quantum error correction. This report briefly presents the background, the motivation, and the topics covered by the seminar, and provides an overview of the invited talks and of three of the breakout sessions that brought together a large number of participants.

Seminar May 20–23, 2024 – https://www.dagstuhl.de/24212
2012 ACM Subject Classification Mathematics of computing → Information theory
Keywords and phrases Fault-tolerant quantum computing, Quantum computing architectures, Quantum error correction, quantum information, Quantum LDPC codes
Digital Object Identifier 10.4230/DagRep.14.5.173

1 Executive Summary

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Background and Motivation: From Classical to Quantum Error Correction and Fault-Tolerance

A fundamental consequence of the mathematical theory of information laid down by Shannon, error correcting codes play a vital role in ensuring the integrity of data in systems exposed to noise or errors. Classical error correcting codes were crucial to the success of modern

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communications and data storage systems (from the Internet to mobile, satellite, and deep-space communications, and from disk to flash memory storage) and found applications in other areas, such as pattern recognition, group testing, cryptography, or fault-tolerant computing. Likewise, quantum error correcting codes are at the heart of all quantum information processing, from fault-tolerant quantum computing to reconciliation in quantum key distribution, quantum sensing, and reliable optical communications.

Computation in the presence of noise is a long-standing problem, going back to the 1950s and the celebrated works of von Neumann, Elias, Taylor, Kuznetsov, Winograd, Cowan, Dobrushin, Pippenger, and many others. The first attempt to apply general error correction techniques for the design of fault-tolerant computing systems is due to Elias (Computation in the presence of noise, 1962), and one of the first attempts to derive fundamental limits in fault-tolerant computing is due to Winograd and Cowan (Reliable computation in the presence of noise, 1963). These works focused on fault-tolerant classical (Boolean logic based) computation, prior to the advent of ultra-high reliability integrated circuits based on complementary metal-oxide-semiconductor (CMOS) technology, but they still inspire and resonate with current approaches to fault tolerance, *e.g.*, to support the ongoing miniaturization of the emerging data processing and storage devices (technology scaling). In parallel, the last years have seen significant advances in the field of quantum technologies, promising a disruptive impact in information and computing technologies. Basic requirements for quantum computation have been demonstrated in various technologies, including semiconductor or superconductor materials, photons, trapped ions, etc. Nonetheless, for unleashing the full computational power that quantum computers can bring, a critical task is to protect the quantum computation from the inherent quantum noise. The discovery of quantum error correcting codes in the mid-90s paved the way to noise resilient quantum computation, developed through the works of Calderbank, Shor, Steane, Sloane, Gottesman, Knill, Kitaev, Freedman, Meyer, Preskill, and many others. The integration of quantum error correction (QEC) into the quantum computation led to the development of the fault-tolerant quantum computing framework, aimed at countering the effects of noise on stored quantum information, faulty quantum preparation, faulty quantum gates, and faulty measurements. Such an integration of QEC and fault-tolerance techniques in quantum computing systems is key to the development of a universal large-scale quantum computer, achieving its expected exceptional potential.

While classical and quantum error correction may be regarded as different paradigms, involving different ways of thinking and to a certain extent different research communities, it turns out that they are actually closely related. One may mention here the formalism of quantum stabilizer codes, allowing notably to move from a continuous to a discrete model for quantum error correction, among which of particular interest is the Calderbank-Shor-Steane (CSS) construction of a quantum code from a pair of orthogonal classical binary codes. CSS codes can be alternatively described as chain complexes involving three spaces, where the boundary operators are defined (up to a choice of bases) by the two orthogonal classical codes. This homological point of view is essentially the one adopted by topological constructions, where quantum codes are produced based on cellular decompositions of surfaces (*e.g.*, torus), or higher dimensional manifolds. In parallel, the powerful machinery of abstract homological algebra proved to be very efficient in providing new constructions of quantum codes, among which of particular interest are codes with constant weight stabilizer generators, referred to as quantum low-density parity-check (qLDPC) codes. The class of qLDPC codes encompasses the above topological constructions, and is the only class of quantum codes known to contain families of codes with both constant non-zero rate and non-zero fault-tolerant error-correction threshold. It is also worth mentioning here the recent constructions of asymptotically good qLDPC codes (with constant rate and relative minimum distance), auguring for practical constructions with increased error correction capacity or reduced qubit overhead. However, unlike their classical counterparts, which are equipped with efficient message-passing decoding algorithms, qLDPC codes are difficult to decode. The decoding of a qLDPC code requires locating not a single most likely error, but the most likely equivalence class of mutually degenerate errors (degeneracy is an inherent characteristic of any qLDPC code), which tends to inhibit the convergence of message-passing algorithms designed for classical codes. Besides, it is also worth mentioning that the time budget available to perform a single error correction round varies with the quantum technology, but a first-order approximation is a period of hundreds of nanoseconds. Hardware implementations meeting such a time constraint will require massive parallel processing, which has to be enabled by both the structure of the quantum code and the decoding algorithm.

To tackle these challenges, this Dagstuhl Seminar aimed at promoting interactions among coding theorists, quantum physicists, mathematicians, and computer and hardware engineers, to discuss achievements, strategies, and remaining gaps in the integration of QEC and fault-tolerance techniques into practical quantum computers, towards a comprehensive and mutual understanding of theory and engineering practice.

Topics Covered by the Seminar

Classical and Quantum LDPC codes. The quest for low-complexity decoders of classical LDPC codes has resulted to the emergence of soft-decision iterative message passing decoders, *e.g.*, based on belief-propagation (BP) or min-sum (MS) algorithms. In the quantum case, decoding a CSS qLDPC code boils down to decoding the two constituent classical LDPC codes (*e.g.*, assuming separate decoding of X and Z errors, which does not preclude taking into account the possible correlations between the two types of error). In homological terms, the goal of the decoder is to find the most likely chain (error) – or more specifically, the most likely class of chains – corresponding to a given boundary (syndrome), where two chains are equivalent if their sum is in the trivial homology class. Maximum-likelihood decoders exist for the toric code (yet, their complexity is too high for practical applications), but they are out of reach for arbitrary topological or qLDPC codes. Developing new approaches to accurate and hardware friendly decoding of quantum codes is a crossroad of theory and practice, and of classical and quantum coding. Presumably, classical-quantum synergies can provide meaningful insights to the theory and practice of qLDPC codes. There are many examples where the theory and practice of qLDPC codes may benefit from classical-quantum synergies, such as devising optimized constructions for short qLDPC codes, improving the decoding performance through modified message-passing or smart post-processing techniques, using knowledge of quantum trapping sets to cope with the code degeneracy, devising machine learning based decoding solutions, conceiving efficient decoding algorithms to exploit soft information on measurement errors, or developing codes and decoding algorithms amenable to single-shot error correction.

Particular challenges discussed during the seminar were broadly related to novel constructions of qLDPC codes and expanding properties of the associated graph, novel decoding algorithms for topological and qLDPC codes, including message-passing based decoding, tensor network decoding, and machine-learning based decoding, applications of quantum error correction in various areas as quantum computing or quantum networks, and the design of entanglement-assisted quantum codes.

Fault-Tolerant Quantum Computation. Quantum memory with a topological or, more generally, qLDPC stabilizer code can be implemented with repeated syndrome measurements, where errors are detected by the difference between syndromes measured in consecutive rounds. It is also worth noticing that a QEC with a sufficiently short syndrome measurement cycle is needed throughout the operation of a quantum computer, and measurement circuits have to be designed with fault-tolerance in mind, *e.g.*, to prevent a single error to spread on multiple qubits. More generally, when non-trivial gates are executed on the logical subspace, detection events have to be chosen for each particular circuit. The gate error for the hardware in use, as well as the specific choice of the circuit and of the detection events determines the error model and the structure of the quantum error-correcting code that has to be decoded. Pauli error channels associated with specific gates on specific qubits are most commonly used for decoding. Actual error probabilities may also depend on the parameters chosen for each qubit (*e.g.*, working frequencies chosen for individual qubits in the case of superconducting qubits), as well as variability of the manufacturing. Other important error types include non-Pauli errors (decay, unitary errors, etc.), as well as leakage from the computational subspace. Furthermore, with some hardware, syndrome measurement may contain additional soft information about the measurement outcome. Taking such information into account may dramatically improve the decoding accuracy. While in theoretical analysis such details can often be ignored, in practice, for a quantum computer operating close to the threshold, a relatively small improvement in the decoding accuracy can reduce the required overhead by orders of magnitude, or even be required to attain fault-tolerance.

Particular challenges discussed during the seminar were broadly related to a variety of Pauli error channels, including those derived from Clifford circuits with gate error models customized for specific hardware, related unification of decoding protocols for qubit-based codes, decoding using soft syndrome information, coherent noise and quantum error correction, subsystem and Floquet codes, effective consideration of geometric and connectivity requirements, fault-tolerant quantum computation, and fault-tolerant design of algorithms and protocols.

From Noisy Intermediate Scale Devices to Large Scale Quantum Computing. While QEC is the only presently known gateway to reap the benefits of computational quantum algorithms, a robust, scalable, and fully functional QEC technique that allows performing fault-tolerant quantum computations has not been demonstrated experimentally yet. Arguably, QEC is the only technology still lacking to realize a vision of useful large-scale quantum computation. However, there are already a few demonstrations of the potential to protect quantum information on noisy intermediate scale quantum (NISQ) processors based on superconducting qubits, such as: i) the experimental implementation of distance-3 surface code on the Zuchongzhi 2.1 superconducting quantum processor showing that by executing several consecutive error correction cycles, the logical error can be significantly reduced after applying corrections (Realization of an Error-Correcting Surface Code with Superconducting Qubits); ii) the experimental demonstration that increasing the code distance leads to a better logical qubit performance using an expanded Sycamore device with 72 transmon qubits (Suppressing quantum errors by scaling a surface code logical qubit). NISQ technology may serve as a first step towards demonstrating a certain number of QEC protocols, suitable to the intermediate scale, but which in the long term may also have useful implications for large-scale quantum technologies. Yet, in a large-scale quantum computer, the QEC decoder design faces significant challenges, arising from the need to integrate various system constraints, such as accuracy, bandwidth, latency, power-consumption, or scalability. QEC decoders need to be powerful enough to accurately correct the quantum errors, fast enough to fight against the qubit decoherence, energy efficient to meet stringent power-consumption requirements, and highly scalable to meet the needs of fault-tolerance. Achieving all these constraints is extremely challenging, and might not be possible with existing solutions. Recent research has focused on the design of hardware architectures capable of efficiently accommodating QEC techniques, where considerations such as timing, latency, power, and wiring between the quantum chip and the QEC processor take a prominent place, as they are critical for creating a viable solution.

The main challenges discussed during the seminar ranged from low-qubit overhead fault-tolerant schemes and efficient implementation of small QEC on NISQ processors to scalable modular quantum computing architectures for quantum error correction and large scale fault tolerance, while also considering software implementation of quality decoders, decoding architectures that lend themselves to high-speed and low energy consumption, and recent progress on the hardware implementation and prototyping of QEC decoders.

Organization of the Seminar

The seminar brought together 30 participants, both senior and talented young researchers, from 13 countries (Denmark, Finland, France, Germany, Great Britain, India, Ireland, Netherlands, Russia, Switzerland, Spain, Taiwan, and the United States), with research expertise in relevant areas, *e.g.*, classical and quantum coding theory, hardware architectures and designs of error correcting codes, quantum information processing and software, fault-tolerant quantum computation and fault-tolerant design of algorithms and protocols, quantum technologies, and quantum computer architecture design.

The primary objective of the seminar was to foster an exchange of ideas on challenges faced by quantum error correction, evolving through presentations as well as discussions aimed at realizing the potential of a large community bring diverse viewpoints to the table. In order to facilitate this, the two and a half day program of the seminar comprised a series of 14 invited talks, organized in seven plenary sessions, as well as five time slots for breakout sessions, giving more time for discussions and the organisation of ad-hoc working groups (bringing together a large part of the participants). The main part of this report includes the abstracts of all talks and three working groups.

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3 Overview of Talks

3.1 Pairwise Transversality of CSS Codes with Applications to Quantum Networks

Alexei Ashikhmin (Bell Labs – Murray Hill, US)

Joint work of Mahdi Bayanifar, Alexei Ashikhmin, Dawei Jiao, Olav Tirkkonen
Main reference Mahdi Bayanifar, Alexei Ashikhmin, Dawei Jiao, and Olav Tirkkonen: "On Transversality Across Two Distinct Quantum Error Correction Codes For Quantum Repeaters", CoRR, Vol. abs/2406.00350, 2024
URL https://arxiv.org/abs/2406.00350

In this work, we study the transversality of pairs of CSS codes and their application in quantum networks employing second-generation quantum repeaters. Motivated by the observation that different stations within a quantum link may encounter different types of errors, we propose utilizing CSS codes tailored to the error models specific to each station. Additionally, we suggest using $[[n, k]]$ codes with $k > 1$ due to their higher efficiency compared to codes with $k = 1$. Quantum networks require that quantum codes used at neighboring stations possess pair-wise transversality. In this work, we establish sufficient and necessary conditions for a pair of CSS codes to be non-local CNOT-transversal. We demonstrate that, unlike the stringent constraints imposed by single CSS code CNOT-transversality, our case requires less restrictive constraints. Further, we establish sufficient and necessary conditions for a code pair to be CZ-transversal. Finally, we demonstrate that our proposed approach yields significant performance gain compared to the conventional approach of employing the same CSS code across all network stations.

3.2 Coherent Errors and Compass Codes

Kenneth R. Brown (Duke University – Durham, US)

Joint work of Balint Pato, Judd Will Staples Jr., Kenneth R. Brown
Main reference Balint Pato, Judd Will Staples Jr., and Kenneth R. Brown: "Logical coherence in 2D compass codes", CoRR, Vol. abs/2405.09287, 2024.
URL https://arxiv.org/abs/2405.09287

Quantum error correction often considers Pauli noise channels where the errors can be described as the random application of Pauli operators. Here we consider a coherent noise channel where all the qubits experience a common rotation around the Z axis of an unknown error. This error model can be studied numerically for the surface code using a transformation from qubits to Majorana fermions. We extend this transformation to compass codes, gauge fixings of the Bacon-Shor code, and develop a family of compass codes where we can analytically determine a threshold rotation angle. We discuss the possibility for extending this result to improve the analytic bound on the coherent error threshold for the surface code.

3.3 Entanglement-Assisted Quantum Error Correction with Qudits

Shayan Srinivasa Garani (Indian Institute of Science – Bangalore, IN)

Joint work of Priya Nadkarni, Shayan Srinivasa Garani
Main reference Priya J. Nadkarni, Shayan Srinivasa Garani: "Non-binary entanglement-assisted stabilizer codes", Quantum Inf. Process., Vol. 20(8), pp. 1–68, 2021.
URL https://doi.org/10.1007/S11128-021-03174-1

Non-binary quantum states also called qudits inherently have a rich quantum information content to be harnessed for applications within quantum communication and computing. Further, qudit systems allow more-complex quantum computational architectures by simplifying certain computational tasks and circuits. The use of pre-shared qudit entangled states within a quantum transceiver system can increase the error correction ability of the system. In this talk, we discuss the ideas behind entanglement-assisted quantum error correction over qudits along with coding-theoretic bounds and encoding circuits.

3.4 Fault-Tolerant Quantum Input-Output

Ashutosh Goswami (University of Copenhagen, DK)

Joint work of Ashutosh Goswami, Matthias Christandl, Omar Fawzi

Standard models of computation and communication concern the design of algorithms and protocols that make use of black boxes, i.e. fixed input-output relations, such as oracles or communication channels. The design of such algorithms and protocols focuses typically on aspects of efficiency, both in terms of complexity and capacity. Whereas this focus is justified in the classical realm, the noise in quantum encoding and decoding devices may put the entire model in doubt; at the least, it will require the quantum designer to come up with noise-robust procedures. In the context of quantum Shannon theory, such procedures have recently been proposed (Christandl and Müller-Hermes, IEEE Trans. Inf. Th. 70, 282 (2024)). Working in Kitaev's framework for fault-tolerant computation, we present general criteria and tools for the fault-tolerant design of algorithms and protocols, which make use of fixed quantum black boxes. Applications of our work can be found in the design of quantum networks or the solution of quantum learning tasks.

3.5 Lowering Connectivity Requirements For Bivariate Bicycle Codes Using Middle-Out Circuits

Mackenzie Hooper Shaw (TU Delft, NL), Barbara Terhal (TU Delft, NL)

Main reference Mackenzie H. Shaw, Barbara M. Terhal: "Lowering Connectivity Requirements For Bivariate Bicycle Codes Using Morphing Circuits", arxiv preprint, CoRR, Vol. abs/2407.16336, 2004
URL https://doi.org/10.48550/arXiv.2407.16336

Recent work by Bravyi et al. [1] proposed a set of small LDPC codes and corresponding syndrome extraction circuits that achieve a similar logical error rate to the surface code under circuit-level noise, but with a much denser encoding of logical qubits. The codes are part of a family of LDPC codes called Abelian two-block group algebra (2BGA) codes with the additional property that the stabilisers have weight six. In this work, we propose a new set of small Abelian 2BGA codes and syndrome extraction circuits with identical $[[n, k, d]]$ parameters to those of Ref. [1] but requiring a connectivity graph with degree five instead of six. Intriguingly, each of our new codes has a depth-7 syndrome extraction circuit – the same depth as those in Ref. [1] – despite the fact that our new codes have weight-9 stabilisers. Our new codes are derived from the codes in Ref. [1] using the "middle-out circuit" construction from Refs. [2, 3]: half-way through the syndrome extraction circuit, the joint code encoded between the data and ancilla qubits corresponds precisely to one of the codes in Ref. [1]. One can therefore perform logical gates by implementing half of the syndrome extraction circuit, followed by the procedures already detailed in Ref. [1]. Finally, we present preliminary numerical results comparing our new codes with those in Ref. [1] under circuit-level noise decoded using BP-OSD.

References

1 S. Bravyi et al., "High-threshold and low-overhead fault-tolerant quantum memory", Nature 627, 778-782 (2024).
2 C. Gidney and C. Jones, "New circuits and an open source decoder for the color code", arXiv:2312.08813.
3 M. McEwen, D. Bacon, and C. Gidney, "Relaxing hardware requirements for surface code circuits using time-dynamics", Quantum 7, 1172 (2023).

3.6 Constant-Overhead Fault-Tolerant Quantum Computation with Reconfigurable Atom Arrays

Liang Jiang (University of Chicago, US)

Joint work of Qian Xu, J. Pablo Bonilla Ataides, Christopher A. Pattison, Nithin Raveendran, Dolev Bluvstein, Jonathan Wurtz, Bane Vasic, Mikhail D. Lukin, Liang Jiang, Hengyun Zhou
Main reference Qian Xu, J. Pablo Bonilla Ataides, Christopher A. Pattison, Nithin Raveendran, Dolev Bluvstein, Jonathan Wurtz, Bane Vasic, Mikhail D. Lukin, Liang Jiang, and Hengyun Zhou: "Constant-Overhead Fault-Tolerant Quantum Computation with Reconfigurable Atom Arrays", Nat. Phys. 20, 1084–1090 (2024)
URL https://doi.org/10.1038/s41567-024-02479-z

Quantum low-density parity-check (qLDPC) codes can achieve high encoding rates and good code distance scaling, providing a promising route to low-overhead fault-tolerant quantum computing. However, the long-range connectivity required to implement such codes makes their physical realization challenging. Here, we propose a hardware-efficient scheme to perform fault-tolerant quantum computation with high-rate qLDPC codes on reconfigurable atom arrays, directly compatible with recently demonstrated experimental capabilities. Our approach utilizes the product structure inherent in many qLDPC codes to implement the non-local syndrome extraction circuit via atom rearrangement, resulting in effectively constant overhead in practically relevant regimes. We prove the fault tolerance of these protocols, perform circuit-level simulations of memory and logical operations with these codes, and find that our qLDPC-based architecture starts to outperform the surface code with as few as several hundred physical qubits at a realistic physical error rate of 10^{-3}. We further find that less than 3000 physical qubits are sufficient to obtain over an order of magnitude qubit savings compared to the surface code, and quantum algorithms involving thousands of logical qubits can be performed using less than 10^5 physical qubits. Our work paves the way for explorations of low-overhead quantum computing with qLDPC codes at a practical scale, based on current experimental technologies.

3.7 How to Fault-Tolerantly Realize any Quantum Circuit with Local Operations

Robert König (TU München, DE)

Joint work of Shin Ho Choe, Robert Köenig
Main reference Shin Ho Choe, Robert Köenig: "How to fault-tolerantly realize any quantum circuit with local operations", CoRR, Vol. abs/2402.13863, 2024.
URL https://arxiv.org/abs/2402.13863

We show how to realize a general quantum circuit involving gates between arbitrary pairs of qubits by means of geometrically local quantum operations and efficient classical computation. We prove that circuit-level local stochastic noise modeling an imperfect implementation of our derived schemes is equivalent to local stochastic noise in the original circuit. Our constructions incur a constant-factor increase in the quantum circuit depth and a polynomial overhead in the number of qubits: To execute an arbitrary quantum circuit on n qubits, we give a 3D quantum fault-tolerance architecture involving $O(n^{3/2} \log^3 n)$ qubits, and a quasi-2D architecture using $O(n^2 \log^3 n)$ qubits. Applied to recent fault-tolerance constructions, this gives a fault-tolerance threshold theorem for universal quantum computations with local operations, a polynomial qubit overhead and a quasi-polylogarithmic depth overhead. More generally, our transformation dispenses with the need for considering the locality of operations when designing schemes for fault-tolerant quantum information processing.

3.8 Correcting Phenomenological Quantum Noise via Belief Propagation and its Extension to Circuit-Level Noise

Ching-Yi Lai (National Yang Ming Chiao Tung University – Hsinchu, TW)

Joint work of Kao-Yueh Kuo, Ching-Yi Lai
Main reference Kao-Yueh Kuo, Ching-Yi Lai: "Correcting phenomenological quantum noise via belief propagation", CoRR, Vol. abs/2310.12682, 2023.
URL https://doi.org/10.48550/ARXIV.2310.12682

Quantum stabilizer codes often face the challenge of syndrome errors due to error-prone measurements. To address this issue, multiple rounds of syndrome extraction are typically employed to obtain reliable error syndromes. In this paper, we consider phenomenological decoding problems, where data qubit errors may occur between two syndrome extractions, and each syndrome measurement can be faulty. To handle these diverse error sources, we define a generalized check matrix over mixed quaternary and binary alphabets to characterize their error syndromes. This generalized check matrix leads to the creation of a Tanner graph comprising quaternary and binary variable nodes, which facilitates the development of belief propagation (BP) decoding algorithms to tackle phenomenological errors. Importantly, our BP decoders are applicable to general sparse quantum codes. Finally we extend this method to handle circuit-level noises by constructing a parity-check matrix over mixed alphabets for the syndrome extraction circuit.

3.9 Maximally Extendable Sheaf Codes

Pavel Panteleev (Moscow State University, RU)

Joint work of Pavel Panteleev, Gleb Kalachev
Main reference Pavel Panteleev, Gleb Kalachev: "Maximally Extendable Sheaf Codes", CoRR, Vol. abs/2403.03651, 2024
URL https://arxiv.org/abs/2403.03651

Sheaf codes are linear codes with a fixed hierarchical collection of local codes, viewed as a sheaf of vector spaces on a finite topological space. Many existing codes, such as tensor product codes, Sipser-Spielman codes, and their more recent high-dimensional analogs, can be naturally represented as sheaf codes defined on simplicial and cubical complexes. We introduce a new property called maximal extendibility, which ensures that within a class of codes on the same space, we encounter as few obstructions as possible when extending local sections globally. It is possible to show that in every class of sheaf codes defined on the same space and parameterized by parity-check matrices with polynomial entries, there always exists a maximally extendable sheaf code. As it turns out, maximally extendable tensor product codes are good coboundary expanders, which allows one to generalize the recent constructions of good quantum low-density parity-check codes to more than two dimensions, and potentially could be used to attack the qLTC conjecture.

3.10 Tensor Network Decoding Beyond 2D

Joseph M. Renes (ETH Zürich, CH)

Joint work of Christophe Piveteau, Christopher T. Chubb, Joseph M. Renes
Main reference Christophe Piveteau, Christopher T. Chubb, and Joseph M. Renes: "Tensor Network Decoding Beyond 2D", preprint, CoRR, Vol. abs/2310.10722, 2023
URL https://arxiv.org/abs/2310.10722

Decoding algorithms based on approximate tensor network contraction have proven tremendously successful in decoding 2D local quantum codes such as surface/toric codes and color codes, effectively achieving optimal decoding accuracy. In this work, we introduce several techniques to generalize tensor network decoding to higher dimensions so that it can be applied to 3D codes as well as 2D codes with noisy syndrome measurements (phenomenological noise or circuit-level noise). The three-dimensional case is significantly more challenging than 2D, as the involved approximate tensor contraction is dramatically less well-behaved than its 2D counterpart. Nonetheless, we numerically demonstrate that the decoding accuracy of our approach outperforms state-of-the-art decoders on the 3D surface code, both in the point and loop sectors, as well as for depolarizing noise. Our techniques could prove useful in near-term experimental demonstrations of quantum error correction, when decoding is to be performed offline and accuracy is of utmost importance. To this end, we show how tensor network decoding can be applied to circuit-level noise and demonstrate that it outperforms the matching decoder on the rotated surface code.

3.11 A new family of Floquet codes: Dynamical Logical Qubits in the Bacon-Shor Code

Eleanor Rieffel (NASA – Moffett Field, US)

Joint work of M. Sohaib Alam, Eleanor Rieffel
Main reference M. Sohaib Alam, Eleanor Rieffel: "Dynamical Logical Qubits in the Bacon-Shor Code", CoRR, Vol. abs/2403.03291, 2024
URL https://arxiv.org/abs/2403.03291

I discuss work, joint with Sohaib Alam, on dynamical logical qubits in the Bacon-Shor code. We choose measurement schedules on a $d \times d$ square lattice that at each round is a subset of the Bacon-Shor code checks. These measurement schedule results in a Floquet code with several dynamical logical qubits. In this talk, I briefly review Bacon-Shor subsystem codes, and then discuss the new family of Floquet codes. This work is part of a larger program trying to understand when one can define Floquet codes, when it is useful to do so, and subtleties with regard to defining their distance. The talk concludes with the statement of some specific open problems.

3.12 Color Codes with Twists: Construction and Universal-Gate-Set Implementation

Pradeep Sarvepalli (Indian Institute of Techology Madras, IN)

Joint work of Manoj G. Gowda, Pradeep Kiran Sarvepalli
Main reference Manoj G. Gowda, Pradeep Kiran Sarvepalli: "Color codes with twists: Construction and universal-gate-set implementation", Phys. Rev. A, Vol. 104, p. 012603, American Physical Society, 2021.
URL https://doi.org/10.1103/PhysRevA.104.012603

Twists are defects in a lattice that can be used to perform encoded computations. Three basic types of twists can be introduced in color codes: twists that permute color, charge of anyons, and domino twists that permute the charge label of an anyon with a color label. In this talk, we look at a subset of these twists from a coding theoretic viewpoint. Specifically, we present a systematic construction of charge permuting and color permuting twists in color codes. We show that by braiding alone, Clifford gates can be realized in color codes with charge permuting twists. We also present the implementation of a non-Clifford gate by state injection, thus completing the realization of a universal gate set. We finally discuss implementing single-qubit Clifford gates by a Pauli frame update and CNOT gate by braiding holes around twists in color codes with color permuting twists.

3.13 On Some Quantum Internet Information Rates

Emina Soljanin (Rutgers University – Piscataway, US)

Joint work of Dunbar Birnie, Christopher Cheng, Emina Soljanin
Main reference Dunbar Birnie IV, Christopher Cheng, Emina Soljanin: "Information Rates With Non Ideal Photon Detectors in Time-Entanglement Based QKD", IEEE Trans. Commun., Vol. 71(4), pp. 2246–2259, 2023.
URL https://doi.org/10.1109/TCOMM.2023.3244244

This talk discusses information rates in two quantum internet building blocks concerning quantum (conference) key distribution (QKD). We first focus on QKD based on time-entangled photon pairs. These systems extract key bits from photon arrival times and thus promise to deliver more than one bit per photon instead of polarization-entanglement QKD, where each entangled photon pair contributes at most one bit to the secret key. However, realistic photon detectors exhibit time jitter and require non-zero time to recover upon registering a photon arrival. We model and evaluate the effect of these impairments on information rates generated based on photon arrival times and ask whether time-entanglement-based QKD can live up to its promise. We next ask whether quantum network multicast can make conference key agreements more efficient. Since there is no quantum information without physical representation (e.g., by photons), the problem of quantum multicast initially seems nothing more than the multi-commodity flow problem of shipping a collection of different commodities through a shared network. However, we show that besides the apparent similarity to the multi-commodity flow problems, quantum networks, to a certain extent, behave as classical information networks. In particular, we show that lossless compression of multicast quantum states is possible and significantly reduces the link capacity requirements of the multicast.

3.14 A Study of the Decoding Radius of Fast Renormalisation Decoders for the Kitaev Code

Gilles Zémor (University of Bordeaux, FR)

Joint work of Wouter Rozendaal, Gilles Zémor
Main reference Wouter Rozendaal, Gilles Zémor: "Analysis of the Error-Correcting Radius of a Renormalisation Decoder for Kitaev's Toric Code", CoRR, Vol. abs/2309.12165, 2023.
URL https://doi.org/10.48550/ARXIV.2309.12165

The renormalisation decoders for Kitaev's toric code introduced by Duclos-Cianci and Poulin exhibit one of the best trade-offs between accuracy and efficiency, with a time complexity in $n \log n$. One question that was left open is how they handle worst-case or adversarial errors, *i.e.*, what is the order of magnitude of the smallest weight of an error pattern that will be wrongly decoded. We initiate such a study involving a simple hard-decision and deterministic version of the Duclos-Cianci and Poulin decoder.

4 Working groups

4.1 Scalable Modular Quantum Computing Architectures

Carmen G. Almudéver (Technical University of Valencia, ES)

To build a universal fault-tolerant quantum computer and achieve the full computational power quantum computing will provide, it is necessary to scale quantum machines up. This requires not only to increase the number of qubits but also to incorporate quantum error correction (QEC) protocols and fault-tolerant (FT) techniques. Modular quantum computing architectures have emerged as one of the most promising approaches for scaling up quantum computers. The main idea is to combine multiple quantum processing units (QPUs) or quantum modules and connect them via classical communication links and ultimately quantum communication technologies.

In this breakout session, led by Dr. Carmen G. Almudéver, after providing an overview on modular quantum computing architectures (e.g. IBM roadmap) we focused on the need for introducing QEC to achieve FT computation. The discussion revolved around what kind of QEC code is more suitable for modular quantum computing architectures and how many logical qubits should be allocated per QPU or even if it will make sense to spread a logical qubit among different QPUs. It was also mentioned that given the structure of these architectures it might be possible that they combine different qubit implementation technologies such as superconducting qubits, neutral atoms or photonic processors. In addition, in these systems different kinds of connections with different losses can be found and therefore some works proposed to reencoding in a different code for transmission or even for computations. In other words, a specific quantum error correction code is used for memory and a different one for transmission and computation. Communication between modules might be the main bottleneck of modular architectures and it will be crucial to optimize logical gates based on the inter-core communication. Furthermore, using the error information provided by the system can help to further improve the error decoding accuracy and the performance of the system. It was pointed out that different schemes of modularity need to be considered depending on the qubit and communication technology. Another important aspect is to properly model the errors in the chip (i.e. spatially correlated errors) when increasing the qubit counts for comparing monolithic (single-chip) with multi-core architectures.

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4.2 Quantum Resource Estimation

Alexandru Paler (Aalto University, FI)

In this breakout session, the discussion was led by Dr. Alexandru Paler and focused on the estimation of the resources needed for executing a quantum circuit that has been encoded based on surface code. More precisely, he presented a resource estimator framework that estimates the physical resources needed to execute a quantum algorithm on a modular superconducting architecture. It was shown how the requirements of a surface code-based circuit can de plotted as a space-time volume, which needs to be minimized. It is therefore key to develop a scalable optimization method for optimizing the space-volume graph. It was pointed out that this space-time volume picture does not include the magic sate distillation process. In this session, the need of improving the logical error rate by developing more accurate and faster (scalable) decoders than MWPM was also discussed.

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4.3 Photonic Quantum Computing Architectures

Eleanor Rieffel (NASA – Moffett Field, US)

This breakout session, led by Dr. Eleanor Rieffel, focused on the implementation of QEC codes on photonic quantum computing architectures. First, the basics of this qubit platform were introduced. In these quantum processors it is difficult to make photons to interact with each other and therefore photonic systems use measurement-based quantum computation in which high-entangled states are created. They have long-range photonic connectivity and make use of fusions for computation. In this kind of model gates are non-deterministic. It was also discussed what the error rates are for this technology. Furthermore, it was mentioned that the main issue of this technology is the interconnection with the optical fiber. Some of the papers below were discussed. We mostly focused on how to implement surface code in photonic processors, in which lattice-surgery can be used, and what the cost of encoding and performing quantum gates is.

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